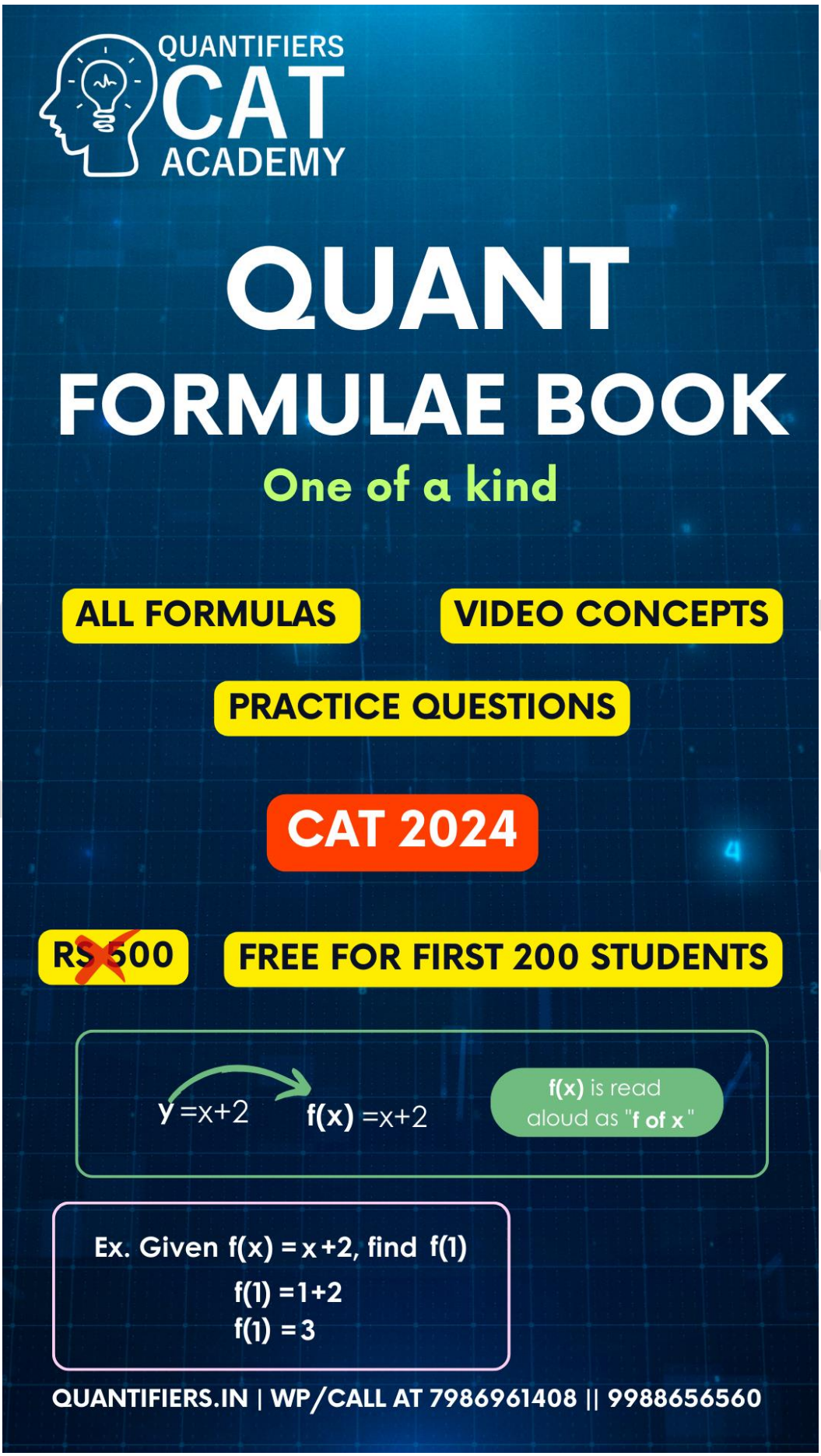
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y =x+2
f(x) =x+2
f(x) is read aloud as "f of x"
Ex. Given f(x) = x +2, find f(1)
f(1) =1+2
f(1) = 3
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# Quantifiers CAT Academy

**Started in 2019**

**Produced 28, 99+ %ilers out of the total ~275 students taught**

**Managed by the three musketeers Rohit Kanwar, Sahil Arora & Jasneet Dua who are the founders as well as the Expert Teaching facility with over 20+ years of experience**

**Helped more than 1 lakh aspirants achieve their dreams through their teachings.**

**Quantifiers is best known for their pedagogy, their emphatic attitude toward every single student, and for the passion with which every single element is driven**

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# Hi Comrade

# I'm Speed – your guide!

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***"I will be guiding you on how to use this book properly, This is a formula book, but this book has many important resources"***

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QUANTIFIERS CAT ACADEMY

# How to use this Formula Book?


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**https://youtu.be/8Bkl7prrRQc**

**What all is present in this book?**
**How to uitlize this book in maximum capacity?**

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*Managed by the three musketeers Rohit Kanwar, Sahil Arora & Jasneet Dua, Quantifiers is one of the most well managed CAT preparation player in India, it has helped more than 1 lakh aspirants achieve their dreams through their teachings.*

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QUANTIFIERS CAT ACADEMY

# Topic- Averages


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https://youtu.be/f7RH9Jjd4rg

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# Arithmetic

## Average

Average means Arithmetic mean of the items and it is = $\frac{\text{Sum of Items}}{\text{Number of Items}}$

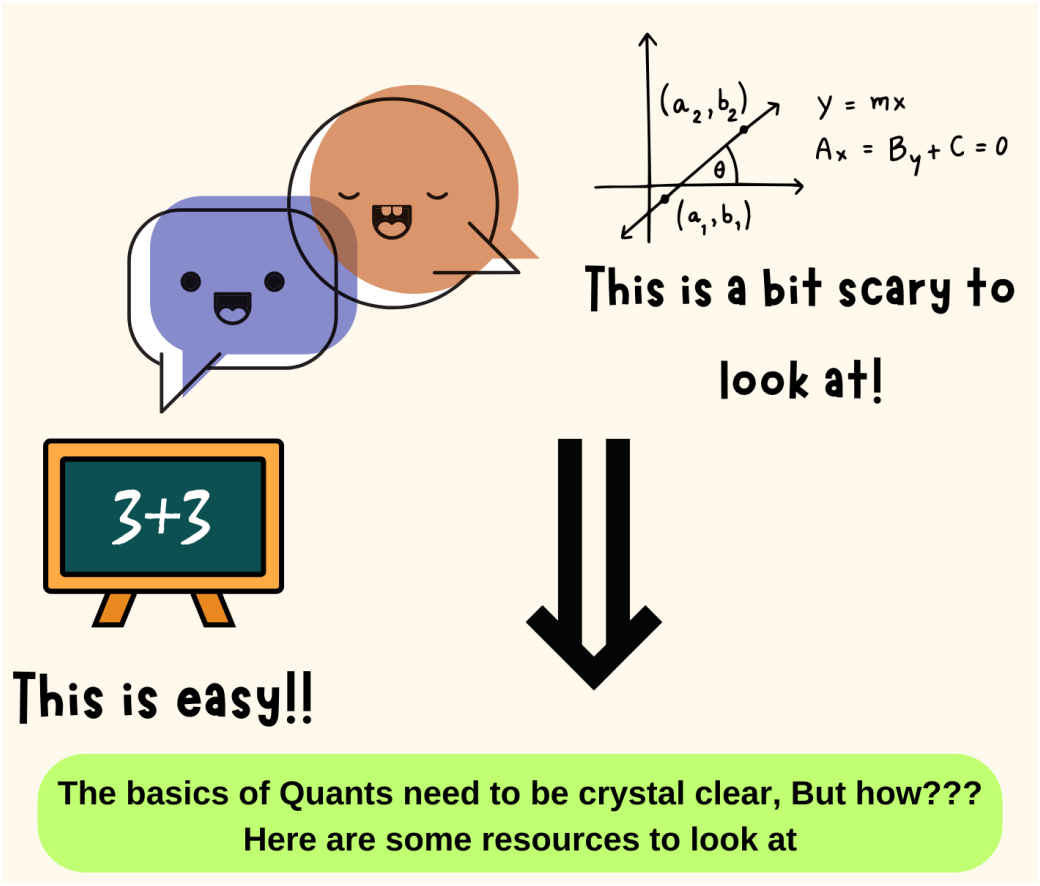
When the difference between the terms is same and the total number of terms are odd (say n), then the middle term = (n + 1)/2th term is the average of all the terms.

When the difference between the terms is same and the total number of terms are even (say n), then n/2 and n/2 + 1 are the two middle terms and the average of all the terms is equal to the average of these two middle terms.

***Average speed:*** If a man covers some journey from A to B at $u$ km/hr. and returns back to A at B uniform speed of $v$ km/hr., then the average speed during the whole journey is $\frac{2uv}{u+v}$ km/hr

1. If $x$ is added in all the items, then average increases by $x$.
2. If $x$ is subtracted from all the items, then average decreases by $x$.
3. If every item is multiplied by $x$, then average also gets multiplied by $x$.
4. If every item is divided by $x$, then average also gets divided by $x$.
5. Weighted average = $\frac{a_1x_1+a_2x_2+....a_nx_n}{a_1+a_2+.....+a_n}$

| | | |
|---|---|---|
| 1 | Onion is sold for 5 consecutive months at the rate of Rs 10, 20, 25, 25, and 50 per kg, respectively. A family spends a fixed amount of money on onion for each of the first three months, and then spends half that amount on onion for each of the next two months. The average expense for onion, in rupees per kg, for the family over these 5 months is closest to (CAT 2021)<br><br>26<br>16<br>18<br>20 | Click here for video solution |
| 2 | In a football tournament, a player has played a certain number of matches and 10 more matches are to be played. If he scores a total of one goal over the next 10 matches, his overall average will be 0.15 goals per match. On the other hand, if he scores a total of two goals over the next 10 matches, his overall average will be 0.2 goals per match. The number of matches he has played is: (CAT 2021) | Click here for video solution |
| 3 | The arithmetic mean of scores of 25 students in an examination is 50. Five of these students top the examination with the same score. If the scores of the other students are distinct integers with the lowest being 30, then the maximum possible score of the toppers is: (CAT 2021) | Click here for video solution |
| 4 | Dick is thrice as old as Tom and Harry is twice as old as Dick. If Dick's age is 1 year less than the average age of all three, then Harry's age, in years, is: (CAT – 2020) | Click here for video solution |
| 5 | A batsman played n + 2 innings and got out on all occasions. His average score in these n + 2 innings was 29 runs and he scored 38 and 15 runs in the last two innings. The batsman scored less than 38 runs in each of the first n innings. In these n innings, his average score was 30 runs and lowest score was x runs. The smallest possible value of x is (CAT 2020)<br><br>1. 1<br>2. 2<br>3. 3<br>4. 4 | Click here for video solution |
| 6 | There are three categories of jobs P.R.T., T.G.T. and P.G.T. The average salary of the teachers who got the job in P.R.T and T.G.T. categories is 26 lakhs per annum. The average salary of the teachers who got the job in T.G.T. and P.G.T. categories is 44 lakhs per annum and the average salary of those teachers who got the job of P.R.T and P.G.T categories is 34 lakhs per annum. The most appropriate range of average salary (in lakhs per annum) of all the three categories (if it is known that each teacher gets only one category of job i.e. P.R.T. or T.G.T. or P.G.T.): | Click here for video solution |
| 7 | Aman and eight of his friends took a test of 100 marks. Each of them got a different integer score and the average of their scores was 86. The score of Aman was 90 and it was more than that of exactly three of his friends. What could have been the maximum possible absolute difference between the scores of two of his friends? | Click here for video solution |




Complete QA Course to cover basics

Complete CAT Quant Course:
https://www.youtube.com/playlist?list=PL1glN7tXDkWgkElsdG7za3cxcomVc9Y5e

Advance level QA Questions
QA Quiz Playlist:
https://www.youtube.com/playlist?list=PL1glN7tXDkWhRYY_0EuzQlCG44BpQZa3g

Some XAT level questions
XAT Series:
https://www.youtube.com/playlist?list=PL1glN7tXDkWj7E3p09aN3ZEpwrb3EPI8d

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QUANTIFIERS CAT ACADEMY

# Topic- Percentage


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**https://youtu.be/haCC5y19DwA**

**How to apply the formula and basics concepts of the topic is covered in this video.**

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## Percentage

The following fractions to percentage conversion chart is helpful while solving the questions of percentage.

| Fraction | %age | Fraction | %age | Fraction | %age | Fraction | %age |
|---|---|---|---|---|---|---|---|
| 1 | 100% | 1/6 | 16.67% | 1/11 | 9.09% | 1/16 | 6.25% |
| 1/2 | 50% | 1/7 | 14.28% | 1/12 | 8.33% | 1/17 | 5.88% |
| 1/3 | 33.33% | 1/8 | 12.5% | 1/13 | 7.69% | 1/18 | 5.56% |
| 1/4 | 25% | 1/9 | 11.11% | 1/14 | 7.14% | 1/19 | 5.26% |
| 1/5 | 20% | 1/10 | 10% | 1/15 | 6.67% | 1/20 | 5% |

**To Increase a Number by x %:**

1. If a number is increased by 10 %, then it becomes 1.1 times of itself.
2. If a number is increased by 20 %, then it becomes 1.2 times of itself.
3. If a number is increased by 30 %, then it becomes 1.3 times of itself.
4. If a number is increased by 40 %, then it becomes 1.4 times of itself.

**To Decrease a Number by x %:**

1. If a number is decreased by 10 %, then it becomes 0.90 times of itself.
2. If a number is decreased by 20 %, then it becomes 0.80 times of itself.
3. If a number is decreased by 30 %, then it becomes 0.70 times of itself.
4. If a number is decreased by 40 %, then it becomes 0.60 times of itself.

**Salary/Weight/Income More**:

If *A*'s income is *R* % more than *B*, then *B* is income is less than that of *A* by $100 \times \frac{R}{100+R}$%. Given below are some of the important results in that context

**Salary/Weight/Income Less**:

If A's income is R% less than *B*, then *B*'s income is more than that of A by $100 \times \frac{R}{100-R}$ %. Given below are some important results in that context.

1. If *A* is $16\frac{2}{3}$ % less than *B*, then *B* is 20 % more than *A*.
2. If *A* is 20 % less than *B*, then *B* is 25 % more than *A*.
3. If *A* is 25 % less than *B*, then *B* is $33\frac{1}{3}$ % more than *A*.
4. If the price is increased then consumption should be decreased by $\frac{100 \times R}{(100+R)}$If the price is decreased, then consumption should be increased by $\frac{100 \times R}{(100+R)}$

**Increase and Decrease by the same % age**.

If a number is increased by *R* %, then this number is decreased by *R* %, then in total there would be a decrease of $\frac{R^2}{100}$ %

**Increase and Decrease by different% age**.

If a number is decreased by *X* %, then this is again increased by *Y* %. Then the total increase in the no. will be $X + Y + \frac{XY}{100}$.

| | | |
|---|---|---|
| 1 | 1000 people voted on a resolution and 10% of the votes were invalid. After some discussion the 1000 people voted again and this time there were 20% invalid votes. The opponents increased by 50% while the motion was now rejected by a majority which is 300% more than that by which it was formerly passed. How many people voted against the resolution after the discussion? | Click here for video solution |

| | | |
|---|---|---|
| 2 | Identical chocolate pieces are sold in boxes of two sizes, small and large. The large box is sold for twice the price of the small box. If the selling price per gram of chocolate in the large box is 12% less than that in the small box, then the percentage by which the weight of chocolate in the large box exceeds that in the small box is nearest to (CAT 2021)<br><br>1. 127<br>2. 124<br>3. 144<br>4. 135 | Click here for video solution |
| 3 | A box has 450 balls, each either white or black, there being as many metallic white balls as metallic black balls. If 40% of the white balls and 50% of the black balls are metallic, then the number of non-metallic balls in the box is: ( CAT 2021) | Click here for video solution |
| 4 | In a tournament, a team has played 40 matches so far and won 30% of them. If they win 60% of the remaining matches, their overall win percentage will be 50%. Suppose they win 90% of the remaining matches, then the total number of matches won by the team in the tournament will be: (CAT – 2021)<br><br>1. 84<br>2. 78<br>3. 86<br>4. 80 | Click here for video solution |
| 5 | Two alcohol solutions, A and B, are mixed in the proportion 1:3 by volume. The volume of the mixture is then doubled by adding solution A such that the resulting mixture has 72% alcohol. If solution A has 60% alcohol, then the percentage of alcohol in solution B is: (CAT – 2020)<br><br>1. 90 %<br>2. 92 %<br>3. 94 %<br>4. 89 % | Click here for video solution |
| 6 | In the final examination, Bishnu scored 52% and Asha scored 64%. The marks obtained by Bishnu is 23 less, and that by Asha is 34 more than the marks obtained by Ramesh. The marks obtained by Geeta, who scored 84%, is (CAT – 2020)<br><br>1. 399<br>2. 417<br>3. 439<br>4. 357 | Click here for video solution |
| 7 | The cost of labor in manufacturing a product is 16.66% of the total cost. Due to improvements in automation, the cost of labor decreased to 12.5% of total cost, all other cost remaining the same. Find the percentage decrease in the cost of labor. | Click here for video solution |

QUANTIFIERS CAT ACADEMY

# Topic- Simple Interest & Compound Interest


**https://youtu.be/f3Z3ImEH_H0**

**How to apply the formula and basics concepts of the topic is covered in this video.**



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## Simple & Compound Interest

**1.** Simple Interest = (P × R × T)/100, where, *P* is the amount invested or borrowed, *T* is the time in years and *R* is the annual interest rate.

**2. A = P + SI**, where, *A* is the amount payable or receivable at the end of period *T*, *P* is the principal and *SI* is the simple interest.

**3.** In case of compound interest, $A = P(1+R/100)^n$, where, *A* is the amount receivable on compound interest basis, *P* is the principal invested, *R* is the annual rate of interest and *n* is the number of years for which the sum has been invested.

**4.** Compound Interest = *A* - *P*, where, *C.I* is the compound interest, *A* is the amount receivable with interest and *P* is the principal invested.

**5.** If the rate percent is per annum and the interest is compounded half yearly, then the amount after 'n' years will be $A = P(1+R/200)^{2n}$

**6.** If the rate percent is per annum and the interest is compounded quarterly, then the amount after 'n' years will be $A = P(1+R/400)^{4n}$.

Therefore, in general, If the rate percent is per annum and the interest is compounded 't' times in a year, then the amount after 'n' years will be $A = P(1+R/t \times 100)^{tn}$

**Interest rate on monthly installment plans**

$R = 24 \times I \times 100/N(F+L)$, where, *R* is the interest, *I* is the installment, *N* is the number of installments, *F* is the principal left before the first installment and *L* is the principal left before last installment.

**Equated Installment**

$X = \frac{A}{1-\left(\frac{100}{100+R}\right)^n} \times \frac{R}{100}$ where, *X* is the equated installment, *A* is the amount borrowed, *R* is the rate of interest and *n* is the number of years.

$\Rightarrow$ $A = P(1+R/200)^{2N}$, if the interest is calculated half-yearly.

$\Rightarrow$ $A = P(1+R/400)^{4N}$, if the interest is calculated quarterly.

**Rule of 72 -** Time taken to double one's investment can be obtained by dividing 72 by the interest rate. This is only in case of compound interest.

**Rule of 69 -** This rule is also used to determine the time taken to double one's invest

# I don't have the motivation to do CAT

Sometimes studying can be very stressful, low scores, backlogs, & multiple responsibilities, etc. can be a lot! Hence you need a mentor who specially drafts everyday tasks for you!

## Meet Rishab Goyal

**IIM C converted**

**CAT: 99.94, QA: 99.94, DILR:99.60**

Click here to connect with Rishabh on Linkedin

**Here is when Quantifiers came into play, you seek direction or motivation, talk to Sahil Sir once and I'm sure you'll have I can do anything attitude in you. I learnt a lot, I improvised a lot, I fell, I rose but the support and motivation was the only constant. QA and DILR were only taught when I joined Quantifiers and still whenever we interact, I asked Sahil Sir that you should have included verbal too. QA and DILR that I learnt, not only prepared me for CAT but the journey transformed as a individual too, making a much more positive person. I discussed doubts at almost every time the clock shows, at 1am, 3am, 5 am, it felt like someone was working on me 24*7, like someone is always pushing me to achieve more and more. The methods, the approaches, and most importantly no fancy stuff made things much more easier, it was like there was a roadmap and we just had to follow it.**
**I would take this opportunity to thank Quantifiers for making the journey to IIM C less difficult than what it could have been.**

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ent. However, it is a more refined formula. Time taken = 0.35 + (69/Interest rate). This is also true in case of compound interest only.

⇒ If $D$ is the difference in CI and SI for 2 years, and $R$ is the rate of interest and $P$ the principal then, $D = R/100 \times R/100 \times P$

⇒ The following table lists the values of an initial investment, P = Re. 1 for certain time periods and rates of interest, calculated at both, simple and compound interest. This would be of great help in time management during the paper, if memorized.

| | | |
|---|---|---|
| 1 | Bank A offers 6% interest rate per annum compounded half yearly. Bank B and Bank C offer simple interest but the annual interest rate offered by Bank C is twice that of Bank B. Raju invests a certain amount in Bank B for a certain period and Rupa invests ₹ 10,000 in Bank C for twice that period. The interest that would accrue to Raju during that period is equal to the interest that would have accrued had he invested the same amount in Bank A for one year. The interest accrued, in INR, to Rupa is: (CAT – 2021)<br><br>1. 1436<br>2. 3436<br>3. 2346<br>4. 2436 | Click here for video solution |
| 2 | A person invested a certain amount of money at 10% annual interest, compounded half-yearly. After one and a half years, the interest and principal together became Rs 18522. The amount, in rupees, that the person had invested is: (CAT – 2020) | Click here for video solution |
| 3 | A few years back Bart borrows a certain sum of money at a certain rate of simple interest. Now the same sum becomes 200% of what he had borrowed. After 2 years the amount will increase by 12.5% compared to the present value. After how many years from now will the amount be 5 times the sum? | Click here for video solution |

| Time ⇓ Rate ⇒ | 5% | | 10% | | 20% | | 30% | |
|---|---|---|---|---|---|---|---|---|
| | **SI** | **CI** | **SI** | **CI** | **SI** | **CI** | **SI** | **CI** |
| 1 year | 1.05 | 1.05 | 1.1 | 1.1 | 1.2 | 1.2 | 1.3 | 1.3 |
| 2 years | 1.10 | 1.1025 | 1.2 | 1.21 | 1.4 | 1.44 | 1.6 | 1.69 |
| 3 years | 1.15 | 1.1575 | 1.3 | 1.331 | 1.6 | 1.728 | 1.9 | 2.197 |
| 4 years | | | 1.4 | 1.4641 | 1.8 | 2.0736 | | |

QUANTIFIERS CAT ACADEMY

# Topic: Profit and Loss


https://youtu.be/9EaTfOHFtak

**How to apply the formula and basics concepts of the topic is covered in this video.**



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## Profit & Loss

1. **Cost Price (CP):** The price paid to buy a particular product is called its cost price. Some overhead expenses such as transportation, taxes etc. are also included in the cost price.
2. **Selling Price (SP):** The sum of money received for the product.
3. **Marked Price (MP):** The price that is listed or marked on the product. This is also known as printed price/quotation price/invoice price/catalogue price.

**Profit:** There is a gain in a transaction if the selling price is more than the cost price. The excess of the selling price to the cost price is called profit.
Therefore, Profit = Selling Price – Cost Price

**Loss:** When the selling price is less than the cost price there is loss in the transaction. The excess of cost price over the selling price is called loss.
Therefore, Loss = Cost Price – Selling Price

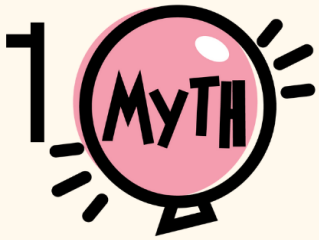
Discount: Discount is the difference of the marked price of an object and the actual selling price. Therefore, Discount = Marked price – Selling Price

**Note:**

1. **Percentage Profit = 100 × Profit/Cost Price**
2. **Percentage Loss = 100 × Loss/Cost price**
3. **Percentage discount = 100 × discount/Marked Price**

4. If two items are sold each at Rs X, one at a gain of p% and the other at a loss of p%, then the two transactions have resulted in an overall loss of $\frac{p^2}{100}$%, and the absolute value of the loss is = Rs. $\frac{2.p^2.X}{100^2-p^2}$

5. If the cost price of two items is X, and one is sold at p% profit and the other at p% loss, then the two transactions have resulted in no gain or no loss.




I need to attempt a maximum number of questions to get 99.99%ile

***You don't have to attempt all the questions! Yes, there are a total of 66 questions, the key here is to attempt a specific amount for example but in CAT 2023, an attempt of total***

***Manali (our student) cracked CAT Exam despite getting 4 correct QA questions and got into IIM C. So can you, believe in yourself.***

| Section | | Section | | Section | | Total | |
|---|---|---|---|---|---|---|---|
| Verbal Ability & Reading Comprehension | | Data Interpretation & Logical Reasoning | | Quantitative Ability | | | |
| Scaled Score | Percentile | Scaled Score | Percentile | Scaled Score | Percentile | Overall Scaled Score | Overall Percentile |
| 51.10 | 99.87 | 23.99 | 98.25 | 11.74 | 87.99 | 86.83 | 99.60 |

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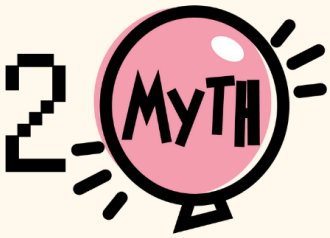
**Trade Discount:** To attract customers, it is a common practice to announce discount on the marked price of an article.

**Note**: The discount is always taken as a % of the marked price, unless otherwise specified. E.g. Let the list price of an article be Rs. 450. A discount of 5% on its list price is announced. Then, the new selling price = 450 × 95/100 = Rs 422.5.

**Cash Discount:** In addition to trade discount, the manufacturer may offer an additional discount called the Cash Discount if the buyer makes full payment within a certain specified time. Cash Discount is usually offered on the net price (the price after subtracting discount from the marked price).Therefore, Cash Price = Net Price - Cash Discount

**Note:** Cash discount is always calculated on net price, unless otherwise specified.

**Wrong Weight**: When a tradesman claims to sell at cost price, but uses a false weight, then the percentage profit earned $= \frac{100 \times Error}{Ture\ Weight - Error}$




CAT is meant for Engineers and only engineer make it to IIMs

***False, There is a diversity factor that every college needs to maintain! Cherry on top, Even from a non-engineer background, you can still ace the CAT exam.***

***Pallav (our student) cracked CAT Exam despite being from commerce background and got into IIM A. So can you, believe in yourself.***

| Section | | Section | | Section | | Total | |
|---|---|---|---|---|---|---|---|
| Verbal Ability & Reading Comprehension | | Data Interpretation & Logical Reasoning | | Quantitative Ability | | | |
| Scaled Score | Percentile | Scaled Score | Percentile | Scaled Score | Percentile | Overall Scaled Score | Overall Percentile |
| 42.17 | 99.46 | 38.36 | 99.90 | 48.82 | 99.93 | 129.35 | 99.98 |

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**6.** If there is an offer of buy x items and get y items free, then the %age discount = $\frac{y}{x+y} \times 100$.

**7.** If C.P of x article is equal to the selling price of y articles then the resultant %age profit or %age loss = $\frac{y-x}{y} \times 100$.

**Successive Discounts**: If a shopkeeper gives two successive discounts of a% and b%, then the overall discount is equal to (a + b – ab/100)%.

| | | |
|---|---|---|
| 1 | Ramu mixes 2 litres of mineral water costing Rs. 15 per litre with 18 litres of milk that he bought for Rs. 900. After selling 2 litres of this mixture to Shamu, Ramu adds tap water so that the ratio of milk to tap water is 9:10. If tap water is free and Ramu claims to sell at cost price of milk, what is his profit%? | Click here for video solution |
| 2 | A shopkeeper buys 120 litre petrol at Rs. 35 per litre and then mixes some kerosene which costs him Rs. 24 per litre. He sells this mixture at Rs. 36/litre but manipulates the meter such that it shows 1 litre when it is actually 0.96 litre. What quantity of kerosene does he mix with petrol if he earns a profit of 25% in all? | Click here for video solution |
| 3 | The ratio of selling prices of three articles A,B and C is 29:27:32. The ratio of percentage profit is 4:2:5 respectively .If the cost price of article A is equal to B and the cost price of article C is Rs.480, then what is the overall gain %? | Click here for video solution |
| 4 | Rani bought more apples than oranges. She sells apples at Rs 23 a piece and makes 15% profit. She sells oranges at Rs10 a piece and makes 25% profit. If she gets Rs 653 after selling all the apples and oranges, find her profit percentage. | Click here for video solution |
| 5 | A person buys tea of three different qualities at Rs. 800, Rs. 500, and Rs. 300 per kg, respectively, and the amounts bought are in the proportion 2 : 3 : 5. She mixes all the tea and sells one-sixth of the mixture at Rs. 700 per kg. The price, in INR per kg, at which she should sell the remaining tea, to make an overall profit of 50%, is<br><br>1. 688<br>2. 675<br>3. 653<br>4. 692 | Click here for video solution |
| 6 | Raj invested Rs.10000 in a fund. At the end of first year, he incurred a loss but his balance was more than Rs. 5000. This balance, when invested for another year, grew and the percentage of growth in the second year was five times the percentage of loss in the first year. If the gain of Raj from the initial investment over the two year period is 35%, then the percentage of loss in the first year is:<br><br>1. 10<br>2. 15<br>3. 70<br>4. 5 | Click here for video solution |
| 7 | A man buys 35 kg of sugar and sets a marked price in order to make a 20% profit. He sells 5 kg at this price, and 15 kg at a 10% discount. Accidentally, 3 kg of sugar is wasted. He sells the remaining sugar by raising the marked price by p percent so as to make an overall profit of 15%. Then p is nearest to<br><br>1. 25<br>2. 22<br>3. 31<br>4. 35 | Click here for video solution |
| 8 | Rani bought more apples than oranges. She sells apples at Rs. 23 apiece and makes 15% profit. She sells oranges at Rs. 10 apiece and makes 25% profit. If she gets Rs. 653 after selling all the apples and oranges, find her profit percentage | Click here for video solution |

QUANTIFIERS CAT ACADEMY

# Topic: Ratios & Proportion + Allegation & Mixture


https://youtu.be/f3Z3ImEH_H0

**How to apply the formula and basics concepts of the topic is covered in this video.**



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## Ratio & Proportion

Ratio is the relation between two quantities of the same kind**.** This relation indicates how many times one quantity is equal to the other. In other words, ratio is a number, which expresses one quantity as a fraction of the other.
Example: Ratio of 12 to 13 is 12/13 or 12: 13.
The numbers forming the ratio are called terms. The numerator, i.e. "12", is known as the antecedent and the denominator, i.e. "13", in this case, is known as the consequent.

The ratio between two quantities a and b if expressed as a/b, is called fractional form and, a: b is called linear form.

If two different ratios, a: b and c: d are expressed in different units, then the two are compounded to obtain a combined ratio.
Compounding of a: b and c: d yields (a × c) /(b × d) .

**1.** If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ , then each of these ratios is equal to $\frac{a+c+e}{b+d+f}$

**2.** If $\frac{a}{b} = \frac{c}{d}$ , then $\frac{b}{a} = \frac{d}{c}$ *(Invertendo)*

**3.** If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$ *(Alterendo)*

**4.** If $\frac{a}{b} = \frac{c}{d}$ , then $\frac{a+b}{b} = \frac{c+d}{d}$ *(Componendo)*

**5.** If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a-b}{b} = \frac{c-d}{d}$ *(Dividendo)*

**6.** If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-c} = \frac{c+d}{c-d}$ *(Componendo dividendo)*

**7.** If a, b and c are positive then

**(i)** If a > b, then $\frac{a+c}{b+c} < \frac{a}{b}$

**(ii)** If a < b, then $\frac{a+c}{b+c} > \frac{a}{b}$

**(iii)** If a > b, then $\frac{a-c}{b-c} > \frac{a}{b}$

**(iv)** If a < b, then $\frac{a-c}{b-c} < \frac{a}{b}$

**8.** Duplicate ratio of a: b is $a^2$: $b^2$

**9.** Sub duplicate ratio of a:b is $\sqrt{a}$: $\sqrt{b}$

**10.** Triplicate ratio of a: b is $a^3$: $b^3$

**11.** Sub triplicate of a: b is $\sqrt[3]{a} : \sqrt[3]{b}$

**Continued Proportion:** Four numbers a, b, c and d are said to be in proportion if a: b = c: d. Here d is called the fourth proportional.
If on the other hand, a: b = b: c = c: d, then the four numbers are said to be in continued proportion.
Third proportional of two number a and b is the third number c such that a: b = b: c.
If we have the ratios, a: b = b: c. Here b is called the mean proportional and is equal to the square root of the product of a and c i.e., $b^2 = a \times c \Rightarrow b = \sqrt{ac}$

If a/b = b/c = c/d etc., then a, b, c, d are in geometric progression.
Let a/b = b/c = c/d = k, then, c = dk; b = ck and a = bk

Since c = dk, b = dk × k = $dk^2$ and a = bk = $dk^2 \times k = dk^3$, implying that they are in geometric progression.

If the three ratios, a: b, b: c, c: d are known, we can find a: d by the multiplying these three ratios a/d = a/b × b/c × c/d

If a, b, c and d are four terms and the ratios a: b, b: c, c: d are known, then one can find the ratio a: b: c: d.

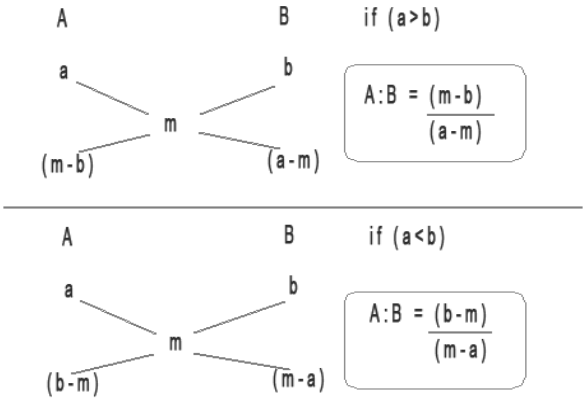
**Variation - Direct, Inverse**

1. If $a$ is directly proportional to $b$, then the ratio of $a$ and $b$ is constant.
2. If $a$ is inversely proportional to $b$, then the product of $a$ and $b$ is constant.
3. If $a \propto b$ and $b \propto c$, then $a \propto c$.
4. If $a \propto b$ and $a \propto c$, then $a \propto (b \pm c)$.
5. If $a \propto b$ and $c \propto d$, then $ac \propto bd$.

| | | |
|---|---|---|
| 1 | Anil, Bobby and Chintu jointly invest in a business and agree to share the overall profit in proportion to their investments. Anil's share of investment is 70%. His share of profit decreases by Rs. 420 if the overall profit goes down from 18% to 15%. Chintu's share of profit increases by Rs. 80 if the overall profit goes up from 15% to 17%. The amount, in INR, invested by Bobby is<br><br>1. 2200<br>2. 2400<br>3. 2000<br>4. 1800 | Click here for video solution |
| 2 | From a container filled with milk, 9 liters of milk are drawn and replaced with water. Next, from the same container, 9 litres are drawn and again replaced with water. If the volumes of milk and water in the container are now in the ratio of 16 : 9, then the capacity of the container, in liters, is: | Click here for video solution |

## Alligation and Mixture

In Mixture problems, different substances are combined, and characteristics of the resulting mixture have to be determined.
In solving mixture problems, we use the Alligation Rule (Alligation means 'linking').The rule states, that "When different quantities of different ingredients are mixed together to produce a mixture of a mean value, the ratio of their quantities is inversely proportional to the differences in their cost from the mean value."




One case in mixtures is, repeated dilution of a mixture with one of the ingredients, by removing, say n litres of the mixture and replacing it with n litres of one of the ingredients. Say there are m litres of water initially. We now remove n litres of the water and replace it by n litres of wine. This operation is done t times. Then,
Quantity of water left in the vessel = $m(1 - n/m)^t$
Where
m = Total quantity
n = Quantity drawn every time
t = No. of times
The above result can also be written as $\frac{Quantity\ of\ the\ original\ liquid\ left}{Total\ liquid} = \left(1 - \frac{n}{m}\right)^t$
If the dilution is done twice then the ratio $\frac{Quantity\ of\ the\ original\ liquid\ left}{Total\ liquid}$ will be equal to a perfect square.
If the dilution is done thrice then the ratio $\frac{Quantity\ of\ the\ original\ liquid\ left}{Total\ liquid}$ will be equal to a perfect cube and so on.

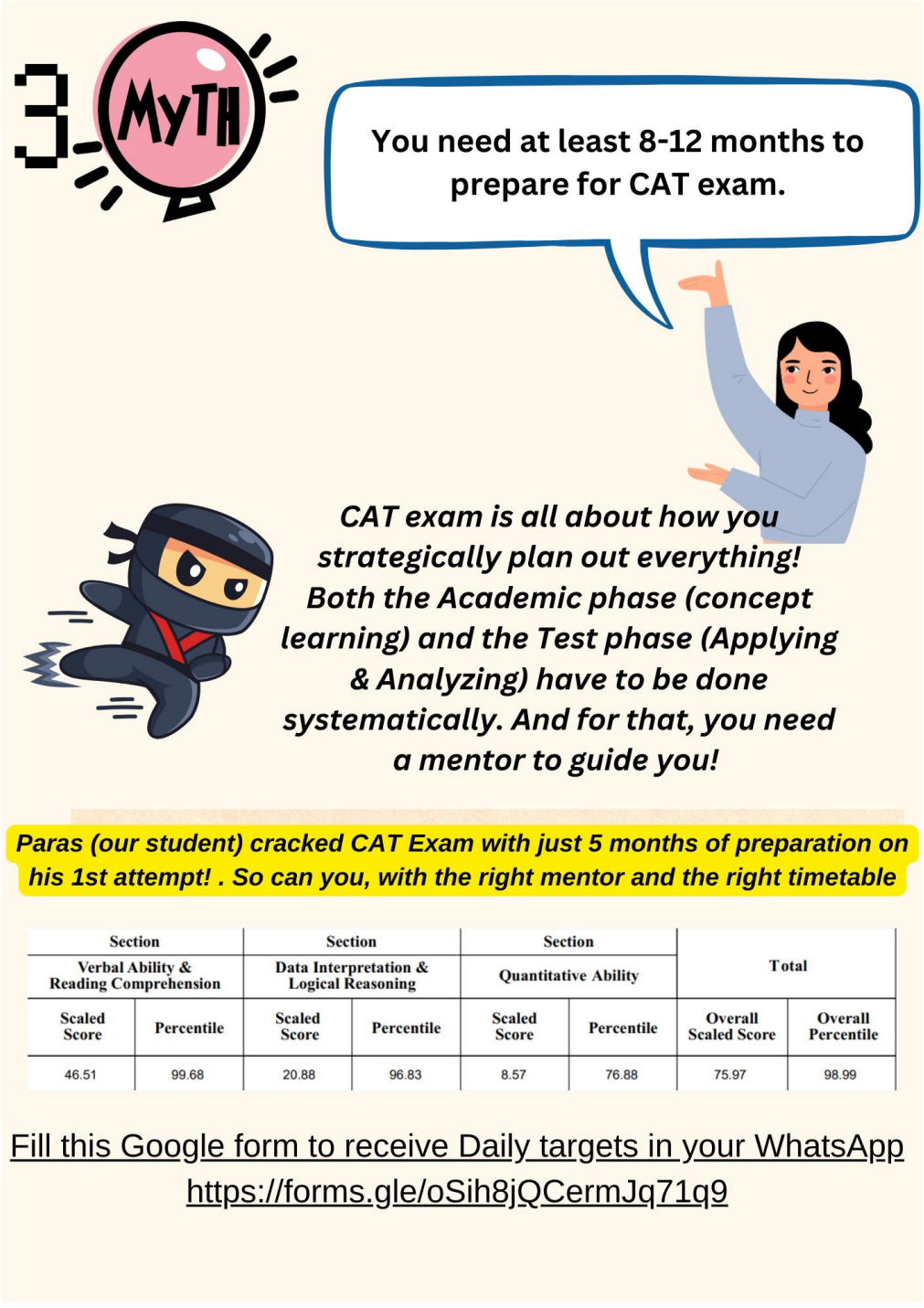
## Partnership

Partnership is an association of two or more persons who invest money together in order to carry out a certain business. Partnerships are of two types:

**(i) Simple Partnership:** When all partners invest in the business at the same time i.e. their capital remains in the business for the same duration it is called simple partnership. In this kind of partnership, the profit is simply distributed amongst the partners, in the ratio of their respective invested capital.

**(ii) Compound Partnership:** When capital of the partners is invested in the business for different time periods, the partnership is known as compound partnership. In this, the profit sharing ratio is calculated by multiplying the capital invested with the unit of time (mostly months).

The profit or loss in the business are shared as per the following rules:

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**Rule 1:** In a simple partnership, the loss or profit is distributed amongst the partners in the ratio of their respective investments.
Example: Say, P and Q invested Rs. a and b for one year in a business. Then, the share of profit or loss will be,

P's profit/ loss: Q's profit/ loss = a: b

**Rule 2:** In a compound partnership, the profit or loss ratio is calculated as capital multiplied by the duration of investment.
Example: P's profit/loss: Q's profit/loss = $a\times t_1$: $b\times t_2$
Where, $t_1$ = P's duration of investment and, $t_2$ = Q's duration of investment

| | | |
|---|---|---|
| 1 | There is 84 liters of milk in a (x + 32) liters of mixture (milk + water) where if (x – 64) liters of mixture is replaced by 4 liters of milk, the quantity of milk will be 60% of the quantity of the resulting mixture. Find the value of 'x'. | Click here for video solution |
| 2 | In a drum of 280 litres, water and alcohol are mixed in certain ratio, if 40 liters alcohol is added to the mixture, the ratio of water and alcohol reverses, what is the initial quantity of alcohol. (write your answer upto decimal points) | Click here for video solution |
| 3 | From a 25 litre solution containing wine and water in the ratio 5:8:5 liter of mixture is taken out and replaced by 10 liter of water, then 9 liter is taken out and replaced by 7 liter of wine. Finally 15 liter is drawn off and replaced by 5 liter of water. Find the wine and water ratio in the resultant mixture? | Click here for video solution |
| 4 | If a certain weight of an alloy of silver and copper is mixed with 3 kg of pure silver, there resulting alloy will have 90% silver by weight. If the same weight of the initial alloy is mixed with 2 kg of another alloy which has 90% silver by weight, the resulting alloy will have 84% silver by weight. Then, the weight of the initial alloy, in kg, is (CAT 2021)<br><br>A. 3<br>B. 2.5<br>C. 3.5<br>D. 4 | Click here for video solution |
| 5 | There are two drums, each containing a mixture of paints A and B. In drum 1, A and B are in the ratio 18: 7. The mixtures from drums 1 and 2 are mixed in the ratio 3: 4 and in this final mixture, A and B are in the ratio 13: 7. What was the ratio of A & B in drum 2? | Click here for video solution |

QUANTIFIERS CAT ACADEMY

# Topic: Time and Work + Time, Speed & Distance (Boats & Streams)


https://youtu.be/7LnfTNMfNFI

**How to apply the formula and basics concepts of the topic is covered in this video.**



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## Time & Work

**1.** If a person can do a certain task in t hours, then in 1 hour he would do $1/t$ portion of the task. A does a particular job in 'a' hours and B does the same job in 'b' hours, together they will take $\frac{ab}{a+b}$ hours.

**2.** A does a particular job in 'a' hours more than A and B combined whereas B does the same job in 'b' hours more than A and B combined, then together they will take $\sqrt{ab}$ hours to finish the job.

**3.** A does a particular job in 'a' hours, B does the same job in 'b' hours and C does the same job in 'c' hours, then together they will take $\frac{abc}{ab+bc+ca}$ hours.

**4.** If A does a particular job in 'a' hours and A & B together do the job in 't' hours, the B alone will take $\frac{at}{a-t}$ hours.

**5.** If A does a particular job in 'a' hours, B does the same job in 'b' hours and A, B and C together do the job in 't' hours, then C alone can do it in $\frac{abt}{ab-at-bt}$ hours.

A and C together can do it in $\frac{bt}{b-t}$hours.

B and C together can do it in $\frac{at}{a-t}$ hours

**6.** If the objective is to fill the tank, then the Inlet pipes do positive work whereas the Outlet pipes do negative work. If the objective is to empty the tank, then the Outlet pipes do positive work whereas the Inlet Pipes do negative work.
If a pipe can fill a tank in 'x' hours then the part filled per hour = $1/x$
If a pipe can empty a tank in 'y' hours, then the part emptied per hour = $1/y$
If a pipe A can fill a tank in 'x' hours and pipe can empty a tank in 'y' hours, if they are both active at the same time, then
The part filled per hour = $1/x - 1/y$ (if $y > x$)
The part emptied per hour = $1/y - 1/x$ (if $x > y$)

**7.** If $M_1$ men can do $W_1$ work in $D_1$ days working $H_1$ hours daily and $M_2$ men can do $W_2$ work in $D_2$ days working $H_2$ hours daily, then $M_1 \times D_1 \times H_1 \times W_2 = M_2 \times D_2 \times H_2 \times W_1$.

| | | |
|---|---|---|
| 1 | A and B work together, they take half a day more than what A, B and C together would take to complete the same work. If B and C work together the number of days they take is 200% more than what A, B and C would take. All three worked together from start to finish and B earned Rs.400 as his share out of the total earnings of Rs.1800. In how many days did A, B and C together complete the work?<br><br>1. 4<br>2. 6<br>3. 8<br>4. 18 | Click here for video solution |
| 2 | 1 man and 3 women and 4 boys can do a work in 96 hours. 2 men and 8 boys can do that work in 80 hours. 2 men and 3 women can do that work in 120 hours. Then how many hours will 5 men and 12 boys take to complete the work. | Click here for video solution |
| 3 | P works 25% more efficiently than Q and Q works 50% more efficiently than R. To complete a certain project, P alone takes 50 days less than Q. If in this project, P alone works for 60 days and then Q alone works for 125 days, then in how many days can R alone complete the remaining work? | Click here for video solution |
| 4 | Work done by A, B, C and D is in arithmetic progression. C worked for 9 days and completed 30% work. A and B worked for 2 days and 3 days only. Find for how many days 'D' work to complete the remaining work if all four together can complete that work in 4 days | Click here for video solution |

| | | |
|---|---|---|
| 5 | The amount Neeta and Geeta together earn in a day equals what Sita alone earns in 6 days. The amount Sita and Neeta together earn in a day equals what Geeta alone earns in 2 days. The ratio of the daily earnings of the one who earns the most to that of the one who earns the least is (CAT – 2021)<br><br>1. 7 : 3<br>2. 11 : 7<br>3. 11 : 3<br>4. 3 : 2 | Click here for video solution |
| 6 | Anu, Vinu and Manu can complete a work alone in 15 days, 12 days and 20 days, respectively. Vinu works everyday. Anu works only on alternate days starting from the first day while Manu works only on alternate days starting from the second day. Then, the number of days needed to complete the work is: (CAT – 2021)<br><br>1. 6<br>2. 7<br>3. 8<br>4. 5 | Click here for video solution |
| 7 | Amar, Akbar and Anthony are working on a project. Working together Amar and Akbar can complete the project in 1 year, Akbar and Anthony can complete in 16 months, Anthony and Amar can complete in 2 years. If the person who is neither the fastest nor the slowest works alone, the time in months he will take to complete the project is: (CAT – 2021) | Click here for video solution |
| 8 | Two pipes A and B are attached to an empty water tank. Pipe A fills the tank while pipe B drains it. If pipe A is opened at 2 pm and pipe B is opened at 3 pm, then the tank becomes full at 10 pm. Instead, if pipe A is opened at 2 pm and pipe B is opened at 4 pm, then the tank becomes full at 6 pm. If pipe B is not opened at all, then the time, in minutes, taken to fill the tank is : (CAT – 2021)<br><br>1. 264<br>2. 144<br>3. 140<br>4. 120 | Click here for video solution |
| 9 | Anil can paint a house in 60 days while Bimal can paint it in 84 days. Anil starts painting and after 10 days, Bimal and Charu join him. Together, they complete the painting in 14 more days. If they are paid a total of Rs. 21000 for the job, then the share of Charu, in INR, proportionate to the work done by him, is (CAT – 2021)<br><br>1. 9100<br>2. 9150<br>3. 9000<br>4. 9200 | Click here for video solution |
| 10 | One day, Rahul started a work at 9 AM and Gautam joined him two hours later. They then worked together and completed the work at 5 PM the same day. If both had started at 9 AM and worked together, the work would have been completed 30 minutes earlier. Working alone, the time Rahul would have taken, in hours, to complete the work is (CAT – 2021)<br><br>1. 12<br>2. 10<br>3. 11.5<br>4. 12.5 | Click here for video solution |
| 11 | Anil can paint a house in 12 days while Barun can paint it in 16 days. Anil, Barun, and Chandu undertake to paint the house for ₹ 24000 and the three of them together complete the painting in 6 days. If Chandu is paid in proportion to the work done by him, then the amount in INR received by him is: | Click here for video solution |
| 12 | A contractor agreed to construct a 6 km road in 200 days. He employed 140 persons for the work. After 60 days, he realized that only 1.5 km road has been completed. How many additional people would he need to employ in order to finish the work exactly on time? | Click here for video solution |

# GOT DOUBTS ?

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## Meet Dhruva

**Dhruva Sareen**
**IIM A**
**CAT: 97.95**
**NMAT:236**
**XAT: 99.96**
**SNAP: 99.93**
**Lawyer**

*Well, I'm not asking you to believe my words but go and witness yourself. I was a member of Quantifiers Whatsapp group, I was simply taken aback by the way every doubt was solved instantaneously, how can someone be awake 24*7. I have personally seen doubts getting solved at 2/3/4 in the night. I took up the course in June and my journey was never the same, I can ensure you the amount of personal attention you get there, you can't get anywhere. I mentioned it to Sahil Sir too that I never felt that there is a teacher teaching at the other side, it was always like, an elder brother, who has been guiding/scolding me, the motivational sessions, the workshops every aspect that you can think of was taken care to perfection.QA was my biggest fear, I remember scoring in single digits in my initial mocks, but thanks to Quantifiers for believing in me, making me realize that nothing is impossible. It will not be an injustice to say that these guys worked harder than us, pushing us every single day, giving us regular tasks, DILR sets, RCs.The most important part is, we were never taught the questions which were irrelevant from exam perspective, sometimes I found some questions in mocks which were alien to me, but thanks to the mentoring, I was always aligned to what actually came in CAT.QA is a tough section for non-engineers they say, I disapprove that, but trust me, the more classes I attended, the less I believed the statement, we were always told that "P&C nahi ayega aisa, Binomial me time waste naai kro ....." Special focus on Arithmetic, Geometry and Algebra played a pivotal role in my scores, The booster sessions were cherry on the cake, it was like every concept knocked our brain and entered.DILR focus was a different level, the variety we practiced made sure that we have covered everything that could be, as a result I was able to solve 4 sets on D-day. Finally, I will again ask you to just join Quantifiers without a doubt and witness the magic.*
*Thanking Quantifiers*
*Adios*

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## Time Speed & Distance

The most important relationship between these three quantities, and possibly the only one which needs to be known is,

**Distance = Speed × Time**
**Speed = Distance traveled/Time**
**Time = Distance traveled/Speed**

**1.** Average speed = Total distance travelled/total time taken.

**(i)** If equal distances are covered at $a$ km/hr and $b$ km/hr, then average speed of the journey is 2ab/(a + b).

**(ii)** If a part of a journey is travelled at speed $S_1$ km/hr in $T_1$ hours and the remaining part at speed $S_2$ km/hr in $T_2$ hours then, total distance travelled = $S_1T_1 + S_2T_2$ km.
Average speed = $\frac{S_1T_1 + S_2T_2}{T_1 + T_2}$ km/hr.

**(iii)** In a journey travelled with different speeds, if the time travelled in each stage is constant, the average speed is the arithmetic mean of the different speeds. Therefore, if a man travelled for a certain distance at x km/hr and for equal amount of time at the speed of y km/hr then the average speed during the whole journey is (x + y)/2 km/hr

**2.** If two bodies are moving in the same direction at a speed of $a$ and $b$ respectively, then their relative speed is the difference of the two speeds i.e., |a - b|.

**3.** If two bodies are moving in the opposite directions at a speed of $a$ and $b$ respectively, then their relative speed is $a + b$.

**4.** If the ratio of the speeds of A and B is a: b, then the ratio of the times taken to cover the same distance is 1/a: 1/b or b: a.

**5.** If the ratio of the speeds of A and B is a: b, then the ratio of distance travelled in equal time intervals is a: b.

**6.** If a person P starts from A and heads towards B and another person Q starts from B and heads towards A and they meet after a time 't' then, t = $\sqrt{x \times y}$, where,
x = time taken (after meeting) by P to reach B and
y = time taken (after meeting) by Q to reach A.

**7.** A and B started at a time towards each other. After crossing each other, they took $T_1$ and $T_2$ hours respectively to reach their destinations. If they travel at constant speeds $S_1$ and $S_2$ respectively all over the journey, then $\frac{S_1}{S_2} = \sqrt{\frac{T_2}{T_1}}$.

**8.** If the length of a train is L meters and the speed of the train is S m/s, then the time taken by the train to pass a stationary man/pole is L/S sec.

**9.** If the length of the train is $L_1$ and its speed is S m/s and the length of a platform (stationary object of comparable length) is $L_2$, then the time taken by the train to cross the platform is $(L_1 + L_2)/S$ sec.

**10.** If the lengths of 2 trains are $L_1$ (faster) and $L_2$ (slower) m, and their speeds are $S_1$ and $S_2$ m/s respectively, then the time taken by the faster train to overtake the slower train is $(L_1+L_2)/(S_1 - S_2)$ sec, and the time taken for the trains to cross each other is $(L_1+L_2)/(S_1+S_2)$ sec.

**11.** If the average speed of a train, without stoppages, is $S_1$ km/hr and the speed with stoppages is $S_2$, then Stoppage time (in min/hr) = $S_1$-$S_2/S_1 \times 60$.

| | | |
|---|---|---|
| 1 | A, B and C run a race on a linear track (All 3 starting at same time). If A beats B by 20 seconds or 180 meters, B beats C by 40 seconds or 240 meters, what is the ratio of speeds of A and C ? | Click here for video solution |
| 2 | Two friends, Ram and Shyam, start at the same point, at the same time. Ram travels straight north at a speed of 10km/hr, while Shyam travels straight east at twice the speed of Ram. After 15 minutes, Shyam messages Ram that he is just passing by a large telephone tower and after another 15 minutes Ram messages Shyam that he is just passing by an old banyan tree. After some more time has elapsed, Ram and Shyam stop. They stop at the same point of time. If the straight-line distance between Ram and Shyam now is 50 km, how far is Shyam from the banyan tree (in km)? (Assume that Ram and Shyam travel on a flat surface.) | Click here for video solution |
| 3 | Everyday a bus leaves Delhi for Ambala and at the same instant another bus leaves Ambala for Delhi. The two buses cross each other at Karnal. Yesterday the bus leaving Delhi started late by 30 minutes, as a result of which it met the bus going in the opposite direction 12 km short of the usual meeting point. Today, the bus leaving Ambala started late by 30 minutes, as a result of which the two buses met 12 minutes later than their usual meeting time. What is the speed of the bus going from Delhi to Ambala? | Click here for video solution |
| 4 | A boy starts from his home at a certain time with a certain speed to pick up his girlfriend from office at 5.00pm. One day his girlfriend left the office at 3.00 pm and start walking to home at the speed of 40 km/hr and meet the boy in the way who left his home at his usual time .They reached home 40 min earlier than their usual time. Find the speed of the boy. | Click here for video solution |
| 5 | After travelling 180km, driver of a train thinks if he will go by same speed then he will be late. Then he increases the speed of train by 6.5 kmph and reaches at the scheduled time. Had he covered whole of the distance at the increased speed, he would have reached early by 40 minutes. If he had travelled whole of the distance at the original speed then he would be late by 14 minutes. Find the total distance | Click here for video solution |
| 6 | A and B start swimming from two ends of a 70 meter long swimming pool simultaneously at uniform speeds, The moment they reach the opposite ends , they turn back and keep on swimming. Speed of A and B is 7m/s and 3m/s respectively. How far will B be from his starting point when he meets A for the 4th time while swimming? | Click here for video solution |
| 7 | The reduction in the speed of an engine is directly proportional to the square of the number of bogies attached to it. The speed of the train is 100 km/hr when there are 4 bogies and 55 kmph when there are 5 bogies. What is the maximum number of bogies that can be attached to the train so that it can move? | Click here for video solution |
| 8 | Two trains cross each other in 14 seconds when running in opposite directions along parallel tracks. The faster train is 160 m long and crosses a lamp post in 12 seconds. If the speed of the other train is 6 km/hr less than the faster one, its length, in m, is<br><br>1. 190<br>2. 184<br>3. 192<br>4. 180 | Click here for video solution |
| 9 | Two trains A and B were moving in opposite DIRECTIONSs, their speeds being in the ratio 5 : 3. The front end of A crossed the rear end of B 46 seconds after the front ends of the trains had crossed each other. It took another 69 seconds for the rear ends of the trains to cross each other. The ratio of length of train A to that of train B is (CAT 2021)<br><br>1. 3 : 2<br>2. 2 : 1<br>3. 5 : 3<br>4. 2 : 3 | Click here for video solution |
| 10 | Mira and Amal walk along a circular track, starting from the same point at the sometime. If they walk in the same direction, then in 45 minutes, Amal completes exactly 3 more rounds than Mira. If they walk in opposite directions, then they | Click here for video solution |

| | | |
|---|---|---|
| | meet for the first time exactly after 3 minutes. The number of rounds Mira walks in one hour is: (CAT 2021) | |
| 11 | Anil, Sunil, and Ravi run along a circular path of length 3 km, starting from the same point at the same time, and going in the clockwise direction. If they run at speeds of 15 km/hr, 10 km/hr, and 8 km/hr, respectively, how much distance in km will Ravi have run when Anil and Sunil meet again for the first time at the starting point? (CAT 2020)<br><br>1. 5.2<br>2. 4.6<br>3. 4.8<br>4. 4.2 | Click here for video solution |
| 12 | A and B are two railway stations 90 km apart. A train leaves A at 9:00 am, heading towards B at a speed of 40 km/hr. Another train leaves B at 10:30 am, heading towards A at a speed of 20 km/hr. The trains meet each other at: (CAT 2020)<br><br>1. 11 : 45 am<br>2. 10 : 45 am<br>3. 11 : 00 am<br>4. 11 : 20 am | Click here for video solution |
| 13 | Vimla starts for office every day at 9 am and reaches exactly on time if she drives at her usual speed of 40 km/hr. She is late by 6 minutes if she drives at 35 km/hr. One day, she covers two-thirds of her distance to office in one-thirds of her usual time to reach office, and then stops for 8 minutes. The speed, in km/hr, at which she should drive the remaining distance to reach office exactly on time is (CAT 2020)<br><br>1. 27<br>2. 28<br>3. 26<br>4. 29 | Click here for video solution |
| 14 | Navin takes 60 mins to reach Manoj's house. Navin leaves his house for Manoj's house with his dog. The speed of the dog is thrice that of Navin's. The dog reaches Manoj's house and comes back to meet Navin and again runs towards Manoj's house and returns to Navin and so on. When will the dog meet Navin for the third time? | Click here for video solution |
| 15 | Train T leaves station X for station Y at 3 pm. Train S, traveling at (6/7)th of the speed of T, leaves Y for X at 5 pm. The two trains pass each other at a station Z, where the distance between X and Z is (5/8) of that between X and Y. How many hours does train T take for its journey from X to Y? | Click here for video solution |
| 16 | Alok started one hour after Bimol from city P towards city Q and crossed Bimol at a distance of 10 km from P. After reaching city Q, Alok immediately started moving back to city P along the same route. On the way back, he again met Bimol, who still needed 1 hour and 12 minutes to reach city Q. What is the distance between city P and city Q if Alok and Bimol travelled at a constant speed throughout the journey and Alok's speed was 5 km/hr? | Click here for video solution |
| 17 | Sounak, Niraj and SK need to reach the lecture hall on time which is 15 km away from their current location. They had only one bike which can accommodate maximum 2 persons at a time. Sounak came up with an idea where all of them can reach at the same time. He took SK with him on bike and asked Niraj to walk until he picks him up. Sounak dropped SK at a point and moved back to pick Niraj, SK walked the remaining distance. If both SK and Niraj walks at a speed of 10 kmph and Sounak drove at a constant speed of 30 kmph then find how much time (in minute) they took to reach the Lecture Hall. | Click here for video solution |
| 18 | On an upward moving escalator Amit, Sanjeev and Vicky take 10 steps, 8 steps and 5 steps respectively to reach the top. On the same upward moving escalator Amit takes 30 steps to come down from the top. Find the ratio of the time taken by Sanjeev and Vicky to reach the top. | Click here for video solution |
| 19 | You travel by Delhi Metro every day from Botanical Garden, Noida to Hauz Khas, Delhi. At Hauz Khas metro station, you use an escalator to get out of the station. The escalator takes 80 seconds. One day, the escalator was not working and you walk up the escalator in 50 seconds. How many minutes does it approximately take you to walk down the working escalator? | Click here for video solution |

## Boats and Stream

Let the speed of the boat in still water is $B$ kmph and the speed at which the stream is flowing is $W$ kmph,

1. When the boat is traveling with the stream, the speed of the boat = $(B + W)$ kmph. It is called the downstream speed.
2. When the boat is traveling against the stream, the speed of the boat = $(B - W)$ kmph. It is called the upstream speed.
3. If the upstream is denoted as $U$ and downstream is denoted as $D$ then $B = (D + U)/2$ and $W = (D - U)/2$

**Important Distance and Time Conversions:**

1 km = 1000 meter
1 meter = 100 cm
1 hour = 60 min
1 min = 60 sec
1 hour = 3600 sec
1 km/hr = $(1 \times 1000/1 \times 3600) = 5/18$ m/sec.

## Races

If A beats B by $x$ metres or $s$ seconds, then the speed of B is x/s metres/sec.

If the length of a circular track is $L$ m, and if A and B take $x$ and $y$ sec. respectively, to complete one round, then both of them will meet at the starting point after LCM $(x, y)$ sec.

If the length of a circular track is $L$ m, and if the speeds of A and B are $x$ m/sec. and $y$ m/sec respectively, then the time after which both of them will meet at a point other than the starting point is $L/|x - y|$ sec, if they are running in the same direction and $L/(x + y)$, if running in the opposite direction.

If the ratio of the speeds of A and B is x: y in lowest form, then the number of points (including the starting point) on the track where they will meet are:

**(i)** $|x - y|$, when they are running in the same direction
**(ii)** $x + y$ when they are running in opposite direction.

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| Section | | Section | | Section | | Total | |
|---|---|---|---|---|---|---|---|
| Verbal Ability & Reading Comprehension | | Data Interpretation & Logical Reasoning | | Quantitative Ability | | | |
| Scaled Score | Percentile | Scaled Score | Percentile | Scaled Score | Percentile | Overall Scaled Score | Overall Percentile |
| 42.17 | 99.46 | 38.36 | 99.90 | 48.82 | 99.93 | 129.35 | 99.98 |

# Transformations we love

**Shreyas**

**IIM A**

## From this

| Section | | Section | | Section | | Total | |
|---|---|---|---|---|---|---|---|
| Verbal Ability & Reading Comprehension | | Data Interpretation & Logical Reasoning | | Quantitative Ability | | | |
| Scaled Score | Percentile | Scaled Score | Percentile | Scaled Score | Percentile | Overall Scaled Score | Overall Percentile |
| 12.6 | 53.01 | 20.54 | 90.49 | 25.07 | 92.25 | 58.21 | 87.34 |

## To this

| Scaled Score | Percentile | Scaled Score | Percentile | Scaled Score | Percentile | Overall Scaled Score | Overall Percentile |
|---|---|---|---|---|---|---|---|
| 42.48 | 98.46 | 18.30 | 87.34 | 38.17 | 99.36 | 98.95 | 99.11 |

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# Topic: Algebra + Logs


https://youtu.be/7LnfTNMfNFI

**How to apply the formula and basics concepts of the topic is covered in this video.**

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# Algebra

## Algebraic Formulae

**1.** $x^n + y^n = (x + y)(x^{n-1} - x^{n-2}.y + x^{n-3}.y^2 - .. + y^{n-1})$ when n is odd. When n is odd, $x^n + y^n$ is divisible by x + y

**2.** $x^n - y^n = (x + y)(x^{n-1} - x^{n-2}.y + ... y^{n-1})$ when n is even. When n is even, $x^n - y^n$ is divisible by x + y.

**3.** $x^n - y^n = (x - y)(x^{n-1} + x^{n-2}.y + .... + y^{n-1})$ for both odd and even n. Therefore, $x^n - y^n$ is divisible by x–y.

**4.** $(a + b)^2 = a^2 + b^2 + 2ab$.

**5.** $(a - b)^2 = a^2 + b^2 - 2ab$.

**6.** $(a^2 - b^2) = (a + b)(a - b)$.

**7.** $(a + b)^2 - (a - b)^2 = 4ab$.

**8.** $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$.

**9.** $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$.

**10.** $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$.

**11.** $(a^3 + b^3) = (a + b)(a^2 + b^2 - ab)$.

**12.** $(a^3 - b^3) = (a - b)(a^2 + b^2 + ab)$.

**13.** $(a + b + c)^2 = [a^2 + b^2 + c^2 + 2(ab + bc + ca)]$.

**14.** $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$. If $a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$.

**15.** $(x + a)(x + b) = x^2 + (a + b)x + ab$.

## Linear Equations

Consider two linear equations:
$A_1 x + B_1 y = C_1$ and $A_2 x + B_2 y = C_2$.
This pair of linear equations has:

**1.** A unique solution if, $\frac{A_1}{A_2} \neq \frac{B_1}{B_2}$.

**2.** Infinite solutions if, $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$.

**3.** No solution if, $\frac{A_1}{A_2} = \frac{B_1}{B_2} \neq \frac{C_1}{C_2}$.

## Quadratic Equations

**1.** General form of a quadratic equation is $ax^2 + bx + c = 0$, $a \neq 0$

**2.** The discriminant of a quadratic equation is $D = b^2 - 4ac$

**3.** The roots of the above quadratic equation are $\frac{-b+\sqrt{D}}{2a}$ and $\frac{-b-\sqrt{D}}{2a}$

**4.** Let α and β be the roots of the above quadratic equation. If $D > 0$, then the roots are real and unequal. The sum of the roots $\alpha + \beta = \frac{-b}{a}$ and the product α × β = $\frac{c}{a}$

**5.** If $D$ is a perfect square, then the roots are rational.

**6.** If $D = 0$, then the roots are real and equal and is equal to $\frac{-b}{2a}$

**7.** If $D < 0$, then the roots are complex and unequal. If $a$, $b$ and $c$ of the quadratic equation are rational, then the roots are conjugates of each other. Ex. if α = $p + qi$, then β = $p - qi$

**8.** If $D \geq 0$, then, $ax^2 + bx + c = a(x - \alpha)(x - \beta)$.

**9.** If $c = a$, then the roots are reciprocal.

**10.** If $b = 0$, then the roots are equal in magnitude but opposite in sign.

**11.** If one of the roots of a quadratic equation with rational coefficients is irrational, then the other roots must be irrational conjugate. If α = $p + \sqrt{q}$, then β = $p - \sqrt{q}$.

**12.** If α, β are the roots of a quadratic equation, then the equation is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.

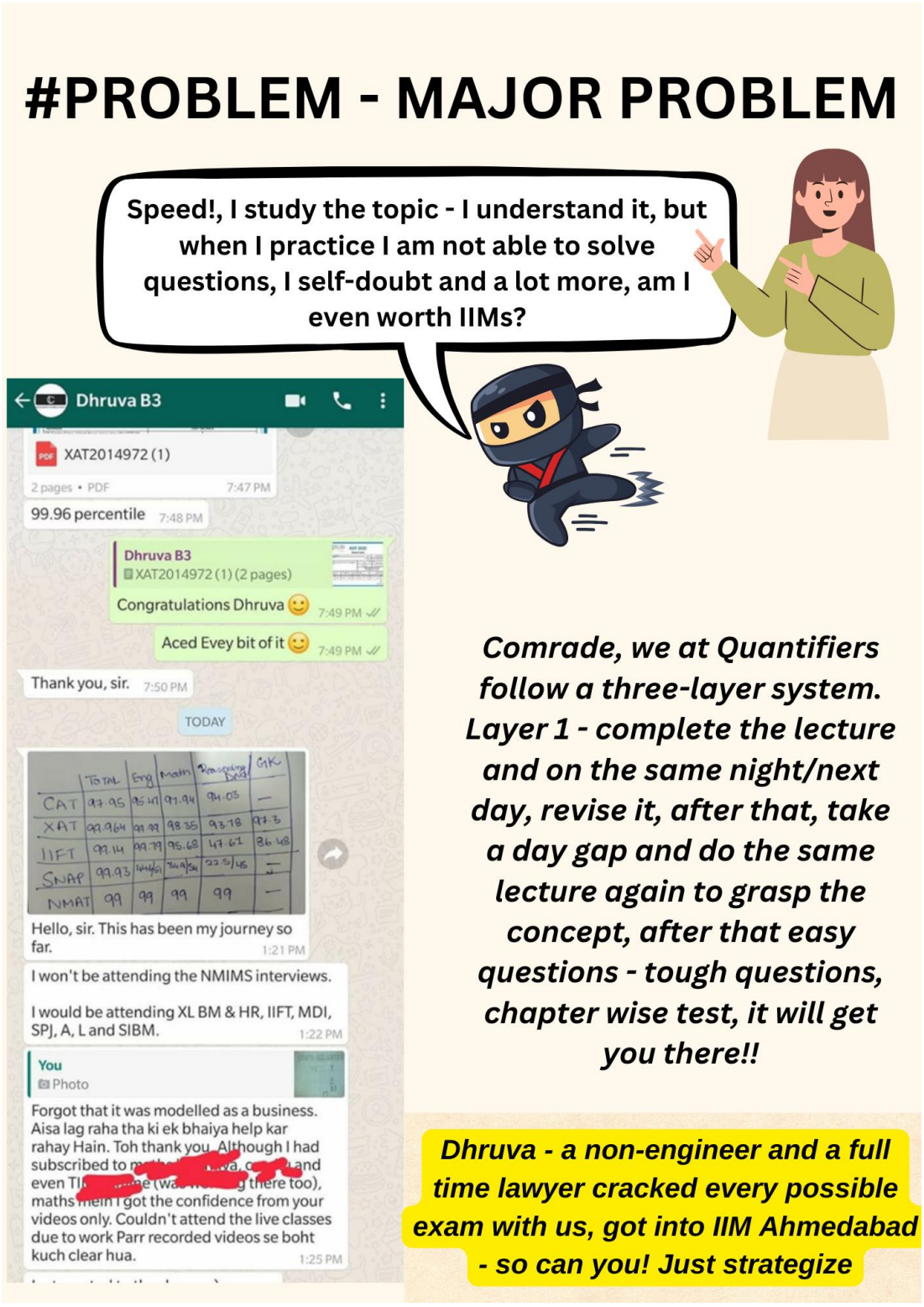
**13.** If a > 0, then the minimum value of $f(x) = ax^2 + bx + c$ is $-(b^2 - 4ac)/4a$ i.e., $-D/4a$ and is obtained at $x = -b/2a$.

**14.** If a < 0, then the maximum value of $f(x) = ax^2 + bx + c$ is $-(b^2 - 4ac)/4a$ i.e., $-D/4a$ and is obtained at $x = -b/2a$.

**15.** A quadratic whose roots are reciprocal of the roots of $ax^2 + bx + c = 0$ is $cx^2 + bx + a = 0$.

**16.** If the roots of an equation are k more than the roots of the equation $ax^2 + bx + c = 0$ then the equation is $a(x - k)^2 + b(x - k) + c = 0$.

| | | |
|---|---|---|
| 1 | If x is a positive real number then the minimum value of (x + 7) (x + 19)/(x + 3) is: | Click here for video solution |
| 2 | If $f(x) = x^2 - 7x$ and g(x) = x + 3, then the minimum value of f(g(x)) − 3x is (CAT 2021)<br>1. -16<br>2. -15<br>3. -12<br>4. -20 | Click here for video solution |
| 3 | If 3x + 2 \|y\| + y = 7 and x + \|x\| + 3y = 1, then x + 2y is: (CAT 2021)<br>1. 8/3<br>2. -4/3<br>3. 1<br>4. 0 | Click here for video solution |
| 4 | Find the range of $(x^2 + 4x + 8)/(x^2 + 4x + 5)$ | Click here for video solution |

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## Inequalities

**1.** If a > b, then b < a.

**2.** If a > b, then a – b > 0 or b – a < 0.

**3.** If a > b, then a + c > b + c.

**4.** If a > b, then a – c > b – c.

**5.** If a > b and c > d, then a + c > b + d.

**6.** If a > b and c > d, then a – d > b – c.

**7.** If a > b and m > 0, then ma > mb.

**8.** If a > b and m > 0, then $\frac{a}{m} > \frac{b}{m}$.

**9.** If a > b and m > 0, then ma > mb.

**10.** If a > b and m < 0, then $\frac{a}{m} < \frac{b}{m}$.

**11.** If 0 < a < b and n > 0, then $a^n < b^n$.

**12.** If 0 < a < b and n < 0, then $a^n > b^n$.

**13.** If 0 < a < b, then $\sqrt[n]{a} < \sqrt[n]{b}$.

**14.** $\sqrt{ab} \leq \frac{a+b}{2}$, where a > 0, b > 0; an equality is valid only if a = b.

**15.** $a + \frac{1}{a} \geq 2$, where a > 0; an equality takes place only at a = 1.

**16.** $\sqrt[n]{a_1 a_2 \ldots . a_n} \leq \frac{a_1+a_2+\ldots.+a_n}{n}$, where $a_1, a_2, \ldots.., a_n > 0$.

**17.** If ax + b > 0 and a > 0, then $x > -\frac{b}{a}$.

**18.** If ax + b > 0 and a > 0, then $x < -\frac{b}{a}$.

**19.** |a + b| ≤ |a| + |b|.

**20.** If |x| < a, then – a < x < a, where a > 0.

**21.** If |x| > a, then x < -a and x > a, where a > 0.

**22.** If $x^2 < a$, then |x| < √a, where a > 0.

**23.** If $x^2 > a$, then |x| > √a, where a > 0.

**24.** If a and b are of same sign then a > b ⇒ 1/a < 1/b

**25.** If a and b are of opposite sign then a > b ⇒ 1/a > 1/b

**26.** If $ax^2 + bx + c < 0$ then (x – m)(x – n) < 0 and if n > m then m < x < n.

**27.** If $ax^2 + bx + c > 0$ then $(x - m)(x - n) > 0$ and if $m < n$ then $x < m$ and $x > n$.

**28.** If product of n positive numbers is constant, then their sum will be minimum when all are equal or close to each other.

**29.** If the sum of n positive numbers is constant, then their product will be maximum when all are equal or close to each other.

| | | |
|---|---|---|
| 1 | Find the sum of all negative integral values of x where $\|x - \| x - 2 \| + 3 \| - 4 < 3$ | Click here for video solution |
| 2 | Consider the function f(x) = (x + 4) (x + 6) (x + 8) ..... (x + 98). The number of integers x for which f(x) is less than 0 are: | Click here for video solution |
| 3 | How many integer values of x will satisfy this $(x^2 - 16\|x\| + 60) / (x^2 - 14x + 49) < 0$ | Click here for video solution |
| 4 | For how many positive integral values of k is (k-11)(k-15)(k − 19)... (k – 99) < 0? | Click here for video solution |
| 5 | The number of integers n that satisfy the inequalities $\|n - 60\| < \|n - 100\| < \|n - 20\|$ is: (CAT 2021) | Click here for video solution |
| 6 | Consider the pair of equations: $x^2 - xy - x = 22$ and $y^2 - xy + y = 34$. If $x > y$, then $x - y$ equals (CAT 2021)<br><br>1. 8<br>2. 6<br>3. 4<br>4. 7 | Click here for video solution |
| 7 | $f(x) = (x^2 + 2x - 15) / (x^2 - 7x - 18)$ is negative if and only if (CAT 2021)<br><br>1. $x < -5$ or $3 < x < 9$<br>2. $-5 < x < -2$ or $3 < x < 9$<br>3. $x < -5$ or $-2 < x < 3$<br>4. $-2 < x < 3$ or $x > 9$ | Click here for video solution |
| 8 | Let k be a constant. The equations $kx + y = 3$ and $4x + ky = 4$ have a unique solution if and only if (CAT 2020)<br><br>1. $k \neq 2$<br>2. $\|k\| = 2$<br>3. $k = 2$<br>4. $\|k\| \neq 2$ | Click here for video solution |

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# Varsha - IIM B

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## Arithmetic Progression

An arithmetic progression is a sequence of numbers in which each term is derived from the preceding term by adding or subtracting a fixed number called the common difference e.g. The sequence 9,6,3,0,-3,.... is an arithmetic progression with –3 as the common difference. The progression –3, 0, 3, 6, 9 is an Arithmetic Progression (AP) with 3 as the common difference.

**1.** The general form of an Arithmetic Progression is a, a + d, a + 2d, a + 3d and so on. Thus, $n^{th}$ term of an AP series is $T_n = a + (n – 1)\, d$. Where $T_n = n^{th}$ term and a = first term. Here d = common difference = $T_m – T_{m-1}$.

**2.** Sometimes the last term is given and either 'd' is asked or 'a' is asked. Then formula becomes $l = a + (n – 1)\, d$

**3.** The sum of first n terms of an AP: $S_n = n/2[2a+(n-1)d]$

**4.** The sum of n terms is also equal to the formula $S_n = n/2(a + l)$ where is the last term.

**5.** When three quantities are in AP, the middle one is called as the arithmetic mean of the other two. If a, b and c are three terms in AP then $b = (a + c)/2$.

## Geometric Progression

A geometric progression is a sequence in which each term is derived by multiplying or dividing the preceding term by a fixed number called the common ratio. The sequence 4, -2, 1, – 1/2 .,.. is a Geometric Progression (GP) for which – 1/2 is the common ratio.

**1.** The general form of a GP is a, ar, $ar^2$, $ar^3$ and so on.

**2.** Thus nth term of a GP series is $T_n = ar^{n-1}$, where a = first term and r = common ratio = $T_m/T_m$

**3.** The formula applied to calculate sum of first n terms of a GP: $S_n = a(r^n-1)/(r – 1)$ where $|r| > 1$ and $S_n = a(1-r^n)/(1 – r)$ where $|r| < 1$.

**4.** When three quantities are in GP, the middle one is called as the geometric mean of the other two. If a, b and c are three quantities in GP and b is the geometric mean of a and c i.e., $b = \sqrt{ac}$.

**5.** The sum of infinite terms of a GP series $S_\infty = a/(1 – r)$

| | | |
|---|---|---|
| 1 | Consider 2 APs 2,6,10..... and 5,12,19.....If S is a set containing the first 100 members of each progression then how many distinct elements are there in S? | Click here for video solution |
| 2 | Three positive integers x, y and z are in arithmetic progression. If y − x greater than 2 and xyz = 5 (x + y + z), then z − x equals: (CAT 2021)<br><br>1. 14<br>2. 10<br>3. 8<br>4. 12 | Click here for video solution |
| 3 | a + b + c + d = 4 All a,b,c,d are integers Find minimum possible value of 1/a + 1/b + 1/c + 1/d | Click here for video solution |
| 4 | If x, y and z are non – zero real numbers and $9^x = 16^y = 36^z$ Find the value of 5[xz/(xy -yz) ] | Click here for video solution |

| | | |
|---|---|---|
| 5 | If a, b, c are non-zero and $14^a = 36^b = 84^c$, then $6b(1/c - 1/a)$ is equal to: | Click here for video solution |
| 6 | How many 3 term geometric progressions can be made from the series 1, 3, $3^2$, $3^3$, ..... $3^{48}$? | Click here for video solution |

## Logarithms

**1.** Definition: $a^x = b$ can be represented in logarithmic form as $\log_a b = x$

**2.** $\log a = x$ means that $10^x = a$.

**3.** $b^{\log_b x} = x$ (The basic logarithmic identity).

**4.** $\log_a(xy) = \log_a x + \log_a y$

**5.** $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

**6.** $\log a^n = n(\log a)$ (Logarithm of a power).

**7.** $\log_x y = \frac{log_m y}{log_m x}$ (Change of base rule).

**8.** $\log_x y = \frac{1}{log_y x}$.

**9.** $\log_x 1 = 0$ $(x \neq 0, 1)$.

**10.** $\log_a b \times \log_b a = 1$

**11.** $\log_{a^b} x = \frac{1}{b}\log_a x$

**12.** The natural numbers 1, 2, 3,... are respectively the logarithms of 10, 100, 1000, ... to the base 10.

**13.** The logarithm of "0" and negative numbers are not defined.
The logarithm of a number to the base "10" is known as common logarithm and the logarithm of a number to the base "e" is known as natural logarithm.

| | | |
|---|---|---|
| 1 | If $\log_a 30 = A$, $\log_a(5/3) = -B$ and $\log_2 a = 1/3$, then $\log_3 a$ equals: (CAT 2020)<br><br>1. $2/(A + B) - 3$<br>2. $2/(A + B - 3)$<br>3. $(A + B)/2 - 3$<br>4. $(A + B - 3)/2$ | Click here for video solution |
| 2 | For a real number a , if $(\log_{15} a + \log_{32} a) / [(\log_{15} a)(\log_{32} a)] = 4$ then a must lie in range<br><br>1. $3 < a < 4$<br>2. $2 < a < 3$<br>3. $4 < a < 5$<br>4. $a > 5$ | Click here for video solution |
| 3 | If $\log_2 [3 + \log_2 \{4 + \log_4 (x - 1)\}] - 2 = 0$, then 4x equals: (CAT 2021) | Click here for video solution |
| 4 | If $5 - \log_{10} \sqrt{(1 + x)} + 4 \log_{10} \sqrt{(1 - x)} = \log_{10} (1/\sqrt{(1 - x^2)})$, then 100x equals | Click here for video solution |
| 5 | $\text{Log}_{(x+3)} (x^2 - x) < 1$ | Click here for video solution |

## Characteristic and Mantissa of Common Logarithms

The integral part of the common logarithm of a number x > 0 is called the ***Characteristic*** and the fractional part is called the ***mantissa***.
E.g., the logarithm of 2 to the base 10 is 0.3010, where 0 is the characteristic and 3010 is the mantissa.
Any positive number "x" can be written in the form $x = a10^n$, where $1 < a < 10$ and n is an integer. The number n is called the order of the number x. e.g. 30 can be written as $3*10^1$ and similarly 300 can be written $3\times10^2$. The same rule applies to fractions as well where the value of n will be negative.
The characteristic of the logarithm of the given number "x" will be "n" and the mantissa will be the logarithm of "a". Therefore, while log 2 = 0.3010, log 20 will be 1.3010 as "n" in this case is "1".
Thus, the value of the characteristic of the logarithm of a number will help determine the number of integral digits the number has = characteristic + 1.

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# Topic: Permutation and combination + Probability


https://youtu.be/90Xh0MymU94

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# Permutation & Combination

## Fundamental Principle of Counting

**Multiplication:** If there are two jobs such that one of them can be completed in $m$ ways, and when it has been completed in any one of these $m$ ways, second job can be completed in $n$ ways, then the two jobs in succession can be completed in $m \times n$ ways.
**Addition:** If there are two jobs such that they can be performed independently in $m$ and $n$ ways respectively, then either of the two jobs can be performed in $(m + n)$ ways.
**Note:** The above principles of counting can be extended to any finite number of jobs.

Permutation of n things taken r at a time, ${}^{n}P_{r} = \frac{n!}{(n-r)!}$ (Includes arrangement)

Combination of n things taken r at a time, ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$ (Includes only selection) ${}^{n}P_{r} = {}^{n}C_{r} \times r!$

Also, ${}^{n}C_{r} = {}^{n}C_{n-r}$

The total number of combinations of n distinct things, taken none or some or all at a time = $2^{n}$

The total number of combinations of n things, r taken at a time, where p things always occur = ${}^{n-p}C_{r-p}$.

The total number of combinations of n things, r taken at a time, where p things will never occur ${}^{n-p}C_{r}$.

The number of ways of dividing n things into various groups, each having p, q, r items = $\frac{n!}{p!\times q!\times r!}$

**Permutation of objects not all distinct:** The number of mutually distinguishable permutations of '$n$' things, taken all at a time, of which p are alike of one kind, $q$ are alike of second such that $p + q = n$ is $\frac{n!}{p!q!}$

If n outcomes can be repeated on r different things, total no. of permutations = $r^{n}$

Circular permutation of n things around a circle = (n – 1)!.

A deck of cards has 52 cards. There are four suits and each suit has 13 cards.
The total number of possible outcomes from a single throw of a perfect dice is 6.
The possible outcomes, of a single toss, of a fair coin are 2 – H, T.

Permutation of n things taken r at a time, in which one particular thing always occurs, is $r \times {}^{n-1}P_{r-1}$.

Circular permutation of n things, if there is no difference between the clockwise and anti-clockwise arrangement, is $\frac{(n-1)!}{2}$.

${}^{n}C_{r-1} + {}^{n}C_{r} = {}^{n+1}C_{r}$.

${}^{n}C_{r} \times {}^{r}C_{k} = {}^{n}C_{k} \times {}^{n-k}C_{r-k}$.

${}^{n}C_{0} + {}^{n}C_{1} + {}^{n}C_{2} + \ldots + {}^{n}C_{n} = 2^{n}$.

${}^{n}C_{0} + {}^{n}C_{2} + {}^{n}C_{4} + \ldots = {}^{n}C_{1} + {}^{n}C_{3} + {}^{n}C_{5} + \ldots = 2^{n-1}$.

**Division of items into groups of unequal sizes**: *Number of ways in which (m + n) items can be divided into two unequal groups containing 'm' and 'n' items is* $\frac{(m+n)!}{m!n!}$.

***Note*:** The number of ways in which $(m + n)$ items are divided into two groups containing '$m$' and '$n$' items is same as the number of combinations of $(m + n)$ things. Thus, the required number = ${}^{m+n}C_{m} = \frac{(m+n)!}{m!n!}$.

***Note*:** The number of ways of dividing $(m + n + p)$ items among 3 groups of size $m$, $n$ and $p$ respectively = (Number of ways to divide) = $\frac{(m+n+p)!}{m!n!p!}$

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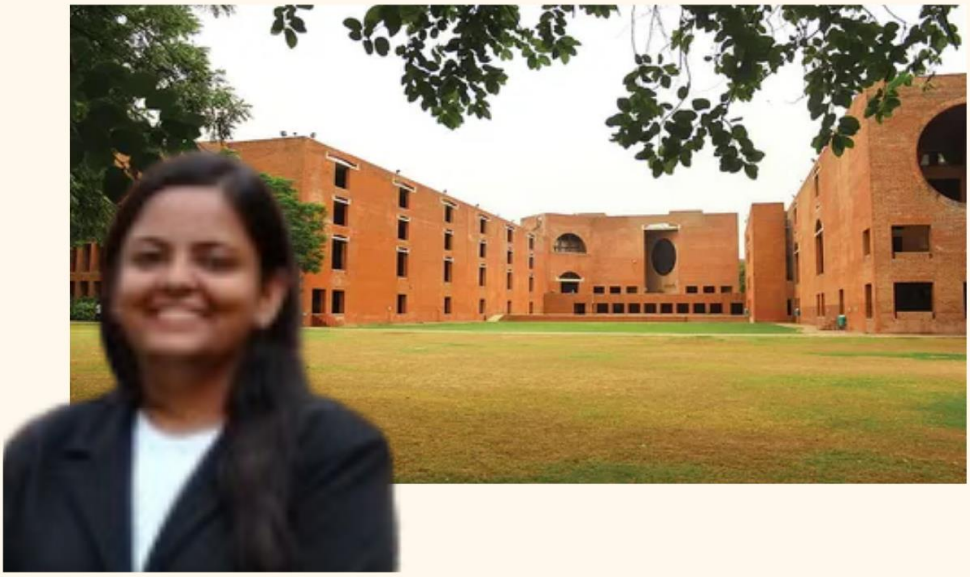
# Anshika - IIM A



**Anisha made it to**
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***Note*:** The number of ways in which *mn* different items can be divided equally into m groups each containing n objects and the order of group is important is $\left\{\frac{(mn)\,!}{(n!)^m} \times \frac{1}{m!}\right\} m! = \frac{(mn)\,!}{(n!)^m}$.
*Note*: The number of ways in which (*mn*) different items can be divided equally into $m$ groups each containing n objects and the order of groups is not important is $\left[\frac{(mn)\,!}{(n!)^m}\right]\frac{1}{m!}$.

The number of non-negative solutions to the equation, $x_1 + x_2 + \ldots + x_r = n$ is ${}^{n+r-1}C_{r-1}$.

The number of positive solutions to the equation, $x_1 + x_2 + \ldots + x_r = n$ is ${}^{n-1}C_{r-1}$.

**Derangement:** If n distinct items are arranged, the number of ways they can be arranged so that they do not occupy their intended spot is:
$D = n!\,[1 - 1/1! + 1/2! - 1/3! + \ldots\ldots (-1)^n/n!]$

| | | |
|---|---|---|
| 1 | In how many ways 10 identical balls be distributed among 5 kids such that all of them get at least one ball and no one receives more than 4 balls? | Click here for video solution |
| 2 | There are 15 identical chairs placed in a row. In how many ways can 4 persons bers seated, each person occupying 1 chair, such that there are atleast 2 empty chairs between any two persons | Click here for video solution |
| 3 | The number of groups of three or more distinct numbers that can be chosen from 1,2, 3, 4, 5, 6, 7 and 8 so that the groups always include 3 and 5, while 7 and 8 are never included together is (CAT 2021) | Click here for video solution |
| 4 | The number of ways of distributing 15 identical balloons, 6 identical pencils and 3 identical erasers among 3 children, such that each child gets at least four balloons and one pencil, is: (CAT 2021) | Click here for video solution |
| 5 | A four-digit number is formed by using only the digits 1, 2 and 3 such that both 2and 3 appear at least once. The number of all such four-digit numbers is: | Click here for video solution |
| 6 | How many integers in the set {100, 101, 102, ..., 999} have at least one digit repeated? (CAT 2020) | Click here for video solution |
| 7 | Find the number of ways in which 179 identical oranges can be distributed into 18 identical boxes such that each box contain different number of oranges not more than 20 and no box is empty? | Click here for video solution |
| 8 | In how many ways can we select 3 different numbers from 1 to 20 such that their sum is divisible by 3? | Click here for video solution |
| 9 | How many 6 digit numbers are there, having exactly 4 distinct digits? | Click here for video solution |
| 10 | In how many ways can 10 couples play a mixed doubles match such that no couple can be in the same team? | Click here for video solution |

# Probability

**1.** Probability = $\frac{Favorable\ Cases}{Total\ Cases}$.
**2.** $0 \leq$ Probability $\leq 1$
**3.** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
**4.** If events A and B are mutually exclusive, then $P(A \cup B) = P(A) + P(B)$
**5.** Probability of an event A not happening = $P(A^c) = 1 - P(A)$
**6.** If A and B are Independent Events, then $P(A \cap B) = P(A) \times P(B)$

**7.** In case of experiments with only 2 outcomes possible (tossing a coin, passing or failing, hitting or not hitting a target etc.), the probability of getting r successes in n trials ($n \geq r$) is ${}^nC_r \times p^r \times q^{n-r}$, where p is the probability of success and q is the probability of failure.

**8.** $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$.

**9.** $P(A \cap B) = P(A) \times P(B)$, if A and B are independent events.

**10.** $P(A \cap B) = 0$, if A and B are mutually exclusive events.

**Mutually Exclusive Events:** Let $S$ be the sample space associated with a random experiment and let $A_1$ and $A_2$ be two events. Then $A_1$ and $A_2$ are mutually exclusive if $A_1 \cap A_2 = \phi$.

**Note 1:** If $A$ and $B$ are two events associated with a random experiment, then
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

**Note 2:** If $A$ and $B$ are mutually exclusive events, then $P(A \cap B) = 0$, therefore $P(A \cup B) = P(A) + P(B)$.

**Note 3:** If $A$, $B$, $C$ are three events associated with a random experiment, then
$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$.

**Note 4:** If $A$ & $B$ are two events associated with a random experiment, then
(i) $P(\overline{A} \cap B) = P(B) - P(A \cap B)$ (ii) $P(A \cap \overline{B}) = P(A) - P(A \cap B)$

Let $A$ and $B$ be two events associated with a random experiment. Then the probability of occurrence of $A$ under the condition that $B$ has already occurred and $P(B) \neq 0$ is called conditional probability and it is denoted by $P(\frac{A}{B})$.

Thus $P(\frac{A}{B})$ = Probability of occurrence of $A$ under the condition that $B$ has already occurred.

$P(\frac{B}{A})$ = Probability of occurrence of $B$ under the condition that $A$ has already occurred.

**Note:** If $A$ & $B$ are two events associated with a random experiment, then $P(A \cap B) = P(A).P(\frac{B}{A})$ if $P(A) \neq 0$ or $P(A \cap B) = P(B) \ . \ P(\frac{A}{B})$ if $P(B) \neq 0$

**Note 2:** If $A_1$, $A_2$, $A_3$, _ _ _ _ $A_n$ are $n$ events related to a random experiment, then $P(A_1 \cap A_2 \cap A_3$ _ _ _ $A_n) = P(A_1) \ . \ P(\frac{A_2}{A_1}) \ . \ P(\frac{A_3}{A_1 \cap A_2}$ _ _ _ _ $P\left(\frac{A_n}{A_1 \cap A_2 \_ A_{n-1}}\right)$

**Total Probability:** Let $S$ be the sample space and let $E_1$, $E_2$, _ _ _ _ $E_n$ be $n$ mutually exclusive and exhaustive events associated with a random experiment. If $A$ is any event which occurs with $E_1$ or $E_2$ or _ _ _ _ _ _ _ $E_n$ then $P(A) = P(E_1).\ P(\frac{A}{E_1}) + P(E_2) \ . \ P(\frac{A}{E_2}) +$ _ _ _ _ _ $+ P(E_n) \ . \ P(\frac{A}{E_n})$.

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# Alirpiya - MDI Gurgaon

## Alirpiya made it to MDI Gurgaon with Quantifiers CAT Academy. So can you

**Baye's Rule:** Let $S$ be the sample space and let $E_1$, $E_2$, _ _ _ _ _ $E_n$ be $n$ mutually exclusive and exhaustive events associated with a random experiment. If $A$ is any event which occurs with $E_1$ or $E_2$, _ _ _ _ or $E_n$, then $P\left(\frac{E_2}{A}\right) = \frac{P(E_i).P(A/E_i)}{\sum_{i=1}^{n} P(E_i).P(A/E_i)}$

**11.** P (A) + P ($A^C$) = 1.

**12.** P ($A^C \cap B^C$) = P [$(A \cup B)^C$] = 1 - P(A ∪ B).

**13.** P ($A^C \cup B^C$) = P [$(A \cap B)^C$] = 1 - P(A ∩ B).

**14.** If probability of an event in defined as the odds against or odds in favour then the logic to solve this is as follows. If the odd in favour of happening of an event are given to be in the ratio of p: q. Then the probability then the event will happen is $\frac{p}{p+q}$.

**15.** If the odds against happening of an event is given to be in the ratio of r: s, then the probability that the event will happen is $\frac{s}{r+s}$. Hence it is to be understood clearly what is being given in the question.

| | | |
|---|---|---|
| 1 | I have five 10-rupee notes, three 20-rupee notes and two 50-rupee notes in my wallet. If three notes were taken out randomly and simultaneously, what is the probability that at least 90 rupees were taken out? | Click here for video solution |
| 2 | You are having 60 kinds of stone out of which exactly two are identical. The stones are transferred into three bags A, B and C equally. What would be the probability that the same Easier stones will be kept in the same bag? | Click here for video solution |

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# Topic: Number System


https://youtu.be/c2kmPJoyuWY

**How to apply the formula and basics concepts of the topic is covered in this video.**



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# Number System

## Types of Numbers

Following are different types of numbers which we generally use in our calculations.

**Natural Numbers**: The numbers 1,2,3,4.... are called natural numbers or positive integers.

**Whole Numbers:** The numbers 0,1,2,3.... are called whole numbers. Whole numbers include "0". Every Natural number is a whole number but the converse is not true.

**Integers**: The numbers .... -3, -2, -1, 0, 1, 2, 3,....are called integers. Every Whole number is an integer but the converse is not true.

**Negative Integers**: The numbers -1, -2, -3, .. are called negative integers.

**Rational Numbers:** Any number which is a positive or negative integer or fraction, or zero is called a rational number. A rational number is one which can be expressed in the following format $\Rightarrow \frac{a}{b}$, where b ≠ 0 and a & b are positive or negative integers. Every Integer is a rational number but the converse is not true.

**Positive Fractions:** The numbers $\frac{2}{3}, \frac{4}{5}, \frac{7}{8}$... are called positive fractions.

**Negative Fractions:** The numbers $-\frac{6}{8}, -\frac{7}{19}, -\frac{12}{47}$... are called negative fractions.

**Note: Between any two different rational numbers *a* & *b* there always exists a rational number calculated by taking the average of *a* and *b* i.e., $\frac{a+b}{2}$**

**Irrational Numbers:** A non-terminating non-recurring decimal number is known as an irrational number. These numbers cannot be expressed in the form of a proper fraction a/b where b ≠ 0. e.g., $\sqrt{2}$, $\sqrt{5}$, π, etc.

**Even Numbers:** The numbers which are divisible by 2 are called even numbers e.g. -4, 0, 2, 16 etc.

**Odd Numbers:** The numbers which are not divisible by 2 are odd numbers e.g. -7, -15, 5, 9 etc.

**Prime Numbers:** Those numbers, which are divisible only by themselves and 1, are called prime numbers. In other words, a number, which has exactly two factors - 1 and itself, is called a prime number. e.g. 2, 3, 5, 7, etc.
*2 is the only even prime number.*
*There are 25 prime numbers up to 100. These are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 & 97. These should be learnt by heart.*

**Composite Number:** A number, which has more than two factors, is called a composite number. e.g. 9, 10, 15, 16, ....
A composite number can be expressed as the product of prime numbers in a unique way. e.g. $100 = 2^2 \times 5^2$

***1 is neither a composite number nor a prime number.***

**Real Numbers:** The above sets of natural numbers, integers, whole numbers, rational numbers and irrational numbers constitute the set of real numbers. Every real number can be represented by a point on a number line.

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# Niharika - MDI Gurgaon

## Niharika made it to MDI Gurgaon with Quantifiers CAT Academy So can you!!!

**Perfect Numbers:** If the sum of all the factors of a number excluding the number itself happens to be equal to the number, then the number is called as perfect number. 6 is the first perfect number. The factors of 6 are 1, 2, 3 & 6. Leaving 6 the sum of other factors of 6 are equal to 6. The next three perfect numbers after 6 are 28, 496 and 8128.

## Operations on Odd & Even Numbers

**1.** Addition or subtraction of any two odd numbers will always result in an even number or zero. E.g. 1 + 3 = 4; 5 - 3 = 2.
**2.** Addition or subtraction of any two even numbers will always result in an even number or zero. E.g. 2 + 4 = 6; 12 - 4 = 8.
**3.** Addition or subtraction of an odd number from an even number will result in an odd number. E.g. 4 + 3 = 7; 10 - 3 = 7.
**4.** Addition or subtraction of an even number from an odd number will result in an odd number. E.g. 3 + 4 = 7; 5 - 2 = 3.
**5.** Multiplication of two odd numbers will result in an odd number. E.g. $3 \times 3 = 9$.
**6.** Multiplication of two even numbers will result in an even number. E.g. $2 \times 4 = 8$.
**7.** Multiplication of an odd number by an even number or vice versa will result in an even number. E.g. $3 \times 2 = 6$.
**8.** An odd number raised to an odd or an even power is always odd.
**9.** An even number raised to an odd or an even power is always even.
**10.** The standard form of writing a number is $m \times 10^n$ where m lies between 1 and 10 and n is an integer. e.g. $0.89713 \Rightarrow 8.9713/10^1 . \Rightarrow 8.9713 \times 10^{-1}$.
**11.** If n is odd. $n(n^2 - 1)$ is divisible by 24. e.g. take $n = 5 \Rightarrow 5(5^2 - 1) = 120$, which is divisible by 24.
**12.** If n is odd prime number except 3, then $n^2 - 1$ is divisible by 24.
**13.** If n is odd. $2^n + 1$ is divisible by 3.
**14.** If n is even. $2^n - 1$ is divisible by 3.
**15.** If n is odd, $2^{2n} + 1$ is divisible by 5.
**16.** If n is even, $2^{2n} - 1$ is divisible by 5.
**17.** If n is odd, $5^{2n} + 1$ is divisible by 13.
**18.** If n is even, $5^{2n} - 1$ is divisible by 13

## Tests of Divisibility

Divisibility rules are very helpful while doing calculations. Following are some important divisibility rules which you should learn from heart.

**1. Divisibility rule of 2** - A number is divisible by 2 when its units place is 0 or divisible by 2. e.g. 120, 138.
**2. Divisibility rule of 3** – A number is divisible by 3 when the sum of the digits of the number is divisible by e.g., 15834 is divisible by 3 as the sum of its digits is 21 which is divisible by 3. Note that if n is odd, then $2^n + 1$ is divisible by 3 and if n is even, then $2^n - 1$ is divisible by 3.
**3. Divisibility rule of 4** - A number is divisible by 4 when the last two digits of the number are 0s or are divisible by 4. As 100 is divisible by 4, it is sufficient if the divisibility test is restricted to the last two digits. e.g. 145896, 128, 18400
**4. Divisibility rule of 5** - A number is divisible by 5, if its unit's digit is 5 or 0. e.g. 895, 100
**5. Divisibility rule of 6** - A number is divisible by 6, if it is divisible by both 2 and by 3. i.e. the number should be an even number and the sum of its digits should be divisible by 3.
**6. Divisibility rule of 8** - A number is divisible by 8, if the last three digits of the number are 0s or are divisible by 8. As 1000 is divisible by 8, it is sufficient if the divisibility test is restricted to the last three digits e.g. 135128, 45000
**7. Divisibility rule of 9** - A number is divisible by 9, if the sum of its digits is divisible by 9. e.g. 810, 92754
**8. Divisibility rule of 11** - A number is divisible by 11, if the difference between the sum of the digits at odd places and sum of the digits at even places of the number is either 0 or a

multiple of 11. e.g., 121, 65967. In the first case 1+1 - 2 = 0. In the second case 6+9+7 = 22 and 5+6 = 11 and the difference is 11. Therefore, both these numbers are divisible by 11.

9. **Divisibility rule of 12** - A number is divisible by 12, if it both divisible by 3 and by 4. i.e., the sum of the digits should be divisible by 3 and the last two digits should be divisible by 4. e.g. 144, 8136.
10. **Divisibility rule of 15 –** A number is divisible by 15, if it is divisible by both 5 and 3.

11. **Divisibility rule of 25** – 2358975 is divisible by 25 if the last two digits of 2358975 are divisible by 25 or the last two digits are 0.
12. **Divisibility rule of 75** - A number is divisible by 75, if it is both divisible by 3 and by 25. i.e., the sum of the digits should be divisible by 3 and the last two digits should be divisible by 25.
13. **Divisibility rule of 125 -** A number is divisible by 125, if its last three right hand digits are divisible by 125 or the last three digits are 0s. e.g. 1254375, 12000.

## Properties of the numbers

Following are some properties of the numbers which are helpful in solving the questions in the examination.

**1.** The sum of 5 successive whole numbers is always divisible by 5.
**2.** The product of 3 consecutive natural numbers is divisible by 6.
**3.** The product of 3 consecutive natural numbers, the first of which is an even number is divisible by 24.
**4.** The sum of a two-digit number and a number formed by reversing its digits is divisible by 11. E.g. 28 + 82 = 110, which is divisible by 11. At the same time, the difference between those numbers will be divisible by 9. e.g. 82 – 28 = 54, which is divisible by 9.
**5.** $\Sigma n = \frac{n(n+1)}{2}$, $\Sigma n$ is the sum of first $n$ natural numbers.
**6.** $\Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$, $\Sigma n^2$ is the sum of first $n$ perfect squares.
**7.** $\Sigma n^3 = \frac{n^2(n+1)^2}{4} = (\Sigma n)^2$, $\Sigma n^3$ is the sum of first n perfect cubes.
**8.** $x^n + y^n = (x + y)(x^{n-1} - x^{n-2}.y + x^{n-3}.y^2 - \ldots + y^{n-1})$ when $n$ is odd. Therefore, when n is odd, $x^n + y^n$ is divisible by $x + y$. E.g., $3^3 + 2^3 = 35$ and is divisible by 5.
**9.** $x^n - y^n = (x + y)(x^{n-1} - x^{n-2}.y + \ldots y^{n-1})$ when $n$ is even. Therefore, when $n$ is even, $x^n - y^n$ is divisible by $x + y$. e.g., $7^2 - 3^2 = 40$, which is divisible by 10.
**10.** $x^n - y^n = (x - y)(x^{n-1} + x^{n-2}.y + \ldots + y^{n-1})$ for both odd and even $n$. Therefore, $x^n - y^n$ is divisible by $x - y$. e.g., $9^4 - 2^4 = 6545$ which is divisible by 7.

## LCM & HCF

1. In case of HCF, if some remainders are given, then first those remainders are subtracted from the numbers given and then their HCF is calculated.
2. Sometimes in case of HCF questions, the same remainder is required is given and the remainder is not given. In such questions, the answer is the HCF of the difference of the numbers taken in pairs.
3. In case of LCM, if a single remainder is given, then firstly the LCM is calculated and then that single reminder is added in that.
4. In case of LCM, if for different numbers different remainders are given, then the difference between the number and its respective remainder will be equal. In that case, firstly the LCM is calculated, then that common difference between the number and its respective remainder is subtracted from that.

***LCM and HCF of Fractions***

**LCM of fractions** $= \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$, **e.g., LCM of** $\frac{3}{4}$ **and** $\frac{1}{2} = \frac{3\ (\text{LCM of numerators})}{2\ (\text{HCF of denominators})}$

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# Riya - IIM Rh

## Riya made it to IIM Rohtak with Quantifiers CAT Academy So can you!!!!

**HCF of fractions** = $\frac{\text{HCF of numerators}}{\text{LCM of denominators}}$, **e.g. HCF of** $\frac{3}{4}$ **and** $\frac{1}{2}$ = $\frac{1\ (\text{HCF of numerators})}{4\ (\text{LCM of denominators})}$

**Note that the product of the two fractions is always equal to the product of LCM and HCF of the two fractions.**

**The product of the two fractions** $= \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$

**The product of the LCM and HCF** = $\frac{3}{2} \times \frac{1}{4} = \frac{3}{8}$

## Fractions

1. Numbers of the form 3/4, 4/5, are called fractions. A fraction can be written as p/q where $q \neq 0$.
2. If the numerator and denominator of a fraction are multiplied / divided by the same number then the value of the fraction does not change.
3. For any positive proper fraction p/q (p<q), the value of the fraction increases when both the denominator and numerator are added by the same positive number.
   E.g., 3/4 = 0.75, (3 + 1)/(4 + 1) = 4/5 = 0.8.
4. For any positive proper fraction p/q (p<q), the value of the fraction decreases when both the numerator and denominator are subtracted by the same positive number.
   e.g., 3/4 = 0.75, (3 – 1)/(4 – 1) = 2/3 = 0.67
5. For any positive improper fraction p/q (p>q), the value of the fraction decreases when both the numerator and the denominator are added to the same positive. E.g., 5/4 = 1.25, adding 1 to the numerator and the denominator, we get (5 + 1)/(4 + 1) = 6/5 = 1.2, which is less than 1.25.
6. For any positive improper fraction p/q (p>q), the value of the fraction increases when both the numerator and denominator are subtracted by the same positive number. E.g., 5/4 = 1.25, by subtracting 1 from both the numerator and denominator we get, (5 – 1)/(4 – 1) = 4/3 = 1.33 > 1.25.

## Types of fractions

Common Fractions: Fractions such 3/4, 32/43 etc are called common or vulgar fractions.
Decimal Fractions: Fractions whose denominators are 10, 100, 1000, ... are called decimal fractions.

**Proper Fraction: A fraction whose numerator is less than its denominator is known as a proper fraction e.g., 3/4.**

Improper Fraction: A fraction whose numerator is greater than its denominator is known as an improper fraction. e.g., 4/3
Mixed Fractions: Fractions which consists of an integral part and a fractional part are called mixed fractions. All improper fractions can be expressed as mixed fractions and vice versa. e.g., $1\frac{3}{4}$.
Recurring Decimals: A decimal in which a set of figures is repeated continually is called a recurring or periodic or a circulating decimal. e.g., 1/7 = 0.142857……..

## Indices

The expression $a^5 = a \times a \times a \times a \times a$
Similarly for any positive integer $n$, $a^n = a \times a \times a \times \ldots..$ $n$ times.
In $a^n$, $a$ is called the base and $n$ is called the index.

***Law of Indices***

Let m and n be positive integers, then

1. $a^m \times a^n = a^{m+n}$
2. $(a^m)^n = a^{mn}$
3. $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$
4. $(ab)^m = a^m \times b^m$

**5.** $a^0 = 1$, where $a \neq 0$

**6.** $a^{-n} = \frac{1}{a^n}$

**7.** $a^{\frac{p}{q}} = \sqrt[q]{a^p}$ i.e., the $q^{th}$ root of $a$ raised to the power of $p$. In particular, $a^{\frac{1}{q}} = \sqrt[q]{a}$

## Remainder Theory

**The basic remainder formula is:**
**Dividend = Divisor × Quotient + Remainder**

If remainder = 0, then it the number is perfectly divisible by divisor and divisor is a factor of the number e.g. when 8 divides 40, the remainder is 0, it can be said that 8 is a factor of 40. There are few important results relating to numbers. Those will be covered one by one in the following examples.

**1. Formulas Based Concepts for Remainder:**

($a^n + b^n$) is divisible by (a + b), when n is odd.
($a^n - b^n$) is divisible by (a + b), when n is even.
($a^n - b^n$) is always divisible by (a - b), for every n.

**2. Concept of Negative Remainder:**
By definition, remainder cannot be negative. But in certain cases, you can assume that for your convenience. But a negative remainder in real sense means that you need to add the divisor in the negative remainder to find the real remainder.

**NOTE:** Whenever you are getting a negative number as the remainder, make it positive by adding the divisor to the negative remainder.

**3. Cyclicity in Remainders:**
Cyclicity is the property of remainders, due to which they start repeating themselves after a certain point.

**4.** Euler Number: Euler number of a number N is the number of numbers less than N and co-prime to N.
Let $N = p^a \times q^b \times r^c \times \ldots$ where p, q, r are distinct prime numbers then the Euler number of N is given by $E(N) = N \times (1 - 1/p)\ (1 - 1/q) \times (1 - 1/r) \times \ldots$.

**5. Role of Euler's Number in Remainders:**
Euler's Remainder theorem states that, for co-prime numbers M and N, Remainder $[M^{E(N)} / N] = 1$, i.e., number M raised to Euler number of N will leave a remainder 1 when divided by N. Always check whether the numbers are co-primes are not as Euler's theorem is applicable only for co-prime numbers.

**6. Wilson's Remainder Theorem** states that for any prime number p, remainder when (p – 1)! is divided by p is p – 1. Further, we can say that (p – 2)!/p will leave remainder 1.

| | | |
|---|---|---|
| 1 | When 1313 is divided by a positive integer x, the remainder is 17. How many values of x are possible? | Click here for video solution |
| 2 | N is a number having exactly 4 factors and is not a perfect cube. Sum of all it's factors excluding the number itself is 2014. Find N. | Click here for video solution |
| 3 | Using all the digits 0,1,2,3,4,5,6 exactly once, Find the greatest number divisible by 55 | Click here for video solution |
| 4 | If a,b,c and d are integers, where a+b+c+d=4 then find the minimum value of 1/a+1/b+1/c+1/d. | Click here for video solution |
| 5 | If $N = 364 \times n^5$, where N is a positive integer, has 182 factors, how many possible values N can assume? | Click here for video solution |
| 6 | How many three-digit numbers are greater than 100 and increase by 198 when the three digits are arranged in the reverse order? | Click here for video solution |

| | | |
|---|---|---|
| 7 | For a 4-digit number, the sum of its digits in the thousands, hundreds and tens places is 14, the sum of its digits in the hundreds, tens and units places is 15, and the tens place digit is 4 more than the units place digit. Then the highest possible 4- digit number satisfying the above conditions is: | Click here for video solution |
| 8 | For all possible integers n satisfying $2.25 \le 2 + 2^{n+2} \le 202$, the number of integer values of $3 + 3^{n+1}$ is: | Click here for video solution |
| 9 | If n is a positive integer such that $(\sqrt[7]{10})\ (\sqrt[7]{10})^2 \ldots\ldots (\sqrt[7]{10})^n$ is greater than 999, then the smallest value of n is | Click here for video solution |
| 10 | Let N, x and y be positive integers such that N = x + y, $2 < x < 10$ and $14 < y < 23$. If N > 25, then how many distinct values are possible for N? | Click here for video solution |
| 11 | If a, b, c are non-zero and $14^a = 36^b = 84^c$, then $6b(1/c - 1/a)$ is equal to: | Click here for video solution |
| 12 | Let m and n be natural numbers such that n is even and 0, 2 less than m/20, n/m, n/11 less than 0.5.<br>Then m – 2n equals:<br>1. 2<br>2. 4<br>3. 3<br>4. 1 | Click here for video solution |
| 13 | How many of the integers 1, 2, ....., 120, are divisible by none of 2, 5 and 7?<br><br>1. 40<br>2. 42<br>3. 41<br>4. 43 | Click here for video solution |
| 14 | $25^{21} - 9^{21}$ is not divisible by which one of the following :<br><br>1. 16<br>2. 98<br>3. 544<br>4. 14896 | Click here for video solution |
| 15 | ABCD is a 4 digit number having all distinct digits such that AB is a 2 digit prime number, CD is also a 2 digit -prime number and BC is a 2 digit perfect square. How many such numbers are possible? | Click here for video solution |
| 16 | Let X be a four-digit number with exactly three consecutive digits being the same and is a multiple of 9. How many such values of X are possible? | Click here for video solution |

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# Topic: Geometry and Mensuration


https://youtu.be/osz2EHN3g2k

**How to apply the formula and basics concepts of the topic is covered in this video.**



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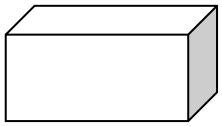
# Geometry & Mensuration

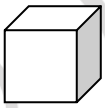
## Rectangular Solids and Cylinders

**Cuboid:** A cuboid is a three-dimensional figure formed by six rectangular surfaces, as shown below. Each rectangular surface is a face. Each solid line segment is an edge, and each point at which the edges meet is a vertex. A rectangular solid has six faces, twelve edges, and eight vertices. Edges mean sides and vertices mean corners. Opposite faces are parallel rectangles that have the same dimensions.
The surface area of a rectangular solid is equal to the sum of the areas of all the faces. The volume is equal to (length) × (width) × (height); in other words, (area of base) × (height).



Body diagonal of a cuboid = Length of the longest rod that can be kept inside a rectangular room is $=\sqrt{L^2+B^2+H^2}$.

**Cube:** A rectangular solid in which all edges are of equal length is a cube. In a cube, just like cuboid, there are six faces, eight vertices & twelve edges.
Volume = $a^3$.



Surface Area = $6a^2$, where $a$ is the side of a cube.
Body Diagonal = Length of the longest rod inside a cubical room = $a\sqrt{3}$

**Right Prism:**
A prism is a solid, whose vertical faces are rectangular and whose bases are parallel polygons of equal area.
A prism is said to be triangular prism, pentagonal prism, hexagonal prism, octagonal prism according to the number of sides of the polygon that form the base.
In a prism with a base of $n$ sides, number of vertices = $2n$, number of faces = $n + 2$.
Surface area of vertical faces of a prism = perimeter of base × height.
Total surface area of a prism = perimeter of base × height + 2 × area of base
Volume of a prism = area of base × height

**Cylinder:** The **surface area** of a right circular cylinder with a base of **radius '$r$'** and **height '$h$'** is equal to $2(\pi r^2) + 2\pi rh$ (the sum of the areas of the two bases plus the area of the curved surface).
The **volume** of a cylinder is equal to $\pi r^2 h$, that is (area of base) × (height).

**Cone:** A cone is having one circle on one of its endings & rest is the curved circle part with a corner on the other end.
**Volume** = $1/3\ \pi r^2 h$ and Surface Area (curved) = $\pi\ r\ l$, where $l$ = slant height.
As per the Pythagoras theorem, $l^2 = r^2 + h^2$.
Surface Area (total) = $\pi\ r\ l + \pi r^2$.
**Frustum of a cone:** A frustum is the lower part of a cone, containing the base, when it is cut by a plane parallel to the base of the cone.
**Slant height**, $L = \sqrt{h^2 + (R-r)^2}$
**Curved Surface area of cone** = $\pi\ (R + r)\ L$.

**Total surface area of frustum** = Base area + Area of upper circle + Area of lateral surface = $\pi\ (R^2 + r^2 + RL + rL)$.

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**Volume of frustum** = $\frac{\pi h}{3}(R^2 + r^2 + Rr)$

**Sphere:** The set of all points in space, which are at a fixed distance from a fixed point, is called a sphere. The fixed point is the centre of the sphere and the fixed distance is the radius of the sphere.
**Volume** = $4/3\pi r^3$. **Surface Area (curved and total)** = $4\ \pi r^2$.

**Hemisphere:** A sphere cut by a plane passing through its centre forms two hemispheres. The upper surface of a hemisphere is a circular region.
**Volume** = $2/3\pi r^3$. Surface Area (curved) =$2\pi r^2$.
**Surface Area (Total)** = $2\pi r^2 + \pi r^2 \Rightarrow 3\pi r^2$.

**Spherical shell:** If $R$ and $r$ are the outer and inner radius of a hollow sphere, then volume of material in a spherical shell = $4/3\pi\ (R^3 - r^3)$.

**Solid ring:** If $R$ and $r$ are the outer and inner radius of a solid ring (can be considered as a cylindrical rod joined end to end), then
**Volume** = $\frac{\pi^2}{4}(R-r)^2(R+r)$.
**Curved Surface area** = $\pi^2(R^2 - r^2)$.

**Pyramid:**

A pyramid is a solid, whose lateral faces are triangular with a common vertex and whose base is a polygon. A pyramid is said to be tetrahedron (triangular base), square pyramid, hexagonal pyramid etc, according to the number of sides of the polygon that form the base.

In a pyramid with a base of $n$ sides, number of vertices = $n$ + 1. Number of faces including the base = $n$ + 1.

**Surface area of lateral faces** = $1/2$ × perimeter of base × slant height

**Total surface area of pyramid** = Base area + ½ × perimeter of base × slant height

**Volume of pyramid** = $1/3$ × Base area × height. A cone is also a pyramid.

## Lines and Angles

**Some important points:**

**(i)** There is one and only one line passing through two distinct points.
**(ii)** Three or more points are said to be collinear if they lie on a line, otherwise they are said to be non-collinear.

**(iii)** Two or more lines are said to be coplanar if they lie in the same plane, otherwise they are said to be non-coplanar.
**(iv)** A line, which is perpendicular to a line segment i.e., intersects at 90° and passes through the midpoint of the segment is called he perpendicular bisector of the segment.

**(v)** Every point on the perpendicular bisector of a segment is equidistant from the two endpoints of the segment.
Conversely, if any point is equidistant from the two endpoints of the segment, then it must lie on the perpendicular bisector of the segment.
If PO is the perpendicular bisector of segment AB, then, AP = PB.
Also, if AP = PB, then P lies on the perpendicular bisector of segment AB.

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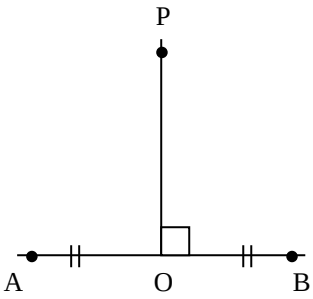
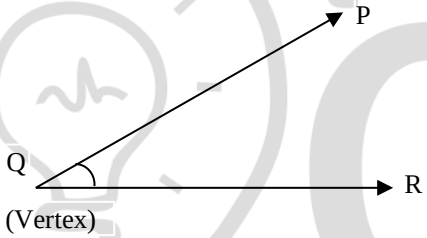



**(vi)** The ratio of intercepts made by three parallel lines on a transversal is equal to the ratio of the corresponding intercepts made on any other transversal by the same parallel lines.

## Angles:

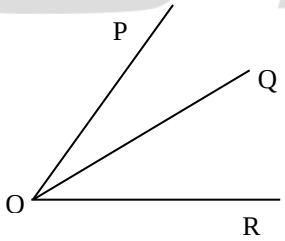
When two rays have the same starting or end points, they form an angle and the common end point is called vertex.
Angle is also defined as the measure of rotation of a ray. For the purpose of Trigonometry, the measure of rotation is termed positive if it is in the anticlockwise direction.




In the figure, ray PQ and QR from angle∠PQR

## Types of Angles:

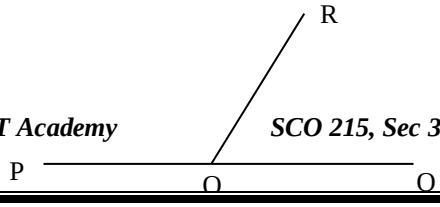
### (i) Adjacent Angles:




$\angle$ POQ and $\angle$ QOR are called adjacent angles, because they have a common side and their interiors are disjoint.

### (ii) Linear Pair:

$\angle$ POR and $\angle$ ROQ are said to form a linear pair because they have a common side and other two sides are opposite rays $\angle$ POR+$\angle$ ROQ =$180^0$
$\Leftrightarrow$ $\angle$ POR and $\angle$ POQ form a linear pair



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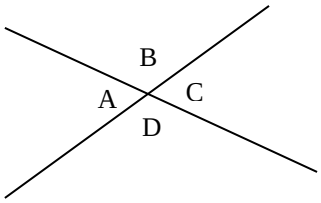
**(iii)** An angle greater than 180°, but less than 360° is called a reflex angle.
**(iv)** Two angles whose sum is 90° are called complementary angles.
**(v)** Two angles having a sum of 180° are called supplementary angles.
**(vi)** When two lines intersect, two pairs of vertically opposite angles are equal. The sum of 2 adjacent angles is 180°.




As given in the above diagram ∠A = ∠C & ∠B = ∠D.
Secondly ∠A + ∠B =180° & ∠C + ∠D = 180°.

**Two lines are parallel to each other if**

**1.** They are parallel to a 3rd line.
**2.** They are opposite sides of a rectangle/ square/ rhombus/ parallelogram.
**3.** If they are perpendicular to a 3rd line.
**4.** If one of them is a side of the triangle & other joins the midpoints of the remaining two sides.
**5.** If one of them is a side of a triangle & other divides other 2 sides proportionately.

**Two lines are perpendicular to each other if**

**1.** They are arms of a right-angle triangle.
**2.** If the adjacent angles formed by them are equal and supplementary.
**3.** They are adjacent sides of a rectangle or a square.
**4.** If they are diagonals of a rhombus.
**5.** If one of them is a tangent & other is radius of the circle through the point of contact.
**6.** If the sum of their squares is equal to the square of line joining their ends.

**Two angles are said to be equal if**

**1.** They are vertically opposite angles.
**2.** Their arms are parallel to each other.
**3.** They are the corresponding angles of two congruent triangles.
**4.** They are the opposite angles of a parallelogram.
**5.** They are the angles of an equilateral triangle.
**6.** They are the angles of a regular polygon.
**7.** They are in same segment of a circle.
**8.** One of them lies between a tangent & a chord thorough the point of contact & other is in the alternate segment, in a circle.

**Two sides are equal to each other if**

**1.** They are corresponding sides of two congruent triangles.
**2.** They are sides of an equilateral triangle.
**3.** They are opposite sides of a parallelogram.
**4.** They are the sides of a regular polygon.
**5.** They are radii of the same circle.
**6.** They are chords equidistant from centre of circle.
**7.** They are tangents to a circle from an external point.

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# Triangles

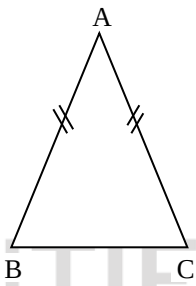
## Types of Triangles

With regard to their sides, triangles are of three types:

**(i) Scalene Triangle:** A triangle in which none of the three sides is equal is called a scalene triangle.

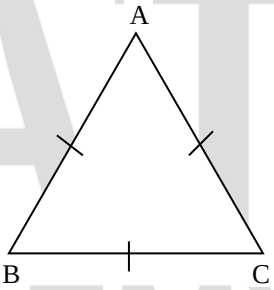
**(ii) Isosceles Triangle:** A triangle in which at least two sides are equal is called an isosceles triangle. In an isosceles triangle, the angles opposite to the congruent sides are congruent. Conversely, if two angles of a triangle are congruent, then the sides opposite to them are congruent.

In Δ ABC, AB = AC,
∠ ABC = ∠ ACB




**(iii) Equilateral Triangle:** A triangle in which all the three sides are equal is called an equilateral triangle. In an equilateral triangle, all the angles are congruent and equal to 60°.

In Δ ABC, AB = BC = AC.
∠ ABC = ∠ BCA = ∠ CAB = 60°




With regard to their angles, triangles are of five types:

**(i) Acute triangle:** If all the three angles of a triangle are acute i.e., less than 90°, then the triangle is an acute-angled triangle.

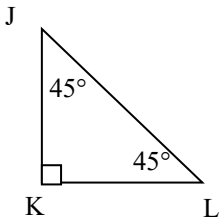
**(ii) Obtuse triangle:** If any one angle of a triangle is obtuse i.e., greater than 90°, then the triangle is an obtuse-angled triangle. The other two angles of the obtuse triangle will be acute.

**(iii) Right Triangle:** A triangle that has a right angle is a right triangle. In a right triangle, the side opposite the right angle is the hypotenuse, and the other two sides are the legs. An important theorem concerning right triangles is the Pythagorean theorem, which states: In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

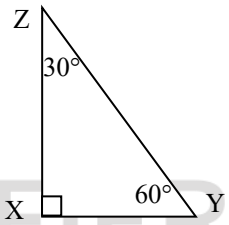
There are some standard Pythagorean triplets given below, which are repeatedly used in the questions. It is better to remember these triplets by heart.
(3, 4, 5), (5, 12, 13), (7, 24, 25), (8, 15, 17), (9, 40, 41), (11, 60, 61), (12, 35, 37), (16, 63, 65), (20, 21, 29), (28, 45, 53).

Any multiple of these triplets will also be a triplet i.e. when we say 3, 4, 5 is a triplet, if we multiply all the numbers by 2, it will also be a triplet i.e. 6, 8, 10 will also be a triplet.

**45°- 45° - 90° Triangle:** If the angles of a triangle are 45°, 45° and 90°, then the perpendicular sides are $1/\sqrt{2}$ times the hypotenuse. In a 45°- 45°- 90° triangle, the lengths of the sides are in the ratio 1: 1: √2. For example, in ΔJKL, if JL = 2, then JK = √2 and KL = √2.

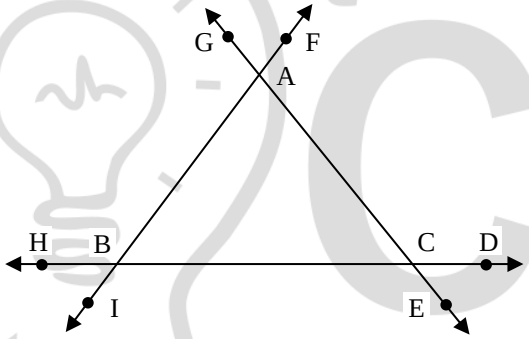



**30°- 60° - 90° Triangle:** In 30°- 60° - 90° triangle, the lengths of the sides are in the ratio 1: √3: 2. For example, in ΔXYZ, if XZ = 3, then XY = 3√3 and YZ = 6. In short, the following formulas can be applied to calculate the two sides of a 30°- 60°-90° triangle, when the third side is given.




Side opposite to 30° = 1/2 of hypotenuse.
Side opposite to 60 ° = √3/2 of hypotenuse.

**Some important properties of triangles:**




**(i)** In Δ ABC, ∠ ABC + ∠ BAC + ∠ ACB = 180°

**(ii)**The sum of an interior angle and the adjacent exterior angle is 180°.
In figure, ∠ ABC + ∠ ABH = 180°
∠ ABC + ∠ CBI = 180°

**(iii)** Two exterior angles having the same vertex are congruent. In figure, ∠ GAB ≅ ∠ FAC

**(iv)** The measure of an exterior angle is equal to the sum of the measures of the two interior angles (called remote interior angles) of the triangle, not adjacent to it.

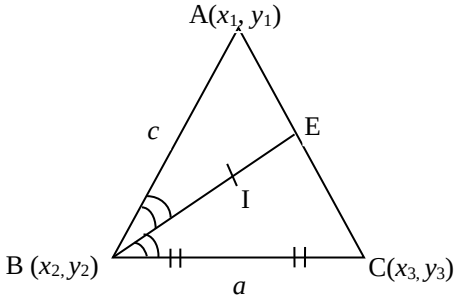
**(vi)** The sum of any two sides of a triangle is always greater than the third side.
In Δ ABC, AB + BC > AC, also AB + AC > BC and AC + BC > AB.

**(vii)** The difference of any two sides is always less than the third side.

## Area of a triangle

Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)} = r \times s = \frac{abc}{4R}$
where, a, b and c are the sides of the triangle,
s = semi perimeter, r = in-radius, R = circum-radius.

**In-center:** Point of intersection of angles bisectors of a triangle is known as the in-center of triangle. The circle drawn from this point, which touches all the three sides of the triangle, is known as in-circle & its radius is called as in-radius. The in-radius is denoted by the letter '*r*'




**(i)** If A($x_1$, $y_1$), B($x_2$, $y_2$) and C($x_3$, $y_3$) are the co-ordinates of a triangle and $a$, $b$, $c$ are the lengths of sides BC, AC and AB respectively then co-ordinates of the in-center are:

$$I = \left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$$

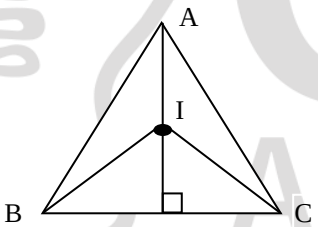
**(ii)** In centre is equidistant for all the three sides, in-circle can be drawn touching all the three sides.

The radius of this 'circle' is termed as 'in-radius'

i.e., $r = \frac{\Delta}{s} = \frac{\text{Area of the triangle}}{\text{Semi perimeter of the triangle}}$

**(iii)** $\frac{AB}{BC} = \frac{AE}{EC}$ (Angle Bisector Theorem)

**(iv)** If I is the in-centre, then $\angle BIC = 90° + \frac{1}{2}\angle A$

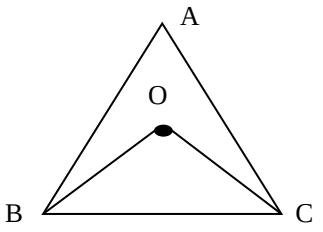



**Circumcenter:** It is the point of intersection of perpendicular side bisectors of the triangle. The circle drawn with this point as center and passing through the vertices is known as circumcircle and its radius is called as circumradius. The circumradius is denoted by the letter '*R*'.

Circumradius = $\frac{abc}{4\Delta}$

$R = \frac{\text{Product of three sides}}{4 \times \text{Area of the triangle}}$

Also $\angle BOC = 2\angle A$




**Centroid:** The segment joining a vertex & midpoint of the opposite side is called median of a triangle. There are three medians & they meet in a single point called, centroid of the triangle. The centroid divides medians in the ratio of 2: 1.

The following formula is applied to calculate the length of the median. The sum of the squares of two sides = 2[median$^2$ + (½ 3$^{rd}$ side)$^2$]

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# Sectional Tests


https://youtu.be/FtH4bZQmhMs

**How to apply the formula and basics concepts of the topic is covered in this video.**



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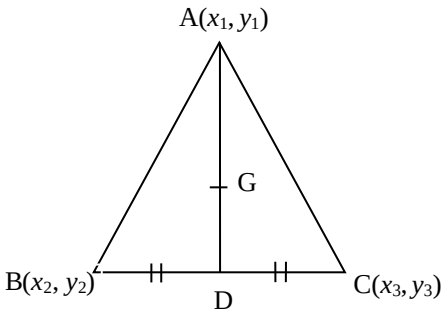
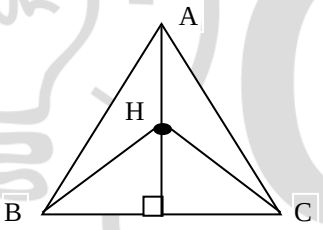



**(i)** If A($x_1$, $y_1$); B($x_2$, $y_2$); C($x_3$, $y_3$) are the co-ordinates of a triangle , then co-ordinates of centroid G are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

**(ii)** The median divides the triangle in two equal parts (not necessarily congruent)

**(iii)** The centroid divides the median in the ratio 2: 1 with the larger part towards the vertex. So, AG: GD = 2 : 1.

**Orthocenter**: The line drawn from any vertex, perpendicular to the opposite side is called the altitude/height. The three altitudes meet in a single point called the orthocenter. The angle made by any side at orthocenter is equal to (180° – vertical angle).




If Δ ABC, is an isosceles triangle with AB ≅ AC, then the angle bisector of ∠ BAC is the perpendicular bisector of the base BC and is also the median to the base.

Area of an isosceles triangle $= \frac{c}{4}\sqrt{4a^2 - c^2}$, where $c$ is the unequal side and $a$ is one of the equal sides.

The altitudes on the congruent sides are equal i.e., BE = CF.

**For an equilateral triangle,**

height $= \frac{\sqrt{3}}{2} \times side$;

$area = \frac{\sqrt{3}}{4} \times (side)^2$, inradius $= \frac{1}{3} \times$ height;

circumradius $= \frac{2}{3} \times$ height and perimeter = 3 × sides

Also, the altitude, median, angle bisector, perpendicular bisector of each base is the same and the ortho-center, centroid, in-center and circum-center is the same.

**Congruency of triangles:** If the sides and angles of one triangle are equal to the corresponding sides and angles of the other triangle, then the two triangles are said to be congruent.

**Two triangles are congruent if**

**1.** Two sides & the included angle of a triangle are respectively equal to two sides & included angle of other triangle (SAS).

**2.** angles & 1 side of a triangle are respectively equal to two angles & the corresponding side of the other triangle (AAS).
**3.** Three sides of a triangle are respectively congruent to three sides of the other triangle (SSS).
**4.** 1 side & hypotenuse of a right-triangle are respectively congruent to 1 side & hypotenuse of another rt. triangle (RHS).

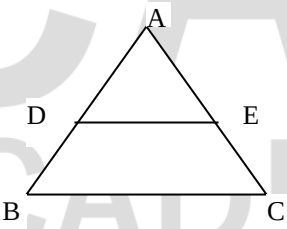
**Similarity of triangles:**
Two triangles are similar if they alike in shape only. The corresponding angles are congruent, but corresponding sides are only proportional. All congruent triangles are similar but all similar triangles are not necessarily congruent.

**Two triangles are similar if**
**1.** Three sides of a triangle are proportional to the three sides of the other triangle (SSS).
**2.** Two angles of a triangle are respectively equal to the two angles of the other triangle (AA).
**3.** Two sides of a triangle are proportional to two sides of the other triangle & the included angles are equal (SAS).

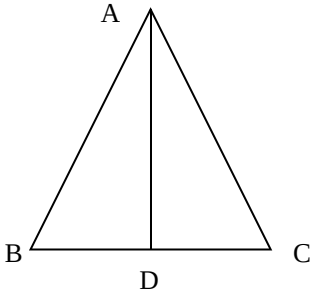
**Properties of similar triangles:**
**1.** If two triangles are similar, ratios of sides = ratio of heights = ratio of medians = ratio of angle bisectors = ratio of inradii = ratio of circumradii.
**2.** Ratio of areas = $b_1h_1/b_2h_2 = (s_1)^2/(s_2)^2$ , where $b_1$ & $h_1$ are the base & height of first triangle and $b_2$ & $h_2$ are the base & height of second triangle. $s_1$ & $s_2$ are the corresponding sides of first and second triangle respectively.

**Some important theorems:**

**(i) Basic Proportionality Theorem:** If a line is drawn parallel to one side of a triangle and intersects the other sides in two distinct points, then the other sides are divided in the same ratio by it. If DE is parallel to BC, then, $\frac{AD}{DB} = \frac{AE}{EC}$

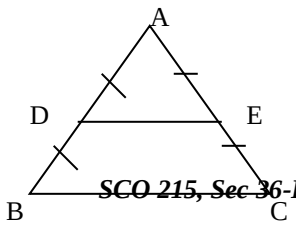



**(ii) The Angle Bisector Theorem:** The angle bisector divides the opposite side in ratio of the lengths of its adjacent arms.




If AD is the angle bisector, then AB/AC = BD/DC.

**(iii) Midpoint Theorem:** The segment joining the midpoints of any two sides of a triangle is parallel to the third side and is half of the third side.
If AD = DB, AE = EC, then DE is parallel to BC and DE = $\frac{1}{2}BC$.



# Circles

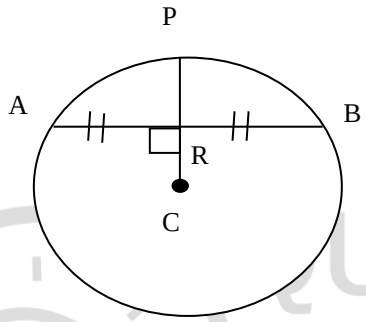
If $r$ is the radius of the circle, then the circumference = $2\pi r$,
The area of a circle of radius r is = $\pi r^2$.

**Congruent Circles:** Circles with equal radii are called congruent circles.

**Concentric Circles:** Circles lying in the same plane with a common centre are called concentric circles.

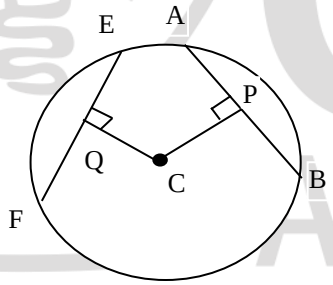
**Some Important Properties of Circles**

**1.** The perpendicular from the center of a circle to a chord of the circle bisects the chord.

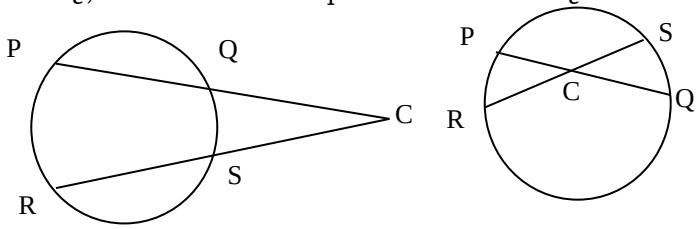



Conversely, the line joining the center of the circle and the midpoint of a chord is perpendicular to the chord.

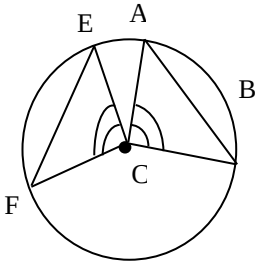
**2.** Equal chords of a circle or congruent circles are equidistant from the center.




Conversely, two chords of a circle or congruent circles that are equidistant from the center are equal.
Two chords PQ, RS intersect at a point then CP × CQ = CR× CS

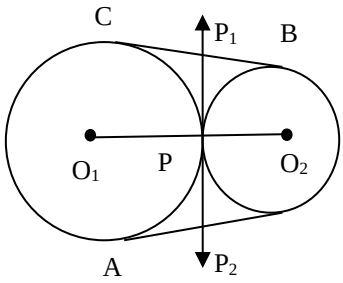



**3.** In a circle or congruent circles, equal chords subtend equal angles at the center.

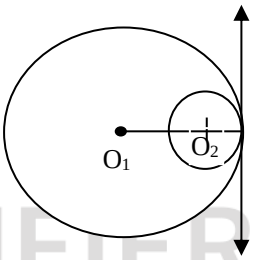


Conversely, chords, which subtend equal angles at the center of the same or congruent circles, are equal.

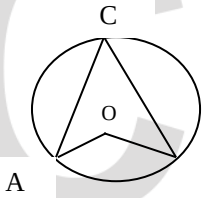
**4.** If the two circles touch each other externally, distance between their centers = sum of their radii.



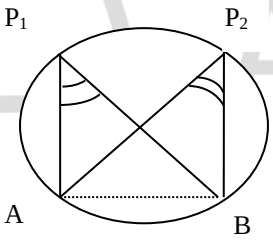

If the two circles touch each other internally,
distance between their centres = difference of their radii.




**5.** The measure of an inscribed angle is half the measure of its intercepted arc. (Inscribed angle theorem). ∠ACB = ½ arc (ADB) = ½ ∠AOB.

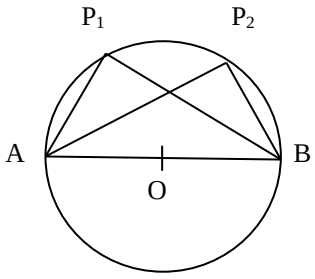



**6.** Angles subtended is the same segment are equal



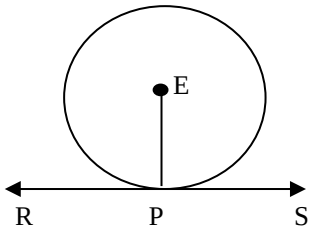

$\angle AP_1B = \angle AP_2B$.

**7.** Angle subtended in a semicircle is $90^0$

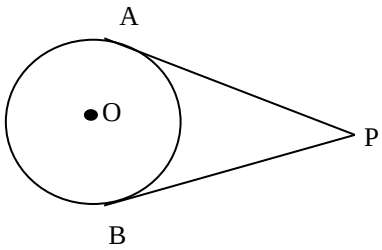



i.e., $\angle AP_1B = 90^0 = \angle AP_2B$

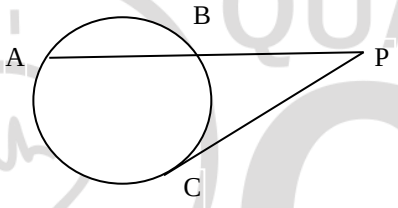
**8.** Tangent is always perpendicular to the line joining the centre and the point of tangency.



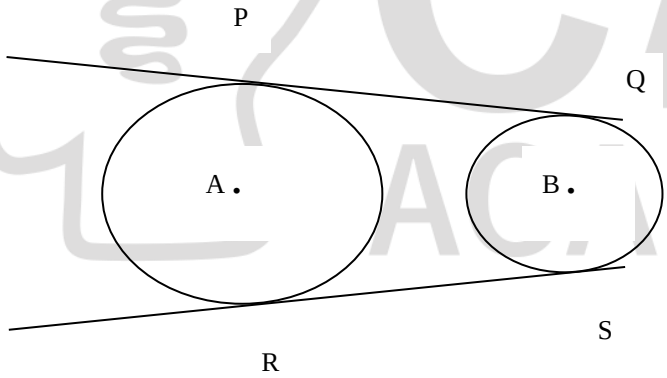

**9.** Two tangents from the same external point are equal in length. PA = PB



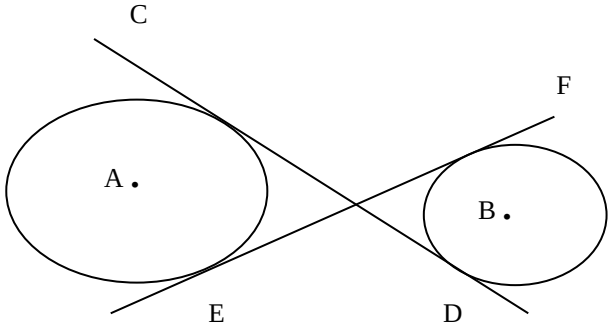

**10.** If AB is any chord of a circle and PC is the tangent (both for a external point P) than PA × PB = $PC^2$



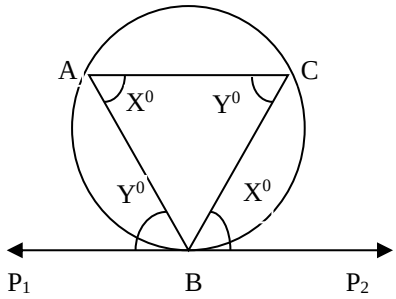

**11.** Length of direct common tangent = $\sqrt{(\text{distance between centers})^2 - (r_1 - r_2)^2}$




**12.** Length of transverse common tangent = $\sqrt{(\text{distance between centers})^2 - (r_1 + r_2)^2}$



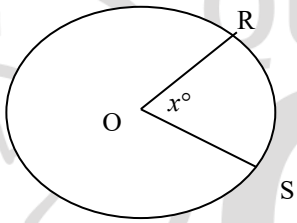
**13.** Alternate Segment Theorem:




Angle between any chord (at the point of tangency) and the tangent is equal to the angle subtended by the chord to any point on the other side of the segment (alternate segment).
In the fig $\angle CBP_2 = \angle CAB = X^0$ & $\angle P_1BA = \angle ACB = Y^0$

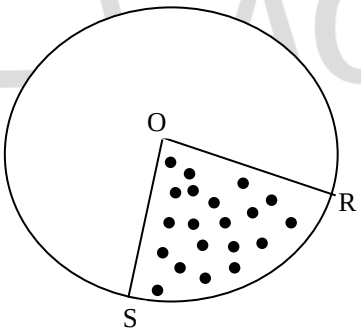
## Sectors of a Circle

The number of degrees of arc in a circle (or the number of degrees in a complete revolution) is 360.




In the circle with center O above, the length of arc RS is $x/360$ of the circumference of the circle; for example, if $x = 60°$, then arc RS has length 1/6 of the circumference of the circle. We can remember the following formulas:

**1.** Length of arc RS = $2\pi r \times x/360$. ∵ the complete circle is having 360 degrees & any part of that shall be equal to $x/360$.




**2.** Area of Sector ORS = $\pi r^2 \times x/360$. ∵ the complete circle is having 360 degrees & any part of that shall be equal to $x/360$.

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# Shreya NMIMS

SVKM'S NMIMS

Deemed to be UNIVERSITY

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**3.** Area of the segment of a circle (QSR) = $r^2\left[\frac{\pi\theta}{360^\circ}-\frac{\sin\theta}{2}\right]$

Perimeter of the segment = $2\pi r\left(\frac{\theta}{360^\circ}\right)+2r\sin\frac{\theta}{2}$

## Quadrilaterals & Polygons

### Quadrilaterals

**1.** Of all the quadrilaterals of the same perimeter, the one with the maximum area is the square.

**2.** The quadrilateral formed by joining the midpoints of the sides of any quadrilateral is always a parallelogram (rhombus in case of a rectangle, rectangle in case of a rhombus and square in case of a square).

**3.** The quadrilateral formed by the angle bisectors of the angles of a parallelogram is a rectangle.

**4.** For a rhombus □ABCD, if the diagonals are AC and BD, then $AC^2 + BD^2 = 4 \times AB^2$.

**5.** If a square is formed by joining the midpoints of a square, then the side of the smaller square = side of the bigger square/√2.

**6.** If P is a point inside a rectangle □ABCD, then $AP^2 + CP^2 = BP^2 + DP^2$.

**7.** The segment joining the midpoints of the non-parallel sides of a trapezium is parallel to the two parallel sides and is half the sum of the parallel sides.

**8.** For a trapezium □ABCD, if the diagonals are AC and BD, and AB and CD are the parallel sides, then $AC^2 + BD^2 = AD^2 + BC^2 + 2 \times AB \times BD$.

**9.** If the length of the sides of a cyclic quadrilaterals area, b, c and d, then it's area = $\sqrt{(s-a)(s-b)(s-c)(s-d)}$, where s is the semi-perimeter.

**10.** The opposite angles of a cyclic quadrilateral are supplementary.

### Quadrilateral Areas

**1.** Square = $a^2$

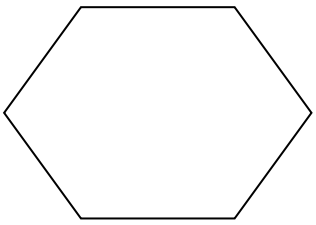
**2.** Rectangle = a × b, where a & b are the length and breadth of the rectangle.

**3.** Parallelogram = b × h, where b is the base and h is the height of the parallelogram.

**4.** For Rhombus = ½ × $d_1$ × $d_2$, where $d_1$and $d_2$ are the diagonals of the rhombus.

**5.** For Trapezium with a and b parallel sides and height h, Area = 1/2 (a + b) × h

## Polygons



1. The sides $a_n$ of regular inscribed polygons, where R is the radius of the circumscribed circle $= a_n = 2R \sin 180^0/n$

**2.** Area of a polygon of perimeter P and radius of in-circle $r = 1/2 \times p \times r$

**3.** The sum of the interior angles of a convex POLYGON, having n sides is $180^o$ (n - 2).

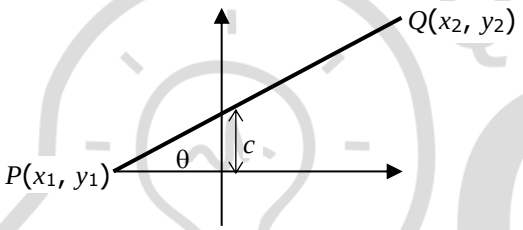
**4.** The sum of the exterior angles of a convex polygon, taken one at each vertex, is $360^o$.

**5.** The measure of an exterior angle of a regular n- sided polygon is $\frac{360^0}{n}$.

**6.** The measure of the interior angle of a regular n-sided polygon is $\frac{(n-2)180^\circ}{n}$.

**7.** The number of diagonals of in an n-sided polygon is $\frac{n(n-3)}{2}$.

## Coordinate Geometry




**1.** Distance PQ $=\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

**2.** Slope of *PQ* = $m = y_2 - y_1/x_2 - x_1 = \tan\theta$

**3.** Equation of *PQ* is $y - y_1/y_1 - y_2 = x - x_1/ x_1 - x_2$ or $y = mx + c$

**4.** The product of the slopes of two perpendicular lines is –1

**5.** The distance between the points $(x_1, y_1)$ and $(x_2, y_2)$ is $\sqrt{[(x_1 - x_2)^2 + (y_1 - y_2)^2]}$.

**6.** If point $P(x, y)$ divides the segment *AB*, where

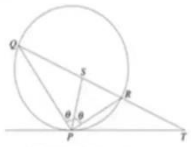
$A \equiv (x_1, y_1)$ and $B \equiv (x_2, y_2)$, internally in the ratio $m : n$, then, $x = \frac{mx_2+nx_1}{m+n}$ and $y = \frac{my_2+ny_1}{m+n}$.

**7.** If *P* is the midpoint, then $x = x_1 + x_2 / 2$ and $y = y_1 + y_2 / 2$.

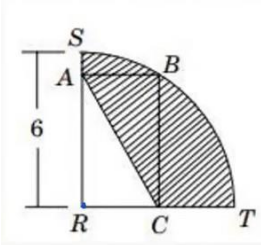
**8.** If $G(x, y)$ is the centroid of triangle *ABC*, $A \equiv (x_1, y_1)$, $B \equiv (x_2, y_2)$, $C \equiv (x_3, y_3)$, then $x = (x_1 + x_2 + x_3) / 3$ and $y = (y_1 + y_2 + y_3) / 3$.

**9.** If $I(x, y)$ is the in-center of triangle *ABC*, $A \equiv (x_1, y_1)$, $B \equiv (x_2, y_2)$, $C \equiv (x_3, y_3)$, then, $x = (ax_1 + bx_2 + cx_3) /(a + b + c)$ and $y = (ay_1 + by_2 + cy_3)/(a + b + c)$ where *a*, *b* and *c* are the lengths of *BC*, *AC* and *AB* respectively.

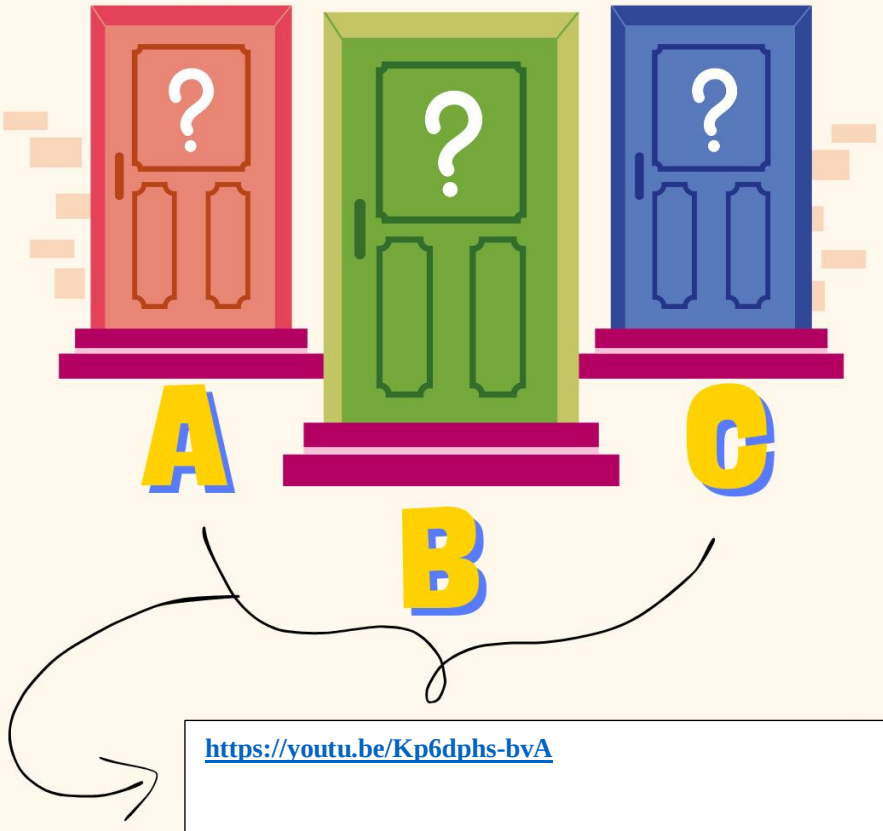
**10.** The equation of a straight line is $y = mx + c$, where *m* is the slope and *c* is the *y*-intercept ($\tan\theta = m$, where $\theta$ is the angle that the line makes with the positive X-axis)

| | | |
|---|---|---|
| 1 | The length of the line joining the midpoints of the non-parallel sides of an isosceles trapezium is 24. If the aforementioned line divides the trapezium in two parts with the areas in ratio 7: 9, then what is the absolute difference (in cm) between the parallel sides of the trapezium? | Click here for video solution |
| 2 | P is any point inside triangle ABC such that PA = 5 units, PB = 6 units and PC = 8 units. Also ∠APB=120 degree and ∠APC=150 degree. The approximate perimeter of triangle ABC is:<br><br>1. 32 Units<br>2. 68 Units<br>3. 50 Units<br>4. 48 Units | Click here for video solution |
| 3 | AF bisects angle EAD in a square ABCD, if FD = 64 cm, and BE = 36 cm, find AE. | Click here for video solution |
| 4 | A square ABCD with side 8 has point E on AD, the value of BE+CE is definitely greater than:<br><br>1. 8√5<br>2. 8√6<br>3. 8√7<br>4. CBD | Click here for video solution |
| 5 | 3 altitudes of triangle PQR have lengths 4,6 and 8 respectively, If P PQR has in-radius of length y, find 13y | Click here for video solution |
| 6 | ABC is an equilateral triangle with O a point in it, with AO, BO and CO being 6,8,10. If triangle has an area of a√3 + b, find a+b. | Click here for video solution |
| 7 | In a triangle ABC, AB = 60, BC= 80, CA= 100. D is a point on AC such that perimeters of triangles ABD and CBD are equal. If the length of BD is AV5, find the value of A. | Click here for video solution |
| 8 | Let P be the point of intersection of the lines 3x + 4y = 2a and 7x + 2y = 2018 and Q the point of intersection of the lines 3x + 4y = 2018 and 5x + 3y = 1. If the line through P and Q has slope 2, the value of a is:<br><br>1. 1<br>2. 1/2<br>3. 4035<br>4. 1009<br>5. 3026 | Click here for video solution |
| 9 | A triangle has its longest side as 38 cm. If one of the other two sides is 10 cm and the area of the triangle is 152 sq.cm, and the length of the third side is AVB,find the value of A+B . | Click here for video solution |
| 10 | The length of sides AB, BC and CA of a triangle ABC is 7 cm, 10 cm and 12 cm respectively. AB is extended to D such that AD = 28 cm. BC is extended to E such that BE = 20 cm. CA is extended to F such that CF = 36 cm. What is the ratio of the area of triangle ABC to the area of triangle DEF? | Click here for video solution |
| 11 | PQR is a right angled triangle where angle R is 90 degree. S is a point on PQ such that PS 29 and QS=1 and RS=13. Find area of triangle QRS. | Click here for video solution |
| 12 | In the diagram, P,Q and R lie on a circle, the tangent at P and the secant QR intersect at T. The bisector of angle QPR meet at S so that QPS = RPS, If SQ=3 and<br><br>  <br><br>PT = 4, Find length of RS | Click here for video solution |
| 13 | In a triangle ABC side AC and the perpendicular bisector of side BC meet in point D and BD bisects angle abc. if ad is 9 DC is 7 find the area of triangle ABD | Click here for video solution |
| 14 | A circle of diameter 8 inches is inscribed in a triangle ABC where angle ABC= 90 degree. If BC = 10 inches then the area of the triangle in square inches is: | Click here for video solution |
| 15 | If the area of a regular hexagon is equal to the area of an equilateral triangle of side 12 cm, then the length, in cm, of each side of the hexagon is:<br><br>1. 6√6<br>2. 2√6<br>3. √6<br>4. 4√6 | Click here for video solution |

| | | |
|---|---|---|
| 16 | Suppose the length of each side of a regular hexagon ABCDEF is 2 cm. It T is the mod point of CD, then the length of AT, in cm, is:<br><br>1. √13<br>2. √15<br>3. √14<br>4. √12m | Click here for video solution |
| 17 | The sides AB and CD of a trapezium ABCD are parallel, with AB being the smaller side. P is the midpoint of CD and ABPD is a parallelogram. If the difference between the areas of the parallelogram ABPD and the triangle BPC is 10 sq cm, then the area, in sq cm, of the trapezium ABCD is:<br><br>1. 40<br>2. 25<br>3. 20<br>4. 30 | Click here for video solution |
| 18 | If a rhombus has area 12 sq cm and side length 5 cm, then the length, in cm, of its longer diagonal is:<br><br>1. √30 + √12<br>2. √37 + √13<br>3. (√13 + √12) / 2<br>4. (√37 + √13) / 2 | Click here for video solution |
| 19 | Let ABCD is a parallelogram. The lengths of the sides AD and the diagonal AC are 10 cm and 20 cm, respectively. If the angle ADC is 30 degree, then the area of the parallelogram, in sq. cm, is<br><br>1. 25 (√5 + √15)<br>2. 25 (√3 + √15)<br>3. 25 (√5 + √15) / 2<br>4. 25 (√3 + √15) / 2 | Click here for video solution |
| 20 | A park is shaped like a rhombus and has area 96 sq m. If 40 m of fencing is needed to enclose the park, the cost, in INR, of laying electric wires along its two diagonals, at the rate of ₹125 per m, is: | Click here for video solution |
| 21 | In a triangle ABC, angle BCA = 50 degree. D and E are points on AB and AC, respectively, such that AD = DE. If F is a point on BC such that BD = DF, then angle FDE, in degrees, is equal to<br><br>1. 72<br>2. 100<br>3. 96<br>4. 80 | Click here for video solution |
| 22 | In ΔABC, ∠B = 900. □PQRS is a square of side 5 units and □WXYZ is a square of side 10 units. MS and SY are the length and breadth respectively of rectangle MNSY. What is the length of MS?<br><br>1. 12 cm<br>2. 15 cm<br>3. 25 cm<br>4. 20 cm | Click here for video solution |
| 23 | The vertices of a triangle are (0, 0), and (4, 0) and (3, 9). The area of the circle passing through these three points is:<br><br>1. 12π/5<br>2. 14π /5<br>3. 205π/9<br>4. 123π /7 | Click here for video solution |
| 24 | In triangle ABC, M is the midpoint of the side AB. N is point in the interior of aking triangle ABC, such that CN is the bisector of angle C and CN is perpendicular to NB. What is the area of triangle BMN and AMN if CN=6, BC= 10cm and AC = 15cm? | Click here for video solution |
| 25 | In a triangle PQR, S is a point on QR and T is a point on PR such that PQ= PR and PS-PT.Angle QPS= 40 degrees. Find angle TSR | Click here for video solution |
| 26 | BD and CE are the medians of triangle ABC, right angled at A. If CE=5V13/2 and BC=10 then length of BD is | Click here for video solution |
| 27 | PQRS is an isosceles trapezium in which PQ is parallel to RS and X is the mid point of PS. If PX= 8 and angle QXR= 90 degrees. Fine the perimeter of the trapezium PQRS. | Click here for video solution |

| | | |
|---|---|---|
| 28 | Length of sides of two rhombuses differ by 2. One of the diagonals for each of the rhombuses is of the same length as that of its side. Product of their areas is 168.75. Find the side of smaller | Click here for video solution |
| 29 | BD and CE are the medians of the triangle ABC right angled at A. If CE = $5\sqrt{13}/2$ and BC = 10, then the length of BD is: | Click here for video solution |
| 30 | If a rhombus has area 12 sq cm and side length 5 cm, then the length, in cm, of its longer diagonal is: | Click here for video solution |
| 31 | ABCD is a rhombus where E is a point on diagonal AC such that AC = 50 and AE = 41. Angle ABE = 90 degree. Find the area of rhombus. | Click here for video solution |
| 32 | Consider triangle with sides in the ratio 1: 1: $\sqrt{n}$ and angles in the ratio 1: 1: m where m, n are distinct natural numbers. What is the value of m + n? | Click here for video solution |
| 33 | In the figure above, arc SBT is one quarter of a circle with center R and radius 6. If the length plus the width of rectangle ABCR is 8, then the perimeter of the shaded part is:   | Click here for video solution |
| 34 | Product of two sides of a triangle is 6. If all three sides are having integral value, then how many such obtuse angle triangles are possible? | Click here for video solution |

# 99.99%ile DILR Strategy




https://youtu.be/Kp6dphs-bvA


**Join our WhatsApp FREE 24/7 Doubt Solving Group (if group is full, follow us on Instagram @quantifiers_cat for the new link)**
**https://chat.whatsapp.com/EX8fGeD7h0zAhs9sDa5eeA**

# CLOCKS & CALENDARS

## CLOCKS

A clock is a complete circle having 360 degrees. It is divided into 12 equal parts i.e. each part is 360/12 = 30°. As the minute hand takes a complete round in one hour it covers 360° in 60 min. In 1 min. it covers 360/60 = 6°/ minute. Also, as the hour hand covers just one part out of the given 12 parts in one hour, this implies it covers 30° in 60 min. i.e. ½° per minute. This implies that the relative speed of the minute hand is 6 – ½ = 5 ½ degrees.

1. Every hour, both the hands coincide once. In 12 hours, they will coincide 11 times. It happens due to only one such incident between 12 and 1'o clock.
2. The hands are in the same straight line when they are coincident or opposite to each other.
3. When the two hands are at a right angle, they are 15-minute spaces apart. In one hour, they will form two right angles and in 12 hours there are only 22 right angles. It happens due to right angles formed by the minute and hour hand at 3'o clock and 9'o clock.
4. When the hands are in opposite directions, they are 30-minute spaces apart.
5. If a clock indicates 8.15, when the correct time is 8, it is said to be 15 minutes too fast. On the other hand, if it indicates 7.45, when the correct time is 8, it is said to be 15 minutes too slow.

**NOTE** If both the hour hand and minute hand move at their normal speeds, then both the hands meet after $65\frac{5}{11}$ minutes.

## Calendars

In an ordinary year there are 365 days, which means 52 × 7 + 1, or 52 weeks and one day. This additional day, is called an odd day. Further, every 100$^{th}$year starting from 1st AD, is a non–leap year, but every 4$^{th}$ century year is a leap year. So, any year divisible by 400 will be a leap year e.g.: 1200, 1600 and 2000. And the years 1800, 1900 will be non-leap years as they are divisible by 100, but not 400.

The concept of odd days is very important in calendars. In a century – i.e. 100 years, there will be 24 leap years and 76 non–leap years. This means that there will be 24 × 2 + 76 × 1 = 124 odd days. Since, 7 odd days make a week, to find out the net odd days, divide 124 by 7. The remainder is 5. This is the number of odd days in a century. You may memorise the following points related to the concepts of calendars to save time during the paper.

100 years give us 5 odd days as calculated above.
200 years give us 5 × 2 = 10 – 7 (one week) – 3 odd days.
300 years give us 5 × 3 = 15 – 14 (two weeks) – 1 odd day.
400 years give us {5 × 4 + 1 (leap century)} – 21} – 0 odd days.

Month of January gives us 31 – 28 = 3 odd days.
Month of February gives us 28 – 28 = 0 odd day in a normal year and 1 odd day in a leap year and so on for all the other months.
In total first six months i.e. January to June give us 6 odd days in a normal year and 7 – 7 = 0 odd days in a leap year. This is going to help, when you want to find a day, which is after 30$^{th}$ June.
In total first nine months i.e. January to September give us 0 odd day in a normal year and 1 odd day in a leap year.

Now, if we start from 1st January 0001 AD; for 0 odd day, the day will be Sunday; for 1 odd day, the day will be Monday; for 2 odd days, it will be Tuesday; for 3 odd days, it will be Wednesday and so on.

QUANTIFIERS CAT ACADEMY

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# Trigonometry

**(I) Measurement of Angles:** Mainly there are three systems for measuring angles.

**Sexagesimal system:** In this system, angle is measured in degrees.

**1 *right angle* = 90⁰, 1⁰ = 60' and 1' = 60".**

**Centesimal system:** In this system angle is measured in grades.

**1 *right angle* = 100ᵍ, 1ᵍ = 100' and 1' = 100".**

**Circular Measure:** In this system, angle is measured in radius.

**The angle subtended at the center of a circle by an arc of length equal to its radius is 1 *radian, written as* 1ᶜ.**

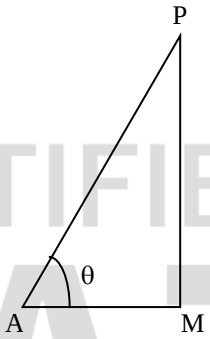
$\pi^c = 180^0 = 200^R$ = 2 right angles

**II. Trigonometric Ratios:**

$\sin\theta = \frac{Perp.}{Hyp.} = \frac{PM}{AP}; cos\,\theta = \frac{Base}{Hyp.} = \frac{AM}{AP};$

$\tan\theta = \frac{Perp.}{Base} = \frac{PM}{AM};$

$\text{cosec}\,\theta = \frac{1}{sin\,\theta}; sec\,\theta = \frac{1}{cos\,\theta}; cot\,\theta = \frac{cos\,\theta}{sin\,\theta}.$




**III. Identities:**

**1.** $\tan\theta = \frac{sin\,\theta}{cos\,\theta}$ **2.** $\cot\theta = \frac{cos\,\theta}{sin\,\theta}$

**3.** $\sin^2\theta + \cos^2\theta = 1$ **4.** $1 + \tan^2\theta = \sec^2\theta$

**5.** $1 + \cot^2\theta = \text{cosec}^2\theta$

**IV. Values of T-Ratios:**

| Angle | sin θ | cos θ | tan θ |
|---|---|---|---|
| $0^0$ | 0 | 1 | 0 |
| $30^0 = \frac{\pi}{6}$ | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{3}}$ |
| $45^0 = \frac{\pi}{4}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | 1 |
| $60^0 = \frac{\pi}{3}$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $\sqrt{3}$ |
| $90^0 = \frac{\pi}{2}$ | 1 | 0 | ∞ |
| $180^0 = \pi$ | 0 | -1 | 0 |

**V. Sign of T-Ratios:**

1st Quadrant: All positive.
2nd Quadrant: sin θ & cosec θ positive.
3rd Quadrant: tan θ & cot θ positive.
4th Quadrant: cos θ & sec θ positive.

| **Remember:** | **I** | **II** | **III** | **IV** |
|---|---|---|---|---|
| | **All** | **sin** | **tan** | **cos** |

**VI. Range of T-Ratios:**

**1.** $-1 \le \sin\theta \le 1$ and $-1 \le \cos\theta \le 1$
Thus, $|\sin\theta| \le 1$ and $|\cos\theta| \le 1$.
The value of $\sin\theta$ *is never greater than 1 and never less than* $-1$.
The value of $\cos\theta$ *is never greater than 1 and never less than* $-1$.
**2.** $\text{cosec}\,\theta \ge 1$ and $\text{cosec}\,\theta \le -1$.
**3.** $\sec\theta \ge 1$ and $\sec\theta \le -1$.
**4.** $\tan\theta$ may assume any value.

**VII. Increasing & Decreasing Functions:**

**1.** The value of $\sin\theta$ increase from $-\frac{\pi}{2}$ *to* $\frac{\pi}{2}$. As $\theta$ increases in this interval, then $\sin\theta$ also increases.
**2.** 1st Quadrant: $\sin\theta$ increases from 0 to 1; $\cos\theta$ decreases from 1 to 0 and $\tan\theta$ increases from 0 to $\infty$.
**3.** 2nd Quadrant: $\sin\theta$ decreases from 1 to 0; $\cos\theta$ decreases from 0 to $-1$; $\tan\theta$ decreases from $\infty$ to 0.
**4.** 3rd Quadrant: $\sin\theta$ decreases from 0 to $-1$; $\cos\theta$ increases from $-1$ to 0; $\tan\theta$ increases from 0 to $\infty$.
**5.** 4th Quadrant: $\sin\theta$ increases from $-1$ to 0; $\cos\theta$ increases from 0 to 1; $\tan\theta$ decreases from $\infty$ to 0.

**VIII. T-Ratios of Negative, Complementary, Supplementary Angles etc.**

**1.** $\sin(-\theta) = -\sin\theta$, $\cos(-\theta) = \cos\theta$, $\tan(-\theta) = -\tan\theta$.

**2.** $\sin(90^0 - \theta) = \cos\theta$; $\cos(90^0 - \theta) = \sin\theta$; $\tan(90^0 - \theta) = \cot\theta$.

**3.** $\sin(90^0 + \theta) = \cos\theta$; $\cos(90^0 + \theta) = -\sin\theta$; $\tan(90^0 + \theta) = -\cot\theta$.

**4.** $\sin(180^0 - \theta) = \sin\theta$; $\cos(180^0 - \theta) = -\cos\theta$; $\tan(180^0 - \theta) = -\tan\theta$.

**5.** $\sin(180^0 + \theta) = -\sin\theta$; $\cos(180^0 + \theta) = -\cos\theta$; $\tan(180^0 + \theta) = \tan\theta$.

**6.** $\sin(360^0 - \theta) = -\sin\theta$, $\cos(360^0 - \theta) = \cos\theta$; $\tan(360^0 - \theta) = -\tan\theta$.

**7.** $\sin(360^0 + \theta) = \sin\theta$, $\cos(360^0 + \theta) = \cos\theta$, $\tan(360^0 + \theta) = \tan\theta$.

**IX. Sum & Difference Formula:**

**1.** $\sin(x + y) = \sin x \cos y + \cos x \sin y$.

**2.** $\sin(x - y) = \sin x \cos y - \cos x \sin y$.

**3.** $\cos(x + y) = \cos x \cos y - \sin x \sin y$

**4.** $\cos(x - y) = \cos x \cos y + \sin x \sin y$.

**5.** $2 \sin x \cos y = \sin(x + y) + \sin(x - y)$.

**6.** $2 \cos x \sin y = (x + y) - \sin(x - y)$.

**7.** $2 \cos x \cos y = \cos (x + y) + \cos (x - y)$.

**8.** $2 \sin x \sin y = \cos (x - y) - \cos (x + y)$.

**9.** $\sin^2 x - \sin^2 y = \sin (x + y). \sin (x - y)$.

**10.** $\cos^2 x - \sin^2 y = \cos (x + y). \cos (x - y)$.

**X. Some More Formulae:**

**1.** $\sin C + \sin D = 2 \sin \left(\frac{C+D}{2}\right) cos \left(\frac{C-D}{2}\right)$

**2.** $\sin C - \sin D = 2 \cos \left(\frac{C+D}{2}\right) sin \left(\frac{C-D}{2}\right)$

**3.** $\cos C + \cos D = 2 \cos \left(\frac{C+D}{2}\right) cos \left(\frac{C-D}{2}\right)$

**4.** $\cos C - \cos D = -2 \sin \left(\frac{C+D}{2}\right) sin \left(\frac{C-D}{2}\right)$

**XI. Tangent Formulae:**

**1.** $\tan (x + y) = \frac{tan\, x + tan\, y}{1 - tan\, x\, tan\, y}$

**2.** $\tan (x - y) = \frac{tan\, x - tan\, y}{1 + tan\, x\, tan\, y}$

**XII. T-Ratios of Multiple Angles:**

**1.** $\sin 2x = 2 \sin x \cos x$

**2.** $\cos 2x = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$.

**3.** $1 + \cos 2x = 2 \cos^2 x$ and $1 - \cos 2x = 2 \sin^2 x$.

**4.** $\sin 3x = 3 \sin x - 4 \sin^3 x$

**5.** $\cos 3x = 4 \cos^3 x - 3 \cos x$

**6.** $\tan 2x = \frac{2 \tan x}{1 - tan^2 x}$

**7.** $\tan 3x = \left(\frac{3 \tan x - tan^3 x}{1 - 3 tan^2 x}\right)$

**8.** $\cos 2x = \frac{1 - tan^2 x}{1 + tan^2 x}$

**9.** $\sin 2x = \frac{2 \tan x}{1 + tan^2 x}$

**XIII. T-Ratios of Sub-multiple Angles:**

**1.** $\sin x = 2 \sin (x/2) \cos (x/2)$

**2.** $\cos x = \cos^2 \frac{x}{2} - sin^2 \frac{x}{2} = 2\, cos^2 \frac{x}{2} - 1 = 1 - 2\, sin^2 \frac{x}{2}$

**3.** $\tan x = \frac{2 \tan (x/2)}{1 - tan^2 (x/2)}$

**4.** $1 - \cos x = 2 \sin^2 \frac{x}{2}$ and $1 + \cos x = 2 \cos^2 \frac{x}{2}$

**5.** $\sin x = \frac{2\tan(x/2)}{1+\tan^2(x/2)}$

6. $\cos x = \frac{1-\tan^2(x/2)}{1+\tan^2(x/2)}$

**XIV. T-Ratios of Some Special Angles:**

**1.** $\sin 15^0 = \frac{\sqrt{3}-1}{2\sqrt{2}} = cos\,7\,5^0$

**2.** $\cos 15^0 = \frac{\sqrt{3}+1}{2\sqrt{2}} = sin\,7\,5^0$

**3.** $\sin 18^0 = \frac{\sqrt{5}-1}{4} = cos\,7\,2^0$

**4.** $\cos 18^0 = \frac{\sqrt{10+2\sqrt{5}}}{4} = sin\,7\,2^0$

**5.** $\sin 36^0 = \frac{\sqrt{10-2\sqrt{5}}}{4} = cos\,5\,4^0$

**6.** $\cos 36^0 = \frac{\sqrt{5}+1}{4} = sin\,5\,4^0$

**7.** $\sin 22\frac{1^0}{2} = \frac{\sqrt{2-\sqrt{2}}}{2}$

**8.** $\cos 22\frac{1^0}{2} = \frac{\sqrt{2+\sqrt{2}}}{2}$