ARITHMETIC

QUANTIFIERS ACADEMY

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QUANTIFIERS CAT ACADEMY

Ratio & Partnership

Ratio:

Ratio is the topic which is used in almost all other topics of arithmetic. It is the first and most important topic along with the percentage when you start the arithmetic part during CAT preparation.

Ratio is the comparison of two quantities of the same type. It simply gives us the comparison that one quantity is how much time of the other quantity. E.g., if there are two different vessels. The first vessel has 1 litre milk and the other vessel has 500 ml milk. Now if we have to compare the volumes of two vessels, first we have to make the units of both the volumes same. So, we can say that the first vessel has 1000 ml milk and the second has 500 ml milk. As the first vessel has double the milk than the second vessel, so the ratio of the volume in first and the second vessel is 2: 1.

Note: As we compare the objects of same type, so, ratio itself is a unitless quantity.

Now, if we say that Rs 1000 is divided among A and B in the ratio 2: 3, then it means that A got 2/3 times the amount that B got or B got 3/2 times the amount that A got. Let us try to find the actual amounts received by A and B.

As the amounts received by A and B are in ratio 2: 3, so, it means that if A gets Rs 2, B will get Rs 3 or if A gets Rs 4, B will get Rs 6 or if A gets Rs 6, B will get Rs 9 and so on.

In general, we can say that if A gets Rs 2x, then B gets Rs 2b and the total amount is Rs 1000.

Therefore, $2x + 3x = 1000 \Rightarrow 5x = 1000 \Rightarrow x = 200$.

Hence, A will get $2x = 2 \times 200$ = Rs 400 and B will get $3x = 3 \times 200$ = Rs 600.

Now, the above calculation can directly be done. For that, simply check the sum of ratio which is 5 here. So, we can say that if total amount is Rs 5, then A will get Rs 2

If the total amount is Rs 1000, then A will get Rs $2/5 \times 1000$ = Rs 400.

Similarly, if total amount is Rs 5, then B will get Rs 3 If the total amount is Rs 1000, then B will get Rs $3/5 \times 1000$ = Rs 600.

Now you can check that it is simply (say for B) = Part of B in ratio/Sum of ratio × Total amount.

The above result can be applied in the questions to find the actual values when the ratio and the total value is given.

Ex. **Rs 15000 are divided among A, B and C in the ratio, 3: 4: 8. Find the amounts received by each of them.**

Sol: Here total amount is Rs 15000.

The ratio of amounts received by A, B and C is 3: 4: 8.

Amount received by A = $3 \times 15000/15$ = Rs 3000

Amount received by B = $4 \times 15000/15$ = Rs 4000

Amount received by C = $8 \times 15000/15$ = Rs 8000

Ex. **The time taken by Anil and Biswas to reach their office from home is in the ratio 5: 8. If Biswas takes 15 minutes more than Anil to reach the office, find the time taken by each of them to reach office.**

Sol: Let the time taken by Anil and Biswas is 5x and 8x minutes respectively.

Given that $8x - 5x = 15 \Rightarrow 3x = 15 \Rightarrow x = 5$

Therefore, time taken by Anil = 5x = 5 × 5 = 25 minutes and time taken by Biswas = 8x = 8 × 5 = 40 minutes.

Alternatively: We can also use a direct approach. As the ratio of times taken is 5: 8, so, assume that Anil takes 5 minutes and Biswas takes 8 minutes. The difference in time is 3 minutes. But the actual difference is 15 minutes. As 15 = 3 × 5, so, to get the required difference, each value should be multiplied by 5. Therefore, Anil will take 5 × 5 = 25 minutes and Biswas will take 8 × 5 = 40 minutes.

Ex. **Divide Rs. 195 among A, B and C such that their shares are in the ratio $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$ respectively.**

Sol: In this question, the ratio is given in fractions. So, first of all try to convert the fractions into integers. For that take the LCM of the denominators of fractions. In this case, it is LCM (2, 3, 4) = 12.

Multiply each term in ratio with 12 to convert the fractions into integers.

We have the required ratio for A, Band C as, $\frac{1}{2}\times 12:\frac{1}{3}\times 12:\frac{1}{4}\times 12$ or 6: 4: 3.

Amount received by A = 6 × 195/13 = Rs 90

Amount received by B = 4 × 195/13 = Rs 60

Amount received by C = 3 × 195/13 = Rs 45.

Ex. **Divide Rs 14100 among A, B and C such that 3 times the amount received is equal to 4 times the amount received by B and 5 times the amount received by C.**

Sol: Now, here the statement of the question does not mean that the ratio of amounts received by A, B and C is 3: 4: 5. It is mentioned that 3A = 4B = 5C. In such questions, take a number which is divisible by all the given numbers in question, e.g., in this question, take a number divisible by 3, 4 and 5. Let us take their LCM which is 60.

So, we have, 3A = 4B = 5C = 60 ⇒ A = 20, B = 15 and C = 12

Hence, the required ratio of A, B and C is 20: 15: 12.

Therefore, Amount received by A = 20 × 14100/47 = Rs 6000

Amount received by B = 15 × 14100/47 = Rs 4500

Amount received by C = 12 × 14100/47 = Rs 3600.

Some important results:

Here are few results which should be kept in mind as they are very helpful while solving the questions of ratios.

1. If two different ratios, a: b and c: d are expressed in different units, then the two are compounded to obtain a combined ratio. Compounding of a: b and c: d yields (a × c)/(b × d)

Ex. **A takes 4 days to complete a job, working 6 hours a day. B takes 7 days to complete the same job, working 5 hours a day. Find is the ratio of the time taken by A and B.**

Sol: Here two different units, number of days and number of hours are used. The ratio of days worked is 4: 7 and the ratio of the number of working hours per day is 6: 5. Compounding these two ratios, we have the ratio of the time taken by A and B = (4 × 6)/(7 × 5) = 24: 35.

2. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$ then each of these ratios is equal to $\frac{a+c+e}{b+d+f}$.

3. If $\frac{a}{b}=\frac{c}{d}$, then $\frac{b}{a}=\frac{d}{c}$

4. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{c} = \frac{b}{d}$ (Alterendo)

5. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+d}{d}$ (Componendo)

6. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a-b}{b} = \frac{c-d}{d}$ (Dividendo)

7. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ (Componendo & Dividendo)

Sometimes, we have questions where we need to combine two ratios. E.g., if a: b and a: c is given and we have to find a: b: c.

In such cases, the first ratio gives us the relation between a and b; the second ratio gives us the relation between b and c, thus the logic says that we should be able to find the relation between a and c because b is common to the two relation. And b being common to the two relations is exactly what we use to find the answer, as done in the following examples.

Ex. **If a: b is 3: 4 and b: c is 5: 6, find the ratio a: b: c.**

Sol: Since b is common to the two ratios, we should make the numeric value of b the same in both the ratios. Also remember that when all terms of a ratio are multiplied with the same constant, the ratio does not change.

Thus, in the first ratio b could be changed to any multiple of 4 and in the second ratio b could be changed to any multiple of 5. Thus, we should make b a multiple of 4 and 5 i.e., 20.

a: b is 3 × 5: 4 × 5 i.e., 15: 20

b: c is 5 × 4: 6 × 4 i.e., 20: 24

Thus, when b is 20, a is 15 and c is 24 and the required ratio of a: b: c is 15: 20: 24.

Ex. **If a: b is 2: 5, b: c is 3: 7 and c: d is 5: 6, find the ratio a: b: c: d.**

Sol: In the ratios of a: b and b: c, if b has to be the same numeric value, it should be a multiple of 5 and 3 i.e., 15. Thus, a: b is 2: 5 which is same as 6: 15 and b: c is 3: 7 which is same as 15: 35. Thus a: b: c is 6: 15: 35

Now c is common to a: b: c and c: d and thus making c equal in the two ratios, c: d is 5: 6 which is same as 35: 42. Thus a: b: c: d is 6: 15: 35: 42.

Ex. **The ratio of the present ages of a father and a son is 14: 3. After six years, the ratio of their ages will be 17: 6. What is the present age of the father?**

Sol: Let the present ages of father and son are 14x and 3x respectively.

Given that $\frac{14x+6}{3x+6} = \frac{17}{6} \Rightarrow 84x + 36 = 51x + 102 \Rightarrow 33x = 66 \Rightarrow x = 2$

Therefore, the present age of father is 14x = 14 × 2 = 28 years.

Partnership:

Partnership is an association of two or more persons who put in their money into a business. The profit earned from the business is then divided among the people depending on the amount of money that each has invested.

Partnership is of two types.

(i) Simple Partnership: When all the partners start the business at the same point of time i.e., their capitals remain in the business for the same duration of time is called simple partnership. In this kind of partnership, the profit is simply distributed in the ratio of their capitals.

(ii) Compound Partnership: When the capitals of the partners are invested in the business for the different time periods the partnership is known as compound

partnership. In this kind the profit-sharing ratio is calculated by multiplying the capital invested with the unit of time (mostly months).

Ex. **Pawan and Vishal started a business with Rs 5000 and Rs 7000 respectively. At the end of the year, the total profit earned was Rs 1440. Find the share of each of them in the profit.**

Sol: Here, the time period of investment is same for Pawan and Vishal. Hence, the profit will be distributed in the same ratio as the ratio of their investments.

The ratio of investment (or profit) is 5000: 7000 or 5: 7.

Therefore, profit earned by Pawan = 5 × 1440/12 = Rs 600

Profit earned by Vishal = 7 × 1440/12 = Rs 840.

Ex. **A and B started a business together. A invests Rs 400 for one year and B invested Rs 700 for 8 months. If they earn a profit of Rs 780 at the end of one year, what is A's share?**

Sol: A's total capital = 400 × 12 = Rs 4800.

B's total capital = 700 × 8 = Rs 5600.

Ratio of profits of A and B = 4800: 5600 = 6: 7

Profit earned by A = 6 × 780/13 = Rs 360 and profit earned by B = 7 × 780/13 = Rs 420.

Let us now discuss few examples:

Ex. **A person divided Rs. 13200 among his 3 sons in the ratio 4: 3: 5. First son kept Rs. 800 for himself, gave Rs. 400 to his wife; and divided the remaining money among his 2 daughters in the ratio 9: 7. Find the amount received by the second daughter.**

Sol: Amount received by the first son 4 × 13200/12 = Rs 4400

Money left with him after keeping for himself and giving to wife = 4400 – 800 – 400 = Rs 3200

Amount received by the second daughter = 7 × 3200/16 = Rs 1400.

Ex. **The monthly incomes of Amit and Suresh are in the ratio 3: 4 and their monthly expenditures are in the ratio of 2: 3. If each of them saves Rs. 1500 a month, then what are their monthly incomes?**

Sol: Let the monthly income of Amit and Suresh is 3x and 4x respectively. As both saves Rs 1500, so, monthly expenditure of Amit and Suresh is 3x – 1500 and 4x – 1500 respectively.

Given that $\frac{3x-1500}{4x-1500} = \frac{2}{3} \Rightarrow 9x - 4500 = 8x - 3000 \Rightarrow x = 1500$.

Therefore, the monthly income of Amit is 3x = 3 × 1500 = Rs 4500 and of Suresh is 4x = 4 × 1500 = Rs 6000.

Ex. **Rs. 885 was divided between A, B and C. If each of them had received Rs. 8 less, their shares would have been in the ratio of 2: 3: 2. How much did B receive from the original amount?**

Sol: If the share of each one is decreased by Rs 8, then the total amount will decrease by Rs 24.

Amount left = 885 – 24 = Rs 861 in which the shares of A, B and C will be 2: 3: 2.

Share of B in Rs 861= 3 × 861/7 = Rs 369.

Therefore, share of B in the original amount = 369 + 8 = Rs 377.

Ex. **Six years back, the ratio of the ages of a husband and his wife was 6: 5. Six years hence, their ages will be in the ratio 10: 9. What is the ratio of their present ages.**

Sol: Let the age of husband and wife, six years back is 6x and 5x respectively.

Given that $\frac{6x+12}{5x+12} = \frac{10}{9} \Rightarrow 54x + 108 = 50x + 120 \Rightarrow 4x = 12 \Rightarrow x = 3$

Present age of husband is 6x + 6 = 6 × 3 + 6 = 24 years and present age of wife = 5x + 6 = 5 × 3 + 6 = 21 years.

The required ratio is 24: 21 or 8: 7.

Ex. **Anil and Kunti, separated by a certain distance start walking towards each other. When they meet Anil has walked 40 meters more than Kunti. If the ratio of the distances covered by Anil and Kunti is 2: 3, then find the original distance between them.**

Sol: Here, the ratio of the distances covered by Anil and Kunti is 3: 2. So, when Anil covered 3 meters, Kunti had covered 2 meters. The difference is of 1 meter. To make this difference of 40 meters, Anil should walk 3 × 40 = 120 m and Kunti should walk 2 × 40 = 80 meter.

So, total distance between them originally = 80 + 120 = 200 meter.

Ex. **The number of Re 1, 50 paise and 25 paise coins in a bag is in the ratio 4: 5: 6. If the bag has Rs. 520, find the number of 50 paise coins.**

Sol: Let the number of Re 1, 50 paise and 25 paise coins is 4x, 5x and 6x respectively.

Total amount is Rs 520.

Therefore, 4x + 5x × 0.5 + 6x × 0.25 = 520

⇒ 4x + 2.5x + 1.5x = 520 ⇒ 8x = 520 ⇒ x = 65.

Number of 50 paise coins = 5x = 5 × 65 = 325.

Ex. **A starts a business with Rs. 4200 and after 5 months, B joins him. At the end of the year, the profit is divided between A and B in the ratio 3: 4. What amount did B invest in the business?**

Sol: Here A joined for complete one year and B joined for 7 months. Let, B invested Rs x.

We have (4200 × 12)/7x = 3/4 ⇒ 600 × 12 × 4 = 3x ⇒ x = Rs 9600.

Ex. **A, B and C rent a pasture. A puts 14 oxen for 5 months, B puts 10 oxen for 6 months and C puts 9 oxen for 5 months for grazing. If the rent of the pasture is Rs. 875, find the share of C in the rent.**

Sol: The rent will be shared in the same ratio in which they used the pasture.

Required ratio in which A, B and C used the pasture is 14 × 5: 10 × 6: 9 × 5 or 14: 12: 9. So, the share of C = 9 × 875/35 = Rs 225.

Ex. **started a business with Rs. 9000 and was joined afterwards by B with Rs. 12000. At the end of the year B got two-fifth of the total profit. When did B join?**

Sol: Let B joined after x months. As he got two-fifth of the total profit, so ratio of profits of A and B is 3: 2.

We have, (9000 × 12)/12000x = 3/2 ⇒ 9/x = 3/2 ⇒ x = 6

Hence, B joined after 6 months.

Ex. **Sanjeev started a business with 20000 and after 4 months Lokesh joined him with 50000. Sanjeev received 28000 including 10% of the profits as a commission for managing the business. What amount did Lokesh receive?**

Sol: The ratio of profits of Sanjeev and Lokesh is 20000 × 12: 50000 × 8 or 3: 5.

Let the total profit is x. So, 0.1x + 3 × 0.9x/8 = 28000 ⇒ 3.5x/8 = 28000 ⇒ x = Rs 64000. Therefore, amount received by Lokesh is 64000 – 28000 = Rs 36000.

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Exercise 1

1. If a – b: b – c: c – d = 1: 2: 3, then what is the ratio of (a + d): c?

1. 1: 2 2. 2: 1 3. 3: 1 4. None of these

2. If a/b = 2/3, b/c = 3/5 and c/d = 5/6, then what is the value of $\frac{3a + 2b + 4c + d}{3d + 4c}$.

1. 19/6 2. 38 3. 1 4. 2

3. If $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a - 3b + 5c}{k}$, where a, b and c are not equal to zero, then 'k' is:

1. 15 2. 17 3. 13 4. Cannot be determined

4. Monthly incomes of X and Y are in the ratio 1: 3 and their expenses are in the ratio 19: 40. X saves Rs. 18,860 less than that Y and in total they save Rs. 36,020. Income of X and Y respectively are:

1. Rs. 10,480 and Rs. 31440
2. Rs. 9,000 and Rs. 27,000
3. Rs. 14,200 and Rs. 42,600
4. Rs. 18,000 and Rs. 31,440

5. A bag contains Rs. 600 in the form of one rupee, 50 paise and 25 paise coins in the ratio of 3: 4: 12. Find the total value (in Rs.) of the 25 paise coins present in the bag.

1. 220 2. 225 3. 210 4. 200

6. The ratio of the first-class fare and second-class fare is 3: 1 and that of the number of passengers traveling by the first class and by the second class is 1: 27. If Rs. 2,700 is collected as fare, then the amount collected from the first-class passengers is:

1. Rs. 300 2. Rs. 729 3. Rs. 270 4. Rs. 96.42

7. The ratio of the present age of a mother to that of her daughter is 9: 4. Ten years ago, the ratio of their ages was 7: 2. What are their present ages?

1. 54 years, 24 years
2. 45 years, 20 years
3. 54 years, 28 years
4. 36 years, 16 years

8. The ratio of the present ages of Chinta and Mani is 14: 19. The average age of Suknaya, Chinta and Mani is 43 years. If the ratio of present age of Suknaya to that of Chinta is 5: 7, find the present age (in years) of Mani.

1. 28 2. 42 3. 57 4. 56

9. The ratio of number of chocolates with Munni and Chameli is 3: 2. If Chameli has 5 chocolates less than Sheela who has three more chocolates than Munni, find the total number of chocolates with three of them.

1. 19 2. 20 3. 23 4. 24

10. In an organization, ratio of the number of males to that of the females is 4: 3. If the number of males is increased by 10% and that of females by 20%. What is the new ratio of the number of males to that of females?

1. 11: 9 2. 10: 9 3. 11: 8 4. 3: 2

11. Three friends A, B and C went on a picnic. A brought 5 apples and B brought 3 apples, and C had Rs. 80. They divided the apples equally among themselves and C gave all his money to A and B for their contributions. What are the respective shares of A and B from that money?

1. 50, 30 2. 70, 10 3. 40, 40 4. 33.33, 66.66

12. A student scored marks in the ratio 5: 4: 6: 8: 7 in five subjects having equal maximum marks. In all, he scored 50% of the maximum marks in all the five subjects

taken together. In how many subjects did he score more than 55% of the maximum marks?

1. 2 2. 1 3. 3 4. Cannot be determined

13. The sum of four numbers is 253. The ratio of the first number to the second one is 2: 3. The ratio of the second number to the third one is 5: 6. The ratio of the third number to the fourth one is 8: 9. What is the average of the second and the third number?

1. 60 2. 64 3. 62 4. 66

14. When three brands of juices are mixed in the ratio of 3: 4: 5 and 4: 5: 6, the cost price of the two-piece mixtures comes out to be Rs. 20 per litre and Rs. 25 per litre respectively. Find the cost price of a litre of mixture of juices in which the three brands of original juices are mixed in the ratio of 6: 7: 8.

1. Rs. 30.7 3. Rs. 30.5 3. Rs. 29.5 4. Rs. 29.7

15. The value of a diamond is directly proportional to the square of its weight. If a diamond weighing 4 kg breaks into 2 pieces, its total value decreases by 37.5%. Weights of the two pieces are:

1. 2 kg, 2 kg 2. 2.5 kg, 1.5 kg 3. 3 kg, 1 kg 4. None of these

16. A family consists of father, mother, son and daughter. The ratio of the weight of the father to that of the son is 3: 2. The ratio of the weight of the son to that of the mother is 5: 6. If the weight of the daughter is 35 kg, which is half the weight of the father, find the weight (in kg) of the mother.

1. 42 2. 56 3. 46.6 4. Cannot be determined

17. The ratios of the incomes and the expenditures of Aishwarya, Babita and Charu are 7: 9: 12 and 8: 9: 15 respectively. If Aishwarya saves one-fourth of her income, then the ratio of their savings is:

1. 56: 99: 69 2. 69: 56: 99 3. 99: 56: 69 4. 99: 69: 56

18. One year ago, the ratio of Harsha's and Mandar's salaries was 3: 5. The ratios of their individual salaries of last year and present year are 2: 3 and 4: 5 respectively. If their combined salary for the present year is Rs.86,000, find the present salary (in Rs) of Harsha.

1. 38,000 2. 24,000 3. 39,000 4. 36,000

19. The numbers of students in Batch A and Batch B were in the ratio 2: 3 in January and 5: 8 in February. The numbers of students in Batch A and Batch B increased from February to March at rates that were twice and thrice, respectively, of rates at which they increased from January to February. If the ratio of the total number of students in these two batches in February and January was 26: 5, then find the ratio of number of students in Batch A and Batch B in March?

1. 25: 64 2. 25: 84 3. 45: 112 4. 35: 92

20. A person buys some apples and mangoes from the market. Cost price of a mango is twice that of an apple and the selling price of a mango is thrice that of an apple. By selling an apple at twice its cost price, he makes 150% profit on the whole. Find the ratio of the number of mangoes to that of apples that he bought from the market.

1. 3: 5 2. 3: 4 3. 1: 2 4. 2: 3

21. Total expenses of a boarding house are partly fixed and partly varying linearly with the number of boarders. The average expense per boarder is Rs. 700 when there are 25 boarders and Rs. 600 when there are 50 boarders. What is the average expense per boarder when there are 100 boarders?

1. 550 2. 580 3. 540 4. 570

22. A is directly proportional to B. B is inversely proportional to C. C is directly proportional to the square of D. D is directly proportional to the cube root of E. Assuming all are positive integers, as A increases, E

1. increases 2. decreases 3. remains constant 4. doubles

23. The pressure of a gas is directly proportional to its temperature and inversely proportional to its volume. The initial pressure, volume and temperature are 150 pascals, 20 cu. m and 100° Kelvin respectively. Find the final temperature if the final volume and pressure of the same gas are 8 cu. m and 150 pascals respectively under the same conditions.

1. 80° Kelvin 2. 50° Kelvin 3. 40° Kelvin 4. 45° Kelvin

24. A, B and C invested their money in the ratio 3: 6: 7. If the total amount invested by them was Rs. 80,000 and the profit earned was 40% of the amount invested. Then what was the sum of the shares (in Rs) of B and C in the profit?

1. 18,000 2. 26,000 3. 26,500 4. 24,000

25. A, B and C invested Rs. 2,000, Rs. 5,000 and Rs. 4,000 respectively in a business. The net profit for the year was Rs. 1,210 which was divided in proportion to investment. Then the amount of profit earned by A, B and C respectively, is

1. Rs. 220, Rs. 550, Rs. 420 2. Rs. 220, Rs. 450, Rs. 440
3. Rs. 220, Rs. 550, Rs. 440 4. Rs. 210, Rs. 450, Rs. 440

26. X, Y and Z are partners in a business with a total capital of Rs. 3,300. The profit at the end of the year is Rs. 1,500 to be divided in proportion to their capitals. If X receives Rs. 450 and Y receives Rs. 550 as their share of profits, then Z's capital (in Rs) is:

1. 910 2. 1,100 3. 1,110 4. None of these

27. Three people A, B and C entered into partnership and the ratio of their investments was 1/2: 1/3: 1/4. The ratio of the time period for which they invested their money was 1: 1/2: 1/4. The profits were shared in the direct proportion to amount invested and time. If A's share is Rs. 30,000, then find B's share.

1. Rs. 10,000 2. Rs. 20,000 3. Rs. 15,000 4. Rs. 25,000

28. A began a business with Rs. 4,500 and was joined after wards by B with Rs. 5,400. If the profits at the end of the year were divided in the ratio 2: 1, then B joined the business after:

1. 5 months 2. 4 months 3. 6 months 4. 7 months

29. Ajay started a firm with a capital of Rs. 28,000. After 5 months, Boman joined him and invested Rs. 40,000 in the firm. Chirag was also added as a new partner with an individual investment of Rs. 56,000 after 7 months of commencement. If at the end of the year, the profit of the firm is Rs. 32,000, what is the share of Boman?

1. Rs. 12,000 2. Rs. 8,000 3. Rs. 14,000 4. Rs. 10,000

30. Abhishek and Rani started a partnership business by investing Rs.1 cr and Rs.2 cr respectively in the beginning of the year. Each of them withdrew one-fourth of their investment after each quarter till 2nd quarter. At this point Susmita and Randeep joined them with Rs.50 lakh and Rs.1 cr respectively. Randeep withdrew his entire investment after 3 months. If the profit at the end of the year is Rs. 41 lakhs, the combined share of Abhishek and Rani is:

1. Rs. 22 lakh 2. Rs. 33 lakh 3. Rs. 8 lakh 4. Rs. 30 lakh

Exercise 2

1. The strength of a class is 70. Which of the following cannot be the ratio of the number of boys and girls in the class?

1. 2: 5 2. 3: 2 3. 11: 3 4. 9: 2

2. The ratio of the number of students in three classes A, B and C is 3: 7: 8. If ten students, are transferred from C to B, B will have 80 students. Find the total number of students in the three classes.

1. 180 2. 150 3. 210 4. 240

3. The ratio of the incomes of A and B is 4: 3. The ratio of the savings of A and B is 5: 6. Find the ratio of the expenditures of A and B, if A spends three-fourth of his income.

1. 3: 2 2. 5: 3 3. 3: 5 4. 2: 3

4. The ratio of the monthly incomes of A and B is 3: 4. The ratio of the monthly expenditures of A and B is 4: 5. Which of the following represents a possible value of the ratio of their savings?

1. 9: 10 2. 3: 4 3. 13: 20 4. 4: 5

5. Three positive numbers p, q and r satisfy $\frac{q+r}{p} = \frac{p+r}{q} = \frac{p+q}{r} = k$, then find the value of k.

1. 3/2 2. 5/2 3. 3 4. 2

6. The area of a square varies with the square of its diagonal. The area of a square whose diagonal measures eight units is 32 sq. units. Find the diagonal of a square of area 72 sq. units (in units).

1. 10 2. 12 3. 15 4. 8

7. A, B and C are three quantities. A varies directly with B when C is constant. A varies directly with C when B is constant. A = 6000 if B = 20 and C = 30. Find A if B = 40 and C = 60.

1. 20000 2. 24000 3. 18000 4. 22000

8. The weights of Bimal and Basu are in the ratio 2 : 3 and the weights of Basu and Bali are in the ratio 4 : 3. What is Basu's weight (in kg) if the sum of the weights of Bimal, Basu and Bali is 203 kg?

1.80 2. 84 3. 72 4. 96

9. A certain number is added to each of a pair of numbers which are in the ratio 4: 7. The sum of the resulting numbers is 75 and their ratio (taken in the same order as mentioned above) is 8: 17. What is the number added?

1. 12 2. – 12 3. 10 4. – 10

10. Rohan and Sohan had certain amounts of money with them. The ratio of the amounts with them is 8: 5. Each day Rohan spends a certain amount and Sohan earns one – sixth of the amount that Rohan spends. After 9 days, the ratio of the amounts with them is 10: 11. After how many more days, will the ratio of the amounts with them be 18: 35?

1. 15 2. 9 3. 6 4. 8

11. Three different types of balls priced at Rs 5, Rs 8 and Rs 13 per piece are displayed in three different boxes by a trader. Mr. Paul bought from this shop all three types of balls spending a total sum of Rs 768. The numbers of the balls he bought, taken in the order in which the prices are mentioned above, are in the ratio 5: 4: 3. How many balls of the costliest variety did he buy?

1. 104 2. 64 3. 48 4. 24

12. A certain amount of money is divided among nine brothers. The second brother gets Rs 2 more than twice the amount given to the first brother. The third brother gets Rs 3 more than thrice the amount given to the first brother, and so on till the ninth brother. If the ratio of the amount with the ninth brother to the amount with the first brother is 10: 1, how much did the fifth brother get (in Rs)?

1. 100 2. 80 3. 90 4. 50

13. Alok distributed a certain number of toffees among his brothers Bala, Chetan and David in the ratio 4: 4: 9. David distributed the toffees that he received among his sisters Amitha, Bama and Chandra in the ratio 1: 7: 8. If Amitha received 18 toffees, find the total number of toffees distributed by Alok.

1. 584 2. 536 3. 544 4. 496

Directions for questions 14 and 15: These questions are based on the data given below.

The amount used to purchase one litre of petrol can be used to purchase three litres of diesel or five litres of kerosene. Out of a certain amount, Rs 510 is spent on diesel.

14. How much is spent on kerosene if equal volumes of the three liquids are purchased with the total amount?

1. Rs 300 2. Rs 306 3. Rs 382 4. Rs 354

15. What will be the amount spent on petrol if the total amount referred in the above question is instead spent to purchase equal volumes of petrol and kerosene only?

1. Rs 1250 2. Rs 1275 3. Rs 1955 4. Rs 1360

16. A number of coins can be divided in the ratio 13: 14: 15. They can also be divided in the ratio 9: 10: 11: 12. If the difference between the greatest parts in the two cases is 6, the number of coins is:

1. 42 2. 84 3. 126 4. 168

17. The mean proportional between two numbers is 9 and the third proportional of the two numbers is 243. Find the larger of the two numbers.

1. 27 2. 81 3. 9 4. 54

18. The volume of a cone varies as the square of the radius of its base when its height is fixed and also varies as its height when radius of its base is fixed. If a radius of 3 cm and height of 7 cm give it a volume of 66 cubic centimetre, then what will be the radius, (in centimetres), if the cone has a height of 6 cm and volume of 308 cubic centimetres?

1. 9 2. 15 3. 7 4. 11

19. A writer gets a fixed amount for his book apart from the royalty he gets per book sold. He gets Rs 46000 and Rs 66000 for 2000 and 3000 books sold respectively. Find his income per book when 6000 books are sold (in Rs).

1. 20 2. 22 3. 23 4. 21

20. The speed of a locomotive without any wagons attached to it is 40 kmph. It diminishes by a quantity which is proportional to the cube root of the number of wagons attached. If the speed of the locomotive is 34 kmph when 27 wagons are attached, what is the maximum number of wagons that can be attached if the condition is that the speed should not fall below 30 kmph?

1. 64 2. 125 3. 216 4. 343

21. A precious stone worth Rs 10872 fell and broke into three pieces, the weights of which are proportional to 1: 2: 3. The value of each stone is directly proportional to the square of its weight. Find the loss in the value caused by the breakage.

1. Rs 3624 2. Rs 4228 3. Rs 6644 4. Rs 7510

22. The amount collected per month from a consumer of electrical power consists of two parts – a fixed charge for providing the service and a variable charge which is directly proportional to the number of units of power consumed. An amount of Rs 700 is collected from the consumer when he consumed 50 units in a month. It is also noticed that when the consumption increases from 100 units per month to 200 units per month, the bill amount increases to 5/4 times that of the former. How much is the fixed charge per month?

1. Rs 600 2. Rs 350 3. Rs 500 4. Rs 400

23. Ajay distributed a total of 60 sweets among his sons Ram, Shyam and Tarun. For every five sweets received by Ram, Shyam received four sweets. For every two sweets received by Shyam, Tarun received three sweets. Find the number of sweets received by Tarun.

1. 24 2. 26 3. 28 4. 30

24. Five vessels have equal capacities. Each vessel contains some milk. The ratio of the quantities of milk in these vessels is 3: 4: 5: 6: 7. The total quantity of milk is three-fifth of the combined capacity of the vessels. How many vessels contain at least half milk?

1. 2 2. 1 3. 3 4. 4

25. Manoj and Shiva, who are colleagues in an office, have their monthly savings in the ratio of 2: 3. Manoj spends two-thirds of his income every month. If the ratio of their monthly incomes is 3: 4, what is the ratio of their expenditures?

1. 3: 2 2. 4: 5 3. 4: 3 4. 5: 3

26. A person had two plots of land. In each plot, he cultivated maize and barley. The ratio of the area under maize cultivation and barley cultivation in the larger plot is 8: 9. The ratio of the total areas under maize cultivation and barley cultivation in the two plots together is 29: 33. The ratio of the areas under maize cultivation and barley cultivation in the smaller plot is 13: 15. The ratio of the area under maize cultivation in the larger plot and that under barley cultivation in the smaller plot is:

1. 8: 15 2. 16: 15 3. 15: 16 4. 15: 8

27. The ratio of Ahmed's age to Mohammed's age is the same as the ratio of the ages of their respective elder brothers. The ratio of the difference of the ages of Ahmed and Mohammed to that of the difference of the ages of their respective brothers is 1: 2. What is the ratio of the sum of the ages of their respective brothers to the sum of the ages of Ahmed and Mohammed?

1. 2: 1 2. 4: 1 3. 3: 1 4. 3: 2

28. Sachin started a business with Rs 20000 and after 4 months Sunil joined him with Rs 40000. Sachin received Rs 39000 as his annual profit share which included a salary of 16% of the annual profit. Find the annual profit share (in Rs) of Sunil.

1. 30000 2. 32000 3. 36000 4. 40000

29. A started a business with Rs 20000. After 3 months, B joined him with Rs 40000. After some more months, C joined them with Rs 100000. B received Rs 18000 out of the total annual profit of Rs 55000. How many months after A started the business did C join?

1. 6 2. 7 3. 5 4. 4

30. Serena and Venus start a business with Rs 50000 each. At the end of 3 months Venus withdraws Rs 10000 from his investment. What percent of the total profit should Venus receive at the end of the year?

1. 33.3% 2. 45 5/6% 3. 50% 4. 45 35/37%

31. Mehta, Mehra and Mihir start a business with investments in the ratios of 1: 2: 3. They make a profit of Rs 600000. If Mihir and Mehra leave after 4 and 8 months respectively, then what is Mehta's share of the total profit (in Rs)?

1. 170000 2. 175000 3. 180000 4. 150000

32. Goyal and Gokul invest Rs 20000 and Rs 30000 respectively and start a business. Since Goyal manages the business, he gets a salary which is equal to 20% of the balance profit after deducting this salary. If Goyal's total earnings are Rs 4800 at the end of one year, what is the total profit made by the business?

1. Rs 4000 2. Rs 5200 3. Rs 9600 4. Rs 6400

33. Ganga and Gayathri start an enterprise by investing Rs 24000 and Rs 36000 respectively. Their agreement is to share half of the total profit equally and then share the remaining half in the ratio of their capitals. If they share the entire profit in the ratio of their capitals, Gayathri would have got Rs 2500 more than what she would have got otherwise. What is the total profit?

1. Rs 75000 2. Rs 72000 3. Rs 108000 4. Rs 50000

34. Gopal, Hari and Karthik started a business with investments of Rs 8000, Rs 12000 and Rs 16000 respectively. Hari and Karthik left the business after x months. Out of the annual profit share, Gopal got more than Hari but less than Karthik. If x is an integer, find the ratio of Gopal's, Hari's and Karthik's shares.

1. 27: 21: 28 2. 24: 21: 28 3. 30: 27: 36 4. 32: 30: 40

35. P and Q started a business. They made an annual profit of Rs 50000. Q being a working partner received 20% of the annual profit as his salary. If the entire profits were divided in the ratio of their investments, P would have received Rs 8000 more as his profit share than what he actually got. Find P's actual profit share (in Rs).

1. 24000 2. 30000 3. 28000 4. 32000

36. A, B, C and D started a business with investments in the ratio 3: 4: 5: 6. As B and C were working partners, they were paid equal salaries. The ratio of B's and C's total annual income is 9: 10. If the total annual profit is Rs 84000, find B's salary (in Rs).

1. 12000 2. 15000 3. 18000 4. 10000

Exercise 3

1. State true or false: If $\frac{a}{b} = \frac{c}{d}$ and $\frac{e}{f} = \frac{g}{h}$, then $\frac{ae+bf}{ae-bf} = \frac{cg+dh}{cg-dh}$

1. Always true
2. Always false
3. Depends on relative values of a and e only
4. Depends on relative values of a, c, e, g only

2. If the square of x varies directly as the cube of y and $x = 3$ when $y = 4$, find the value of y when $x = \frac{1}{\sqrt{3}}$.

1. $\frac{2}{\sqrt{3}}$ 2. $\frac{4}{\sqrt{3}}$ 3. $\frac{2}{3}$ 4. $\frac{4}{3}$

3. If $\frac{y}{x-z} = \frac{y+x}{z} = \frac{x}{y}$, find the ratio of x: y: z.

1. 2: 3: 4 2. 2: 4: 3 3. 4: 3: 2 4. 4: 2: 3

4. An amount of Rs. 735 was divided between A, B and C. If each of them had received Rs. 25 less, their shares would be in the ratio of 1: 3: 2. Find the share of C.

1. Rs. 170 2. Rs.195 3. Rs. 220 4. Rs. 245

5. Rs. 5390 is divided into three parts in such a way that half of the first part, one-third of the second part and one-sixth of the third part are in the ratio 2: 3: 6. Find the second part.

1. Rs. 1796.66 2. Rs. 3960 3. Rs. 1470 4. Rs. 990

6. Rs. 500 is divided among A, B, C and D such that C gets as much as A and B together get, D gets as much as A and C together get and A and D together get as much as B and C together get. Find the share of C.

1. Rs. 200 2. Rs. 150 3. Rs. 125 4. Rs. 100

7. Rs. 3000 is distributed among A, B and C such that A gets $\frac{2^{rd}}{3}$ of what B and C together get and C gets $\frac{1}{2}$ of what A and B together get. Find B's share.

1. 800 2. 1000 3. 1200 4. 1500

8. The ratio of the number of boys and girls in a class is 3: 5. Certain amount was distributed to girls and boys such that the ratio of the amount received by each girl to that received by each boy is 1: 2. If a student on an average receives 27.5 paise, which of the following data can be accurately determined?

I. The actual number of boys and girls in the class

II. The actual amount that each girl and boy receive

III. The total amount distributed

1. All three 2. Only I 3. Only II 4. Only III

9. The incomes of A and B are in the ratio 3: 2 and their expenses are in the ratio 5: 3. If each of them saves Rs. 1000, then find the income of A.

1. Rs. 3000 2. Rs. 4500 3. Rs 5000 4. Rs. 6000

10. In a particular city the demographics is such that there are four men for every three women and five children for every three men. If the city has 81,180 women, find the number of children in the city.

1. 135,300 2. 36,331 3. 180,400 4. 158,400

11. A factory employs skilled workers, semi-skilled workers and unskilled workers in the proportion 5: 2: 3 and the wages of each skilled, semi-skilled and unskilled worker is in the ratio 8: 5: 1. The total wage bill of the factory is Rs. 3975. If it is known that the number of workers that the factory employs is a number between 20 and 50 and the wages paid to each employee of each category is an integral value, find the wages paid to each employee of each category of workers respectively.

1. Rs. 200, Rs. 125, Rs. 25
2. Rs. 120, Rs. 75, Rs. 15
3. Rs. 600, Rs 375, Rs 75
4. Could be either (1) or (2)

12. A and B enter into a partnership investing Rs. 16,000 and Rs. 12,000 respectively. After 3 months, A withdrew Rs. 5,000 while B invested Rs. 5,000 more. After 3 more months, C joins the business with a capital of Rs. 21,000. If the total profit at the end of the year is Rs. 26,400, by what amount does B's share exceed the share of C?

1. Rs. 1200
2. Rs. 2400
3. Rs. 3600
4. Rs. 3000

13. A cat takes 5 leaps for every 4 leaps of a dog, and 3 leaps of the dog are equal to 4 leaps of the cat. And a fox takes 3 leaps for every 5 leaps of the same dog, and 2 leaps of the fox are equal to 3 leaps of the dog. What is the ratio of the speeds of the cat and the fox?

1. 25: 24
2. 5: 3
3. 20: 21
4. 9: 10

14. The value of a diamond is proportional to the square of its weight. A diamond weighing 4 gm falls and breaks into two pieces. What are the weights of the pieces if the value of the diamond reduces by 37.5%?

1. 1 gm, 3 gm
2. 1.5 gm, 2.5 gm
3. 2 gm, 2 gm
4. 1.2 gm, 2.8 gm

15. Certain amount is distributed among A, B and C such that A receives the largest amount and C receives the lowest amount. The ratio of the differences in the amounts of A & B, and B and C is in the ratio 2: 3. If A receives Rs. 125 more than C and B receives Rs. 175, find the total amount distributed.

1. 400
2. 450
3. 500
4. 475

16. If a, b, c and d are in continued proportion, find the ratio $\frac{a^3+b^3+c^3}{b^3+c^3+d^3}$ in terms of b and c.

1. $\frac{b}{c}$
2. $\frac{c}{b}$
3. $\frac{b^3}{c^3}$
4. $\frac{c^3}{b^3}$

17. If a, b, c and d are in continued proportion, find the mean proportional between $a + b$ and $c + d$.

1. $a + d$
2. $b + c$
3. 1
4. $b - c$

18. a, b and c are three positive numbers and $X = \frac{a+b+c}{2}$. If $(X - a)$: $(X - b)$: $(X - c)$ = 2: 5: 7, find the ratio of a: b: c.

1. 7: 4: 2
2. 8: 5: 3
3. 9: 6: 4
4. 12: 9: 7

19. y varies as the sum of two quantities, of which one varies directly as x and the other inversely as x. If $y = 6$ when $x = 4$ and $y = \frac{10}{3}$ when $x = 3$, find the equation between x and y.

1. $y = x + \frac{8}{x}$
2. $y = x + \frac{1}{x}$
3. $y = 2x - \frac{8}{x}$
4. $y = 2x - \frac{1}{x}$

20. y varies as the sum of three quantities of which the first is constant, the second varies as x and the third varies as x^2. $y = 0$ when $x = 1$; $y = 1$ when $x = 2$; and $y = 4$ when $x = 3$. Find the value of y when $x = 7$.

1. 36
2. 9
3. 16
4. 25

21. When a body falls from rest, its distance from the starting point varies as the square of the time it has been falling. If a body falls through 175 m in 5 seconds, how far does it fall in the 10^{th} second?

1. 7 m 2. 700 m 3. 70 m 4. 133 m

22. The expenses of a school are partly constant and partly vary as the number of boys. The expenses were Rs. 10,000 for 150 boys and Rs. 8,400 for 120 boys, what will be the expenses when there 330 boys?

1. Rs.18,400 2. Rs. 19600 3. Rs. 23,100 4. Rs. 22,000

23. The electricity bill of a certain establishment is partly fixed and partly varies as the number of electricity consumed. When in a certain month 540 units are consumed, the bill is Rs. 1800. In another month 620 units are consumed and the bill is Rs. 2040. What will be the bill amount when 500 units of electricity are consumed?

1. Rs. 1666.66 2. Rs. 1650 3. Rs. 1680 4. Rs. 1660

24. The reduction in speed of an engine is directly proportional to the square of the number of bogies attached. If the speed of the engine is 120 kmph when 14 bogies are attached and 100 kmph when 16 bogies are attached, find the speed of the engine when there are 20 bogies attached.

1. 60 kmph 2. 48 kmph 3. 52 kmph 4. 80 kmph

25. The consumption of coal (tones per hour) by a train engine is directly proportional to the square of the speed of the engine. When the speed of the engine is 40 kmph, the consumption of coal per hour is 2 tonnes. Find the total cost of a journey of 100 kms covered at a speed of 50 kmph if the price of coal is Rs. 10 per tonne.

1. Rs. 62.5 2. Rs. 25.6 3. Rs. 12.8 4. Rs. 31.25

26. The cost of a boarding school is partly fixed and partly varies directly with the number of boarders. When there are 400 boarders, the cost per head is Rs. 120 and when there are 500 boarders, the cost per head is Rs. 100. Find the cost per head when there are 800 boarders.

1. Rs. 40 2. Rs. 62.5 3. Rs. 60 4. Rs. 70

Average and Alligation & Mixtures

Average:

Average means balancing of the terms. It means that when we have certain numbers with us and we want to make them all equal without changing their sum, then that equal number will be the average of all the numbers.

Mathematically, Average = $\frac{Sum\ of\ terms}{Number\ of\ terms}$

Or, we can have **Sum of terms = Average × Number of terms.**

The above result is applicable in all the questions of average.

Ex. **Find the average of 12, 15, 21, 28, 34.**

Sol: Here, the average of the numbers = $\frac{Sum\ of\ terms}{Number\ of\ terms} = \frac{12+15+21+28+34}{5} = \frac{110}{5} = 22.$

Ex. **The average weight of 15 men in a group is increased by 1 kg when one of the men who weighs 60 kg is replaced by a new man. Find the weight of new man.**

Sol: We know that, Sum of terms = Average × Number of terms. Now, this formula can be used to get the answer.

Let the average weight of the original 15 men is x, so the average weight of the final 15 men will be x + 1. Let the weight of the new man is y kg

Using the above result, we have, 15x – 60 + y = 15 (x + 1)

⇒ 15x – 60 + y = 15x + 15 ⇒ y = 75 kg.

Note: Following points should be kept in mind while solving the questions of average.

I. ***If the difference in the terms is same and the total number of terms are odd (say 'n'), then the average will be the middle term, i.e., (n +1)/2th term.***

E.g., If we have the numbers 4, 7, 10, 13, 16, 19, 22 then we can see that the difference in these numbers is same. In this case we can directly say that the average will be the middle term. Here, number of terms are 7. So, n = 7. Hence, the middle term is (7 + 1)/2 = 4th term which is 13. So, the average of the given numbers is 13.

II. ***If the difference in the terms is same and the total number of terms are even (say 'n'), then there will be two middle terms n/2th term and (n/2 + 1)th term and the average of all the numbers will be equal to the average of these two middle terms.***

E.g., If we have the numbers 4, 7, 10, 13, 16, 19, 22, 25, then again, the difference in the terms is same and there are two middle terms here, i.e., 13 and 16. So, the average of all the terms is equal to the average of 13 and 16 which is 14.5.

Note: ***In the above 2 pointers, you can see that the terms are in AP and hence the average can be found as (First Term + Last term)/2.***

Ex. **Consider a sequence of seven consecutive integers. The average of the first five integers is n. Find the average of all the seven integers.**

1. n **2. n + 1** **3. n + (2/7)** **4. (n + 2)/7**

Sol: Here, we have consecutive integers, which means the average will be the middle term. Now the middle term of the first five integers will be the 3rd term and it is the average which is given to be equal to 'n'. So, the 4th term will be n + 1. Now the middle term of all the 7 integers will be the 4th term and it will be the required average. Hence, the average of all the seven integers is n + 1.

III. ***If same number is added to each of the given numbers, then the average also increases by the same amount.***

E.g., Average of 10, 20, 30, 40 and 50 is 30. Now If we add same number say 5 to every number, then the numbers will become 15, 25, 35, 45 and 55 and their average will be 35 which is 5 more than the original average.

IV. If same number is subtracted from the given numbers or same number is multiplied to each of the given numbers, then the average also decreases/multiplies by the same amount. The same happens with the division also.

Let us now consider what happens to the average of the given numbers if a number is increased or decreased by some amount.

Let us take an example where there are five children A, B, C, D and E having 10, 20, 30, 40 and 50 marbles respectively. The average number of marbles will be 30 with them. Now, what does it mean? As discussed at the start of this article that average simply means balancing of the terms which means that if we want to equate the number of marbles with these children then everyone will get 30 marbles. Now how will that happen? Here, C has his quota of 30 marbles. B has 10 marbles less than the average and D has 10 marbles more than the average. So, D will give his additional 10 marbles to B and both will have 30 marbles each. Similarly, E will give his additional 20 marbles to A and both will have 30 marbles each. Now, consider that say A got 20 more marbles from his parents. So, what will he do with these additional 20 marbles. Obviously, as they have earlier distributed the marbles equally, so he will divide these additional 20 marbles with the other children. So, everyone will get 20/5 = 4 additional marbles. Therefore, the number of marbles with each of them will be 34 and so as the average. Now, what do we learnt from here?

"If a number is increased by some amount, then that increase will be distributed on all the given numbers equally and whatever is received by each number, the average also increases by the same amount."

You can see that the same happened in the above example also, where the additional 20 marbles obtained by A got distributed equally among all the five children and the average also increased by 4.

Now, what will happen If say E lost his 20 marbles? In this case the loss of E will be shared by all the five children. So, 20/5 = 4 is the number of marbles that will be given to E by A, B, C and D so that everyone has 26 marbles now and that is also the average numbers of marbles with them.

E.g., Let us consider the numbers 13, 16, 20, 21, 35. The average of these numbers is 21. Now if we replace the number 13 with the numbers say 18, then what will be the average?

Well, the answer is very easy. Replacing the number 13 by 18 means that 13 is increased by 5. Now, this increase will be equally divided among all the 5 numbers. So, every number will get 5/5 = 1, so the average will also increase by 1. Therefore, the new average will be 21 + 1 = 22.

Hence, we can say that **"If a number is decreased by some amount, then that decrease will be distributed on all the given numbers equally and whatever is reduced from each number, the average also decreases by the same amount."**

The above discussed two points are useful when we do the questions without making the equations and that really saves the time in the competitive exams like CAT, XAT etc.

In the next part, we will discuss about the change in average if a number is deleted from the group of numbers.

Well, let us again take the same example as above where the children A, B, C, D and E have marbles with them and the average marbles with them is 30. Now, what will happen if say A leaves the group. Will the average increase or decrease?

Remember that as per the concept of average, each one of them has 30 marbles. Now when A will leave the group, then he will return the 20 marbles back to E. So, B, C, D and E will have 20 marbles as surplus with them and they will divide them equally. So, each will get additional 20/4 = 5 marbles and total marbles with them will be 35 each and average will also become 35. So, in this case, the average increased.

Let us now consider another case where E leaves the group. In this case, A will lose the 20 marbles he got from E. Now this loss of A will be shared by A, B, C and D equally. So, everyone will share 20/4 = 5 marbles and they will have 25 marbles each. In this case the average will decrease to 25.

It means that if a number smaller than the average is deleted, then the average will increase and if a number greater than average is deleted then the average will decrease. This increase or decrease in the average is equal to the difference of the deleted number from the average divided by the remaining numbers.

E.g., Let us consider the numbers 13, 16, 20, 21, 35. The average of these numbers is 21. Now if we delete the number 13, then what will be the new average?

Well here the difference of 13 from the average is 8. So, average will increase by 8/4 = 2. The new average will be 21 + 2 = 23.

Weighted Average: The weighted average is the average where the numbers in the data have different degree of importance. While calculating the weighted average, each number is multiplied by the weight assigned to it and then all the values are added.

Let there are n values x_1, x_2, x_3, ……., x_n having the weights w_1, w_2, w_3,……., w_n respectively.

The weighted average = $\frac{w_1x_1+w_2x_2+w_3x_3+\ldots\ldots\ldots w_nx_n}{w_1+w_2+w_3+\ldots\ldots\ldots+w_n}$.

Ex. **A test was given to three sections A, B and C of class 9. The average scores of the sections A, B and C are 30, 35 and 40 respectively. The number of students in sections A, B and C are 20, 25 and 30 respectively. Find the average score of the class 9.**

Sol: Here the number of students in each section is different, so the formula of weighted average will be used to find the average of the class.

Weighted average = $\frac{30\times20+35\times25+40\times30}{20+25+30}=\frac{2675}{75}=35.67$

Therefore, the average of the class is 35.67.

Let us now discuss few examples:

Ex. **The average of Rahul after 50 one day matches is 70. If Rahul's target is to increase his average to 72 after the next one-day match how much should Rahul score in the next match.**

1. 165 **2. 172** **3. 160** **4. 176**

Sol: Total score of Rahul after 50 matches = 50 × 70 = 3500

If his average after 51 matches is 72, then the total will be = 51 × 72 = 3672

Hence, Rahul should score 3672 – 3500 = 172 runs in the next match.

Ex. **From a group of 16 students with average weight 42.5 kg, two students leave the group and because of that the average of the group decreases to 41.25 kg. Find the average weight of the two students who left the group.**

1. 51.25 **2. 51.75** **3. 50.5** **4. 52.25**

Sol: Total weight of 16 students = 16 × 42.5 = 680 kg

Total weight of the remaining 14 students = 14 × 41.25 = 577.5

Therefore, the total weight of the two students who left the group = 680 – 577.5 = 102.5 kg. Hence, the average weight of the two students = 102.5/2 = 51.25.

Ex. **If a student weighing 70 kg joins a group of *n* students, the average weight of the group increases by 1 kg. If the new student weighed 55 kg, the average weight of the group would have declined by 2 kg. Find *n*.**

1. 3 **2. 4** **3. 5** **4. 6**

Sol: Let the average weight of the group is x kg

Now, in first case nx + 70 = (x + 1) × (n + 1)

⇒ nx + 70 = nx + n + x + 1

⇒ n + x = 69 ……..(i)

In the second case, nx + 55 = (x – 2) (n + 1)

⇒ nx + 55 = nx + x – 2n – 2

⇒ x – 2n = 57 …..(ii)

Solving (i) and (ii), we get 3n = 12 or n = 4.

Ex. **The average of 13 consecutive numbers is 324. If there is another series of consecutive numbers which starts with the same number as the previous series, then what is the average of first 20 numbers of this series.**

1. 327 **2. 326.5** **3. 327.5** **4. 328.5**

Sol: Since the numbers are consecutive the average would be the same as the middle term. The middle term of 13 numbers is the 7th number which is 324. Since the next series start with the same number, the 10th number is 327 And 11th number is 328. So, the average of 20 consecutive numbers is the average of 10th and 11th number i.e., average of 20 numbers is equal to (327 + 328)/2 = 327.5.

Ex. **The average age of 20 women is 49 years. The age of the youngest woman is 32 years and that of the oldest woman is 56 years. If two women with ages 45 and 50 years leave the group and three women join the group, the average remains unchanged. What is the average age (in years) of the three women who join the group later?**

1. 48 **2. 32** **3. 49** **4. 45**

Sol: Total age of the group of 20 women = 49 × 20 = 980 years.

Total age of the women who left the group = 45 + 50 = 95 years.

Let, the average age of 3 women who joined later be x years.

Hence, the sum of their ages = 3x.

Since the average age after the leaving and joining of women is still 49 years, therefore,

980 + 3x – 95 = 49 × 21

⇒ 3x + 885 = 1029 ⇒ 3x = 144 ⇒ x = 48 years.

Ex. **A class consists of 20 boys and 30 girls. In the mid-semester examination, the average score of the girls was 5 higher than that of the boys. In the final exam, however, the average score of the girls dropped by 3 while the average score of the entire class increased by 2. The increase in the average score of the boys is:**

1. 9.5 **2. 10** **3. 4.5** **4. 6**

Sol: The average score of 30 girls decreased by 3, so, the total score of the girls decreased by 30 × 3 = 90. The average of the class still increased by 2, i.e., the total score of the class increased by 50 × 2 = 100 and this increase happened in spite of the

decrease in the marks of the girls. It means that all the increase happened because of the marks of the boys and they have compensated for the decrease in marks of the girls also. Hence, the marks of boys must have been increased by 90 + 100 = 190 and their average increased by 190/20 = 9.5.

Ex. **In an apartment complex, the number of people aged 51 years and above is 30 and there are at most 39 people whose ages are below 51 years. The average age of all the people in the apartment complex is 38 years. What is the largest possible average age, in years, of the people whose ages are below 51 years?**

1. 26 **2. 27** **3. 28** **4. 25**

Sol: Let there are 'n' people below the age of 51 years.

The total age of all the people = (30 + n) × 38 years.

Now, the average age of the people below the age of 51 years will be maximum if the average age of the people above 51 years of age is minimum i.e., their average age is 51 years. Hence, the total of the age of people below 51 years = (30 + n) × 38 – 51 × 30 = 38n – 390.

Average age of the people below 51 years of age = (38n – 390)/n = 38 – 390/n

This average will be maximum if 390/n is minimum i.e., n is maximum and maximum value of n = 39. Hence, the maximum average age = 38 – 390/39 = 28 years.

QUANTIFIERS CAT ACADEMY

Exercise - 4

1. The average age of a group of 23 persons increases by 4 months, when a person of 20 years is replaced by a new person. The age of the new person is:

1. 28 years
2. 27 years 8 months
3. 27 years
4. 26 years 6 months

2. The average age of a group of officers goes down to 56 years from 58 years, when an old member is replaced by a young member. What is the total number of members in that team if the age difference between those two members is 30 years?

1. 15
2. 20
3. 18
4. 30

3. The average height of 13 people reduces by 2 cm if a person of height 184 cm is replaced by a new person. Find the height of the new person?

1. 154 cm
2. 159 cm
3. 197 cm
4. 158 cm

4. There are 24 students in a class whose average marks in a subject, the maximum marks of which is 100, is 89. If 3 students leave the class, then what is the maximum by which the average could go up?

1. 10.7
2. 10.5
3. 11.2
4. 11

5. A shipping clerk has five boxes of different but unknown weights each weighing less than 100 kg. The clerk weights the boxes in pairs. The weights obtained are 110, 112, 113, 114, 115, 116, 117, 118, 120 and 121 kg. What is the weight of the heaviest box?

1. 60 kg
2. 65 kg
3. 64 kg
4. 62 kg

6. There are 40 boys in a class. One of them weighing 100 kg goes away. A new boy joins the class at the same time. Consequently, the average weight of the boys is increased by 0.25kg. Find the weight of the new boy.

1. 98.5 kg
2. 110 kg
3. 105 kg
4. 99 kg

7. The average rainfall for Monday, Tuesday and Wednesday is 4.3 cm. The average rainfall for Friday, Saturday and Sunday is 3.9 cm. If the average rainfall for the total week is 3.7 cm, then what is the rainfall recorded on Thursday?

1. 1.4 cm
2. 1.3 cm
3. 1.2 cm
4. 1.1 cm

8. The average age of 5 members of a committee is the same as it was 3 years ago, because an old member has been replaced by a new member. The difference between the ages (in years) of old and new member is:

1. 12
2. 15
3. 10
4. 14

9. The average marks obtained by the candidates who passed and those who failed in an examination are 60 and 25 respectively. The average marks obtained by all the candidates who appeared for the examination is more than 43 but not more than 49. The number of candidates who passed as a fraction of the total number of candidates who appeared for the examination could never be:

1. 3/5
2. 4/7
3. 18/35
4. 24/35

10. The average age of a class of 60 students is 12 years. If the number of girls is increased by 3/4th and the number of boys is decreased by half, the total number of students does not change; nor does the average age of either the boys or the girls change. But the combined average age changes to 13.5 years. The average age of the boys (in years) is:

1. 8
2. 10
3. 12
4. Cannot be determined

11. The average age of the students in an institute was 15 years. Then 10 students of average age 12 years joined the institute and the average age of the students of this

institute became 14.8 years. What was the initial number of students in that institute?

1. 140 2. 120 3. 150 4. None of these

12. Ramu appears in six different papers in his semester examination, where the maximum marks were 50 for each paper. His marks in these papers are in the proportion 8: 9: 10: 13: 14: 15. Considering his aggregate in all the papers together, he fails to obtain 50% of the total marks. What is the minimum possible additional marks Ramu should get to obtain 50% of the total marks, given that he got integral marks in each paper?

1. 81 2. 57 3. 12 4. 18

13. The average age of a group of friends is 37 years. If 6 new friends whose average age is 35 years join them, the average age of the entire group becomes 36 years. How many people were there in the group initially?

1. 5 2. 8 3.10 4. 6

14. The average age of the boys in a class is 18 years. The average age of the girls in that class is 12 years. If the ratio of the numbers of boys and girls in that class is 3: 2, find the average age (in years) of the class.

1. 14.5 2. 15.6 3. 16.5 4. 14.8

15. The average weight of all the students in a class equals the average of the average weight of the boys and that of the girls, which of the following holds true?

1. The numbers of boys and girls in the class are equal.
2. The average weights of the boys and the girls are equal.
3. At least one of option 1 or 2
4. Neither option 1 nor 2

16. The temperatures on Sunday, Monday and Tuesday were 28° C, 30° C and 32° C respectively. If the temperature of each day from Tuesday to Friday (both days inclusive) was 1° C less than that of the following day, what was the difference in the average temperatures of the first four days of the week and that of the last three days of the week?

1. 4.5° C 2. 4.25° C 3. 0.5° C 4. 4.75° C

17. The average age of a board of 10 advisors of a company is the same as it was 3 years ago, on account of the replacement of one of the older advisors by a younger man. How much younger is the new man than the director whom he replaced?

1. 24 years 2. 40 years 3. 52 years 4. None of these

18. The average of n numbers is a. If one of the numbers, y, is replaced by x, the average becomes b. Find the relation between n, a, x, y and b.

1. $\frac{1}{n} = \frac{x-y}{a-b}$ 2. $\frac{1}{n} = \frac{a-b}{y-x}$ 3. $\frac{1}{n} = \frac{x+y}{a+b}$ 4. $\frac{1}{n} = \frac{a+b}{x+y}$

19. The average weight of all the students of classes I and II equals the average of the average weight of the students of the two classes. There are twice as many students in class II as in class I. The sum of twice the average weight of the students of class I and the average weight of the students of class II is 60 kg. Find the average weight of class I (in kg).

1. 18 2. 20 3. 15 4. 25

20. There are N students in a class. Their class teacher gave them a task of finding the average of the first N natural numbers and each of them left out a different number and found the average of the remaining numbers. The average of the averages obtained by all the students was 21. Find N.

1. 39 2. 40 3. 41 4. 38

21. A's weight equals the average weight of B, C and D. B's weight equals the average weight of A, C and D. The average weight of C and D is 30 kg. Find the average weight of A and B.

1. 15 kg 2. 30 kg 3. 60 kg 4. 45 kg

22. In a group of 11 people, x is 32 years old and y is 4 years younger than x. If x and y are replaced by two other people, the average age of the group drops by 1 year. Find the average age of the two people replacing x and y.

1. 26 years 2. 28 years 3. 29 years 4. 24.5 years

23. A basketball player played nine matches. The average number of points he scored per match was 16. His points in the i^{th} match were two less than that in the $(i-1)^{th}$ match. Find the average number of points scored in the second and the eighth matches.

1. 8 2. 12 3. 16 4. 18

24. There are eleven people in a group. The average age of the oldest and the youngest is 11 years. If any one person leaves the group, the maximum and the minimum average of the remaining are 12 years and 11 years respectively. Find the average age of the entire group, in years.

1. 9 2. 10 3. 11 7/11 4. 11 5/11

25. The average of the ages of a man and his only son exceeds the average of the ages of his wife and his son by 25%. The age of his son is 30 years less than the age of his wife whose age is 10 years less than his age. Find the age of his son (in years).

1. 5 2. 8 3. 10 4. 9

26. There are two classes A and B. The average weight of the students in class A is 40 kg. The average weight of the students in class B is 60 kg. A student, whose weight is x kg left A and joined B. As a result, the average weight of A, as well as that of B decreased. Which of the following must be true?

1. $35 < x \leq 40$ 2. $40 < x < 60$ 3. $60 \leq x < 65$ 4. $30 < x \leq 35$

27. The average cost per fruit of 5 apples and 4 mangoes is Rs 48 and the average cost per fruit of 7 apples and 8 mangoes is Rs 60. Find the total cost of 24 apples and 24 mangoes.

1. Rs 1332 2. Rs 2424 3. Rs 2664 4. Rs 3996

28. The students of three classes P, Q and R take a test. The average mark per student of P and Q put together is 60. The average mark per student of Q and R put together is 64. The average mark per student of P and R put together is 72. If the average mark per student of all the classes put together is x, which of the following is a possible value of x?

1. 61.5 2. 61.9 3. 62.5 4. 68.3

Alligation

Alligation and mixtures is one of the important topics of CAT, XAT and other competitive examinations. Alligation is the method used to find out the ratio in which two items of the same type should be mixed to get the desired mixture. Mathematically, alligation is just the weighted average.

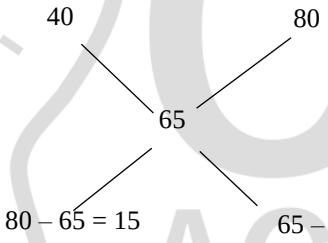
Let us take an example. In what ratio, the two varieties of rice having cost price of Rs 40/kg and Rs 80/kg should be mixed so that the resulting mixture has cost of Rs 65/kg?

Now, the above problem can be solved by using the concept of weighted average. Let us take m kg of the first variety and n kg of the second variety.

Total cost price = 40m + 80n and total weight is m + n kg. As the average cost of the mixture is Rs 65, we have, $\frac{40m+80n}{m+n} = 65 \Rightarrow 40m + 80n = 65m + 65n \Rightarrow 80n - 65n = 65m - 40m \Rightarrow 15n = 25m \Rightarrow m/n = 15/25$ or $m/n = 3/5$. So, the two varieties should be mixed in m: n or 3: 5.

So, in the above example, it can be seen that the two varieties should be mixed in 3: 5. So you can take 3 kg of the first and 5 kg of the second variety, or 6 kg of the first and 10 kg of the second variety and so on.

Pictorial representation of alligation rule: The above example of alligation can also be represented in a pictorial way as shown in the in the diagram below. While making this diagram, we should keep the given extreme values above in the diagram and the required value (weighted average value) in the middle. Then we take the cross differences and write down those differences. The ratio of these cross differences is the required ratio. Let us draw the diagram for the above-mentioned example as below.



Therefore, the two varieties should be mixed in ratio 15: 25 or 3:5.

Note: While taking the cross difference, always subtract the smaller value from the larger one.

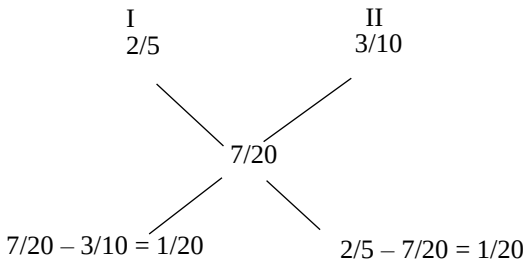
The alligation rule can be applied to many problems of mixtures or whenever we are mixing two items of the same type in different proportions to get some desired value. It can be used in the questions of simple interest, average, profit and loss etc. Let us discuss few examples to understand it better.

Ex. **Two solutions of contain milk and water in the ratio 2:3 and 3: 7. In what ratio the two solutions should be mixed so that the ratio of milk and water in the final solution is 7: 13?**

Sol: Before, we solve this question, you should remember one thing that the rule of alligation can be applied to the items of same type. So here, either the rule of alligation can be applied on milk or on water. So, we will solve it in two ways: first by milk and then by using water.

Method I: Using milk:

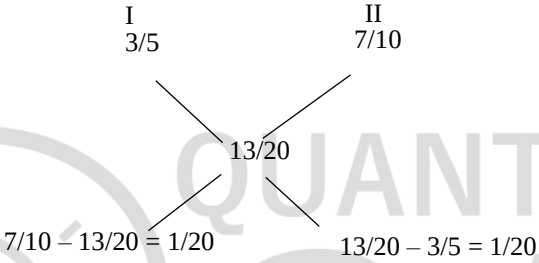
Here the ratio of milk and water in the first solution is 2: 3 ⇒ Milk = 2/5 of the total solution. Similarly, milk in the second solution is 3/10 of the total solution and in the final solution, milk is 7/20 of the total solution. Now, using the rule of alligation, we have



Hence, the two solutions should be mixed in the ratio 1/20: 1/20 or 1: 1.

Method II: Using water: We can also solve the above problem by taking water.

The water in the first solution is 3/5 of the total, in the second solution, the water is 7/10 of the total and in the final solution, it is 13/20 of the total. Therefore, by alligation, we have,



Therefore, the required ratio is 1: 1.

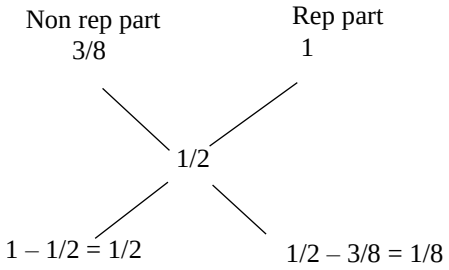
So, we can solve the question by any method and the result will not change.

Ex. **The ratio of milk and water in a solution is 3: 5. What proportion of this solution should be replaced with pure milk so that the ratio of milk and water in the final solution becomes 1: 1?**

Sol: This question is based on the replacement of the solution with some pure milk. We can again use either milk or water to solve it.

Method I: By using milk: The milk in the original solution is 3/8 of the total mixture and we are replacing the mixture with pure milk. So, we will take the second extreme value as 1. The amount of milk in the final solution is 1/2 of the total.

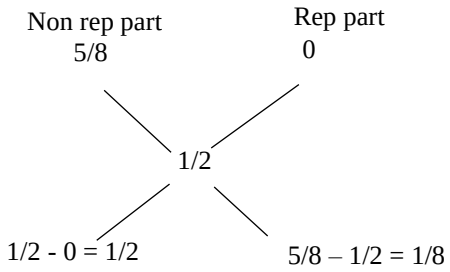
By alligation, we have,



Therefore, the ratio of the non replaced and replaced part is 1/2: 1/8 or 4: 1. This ratio means that if there are 5 (4 + 1) parts in total, then we should replace only one part out of the total five. Hence, $1/5^{th}$ of the total solution should be replaced by pure milk to get ratio of milk and water as 1: 1.

Method II: By using water: The water in the original solution is 5/8 of the total and as we are replacing the mixture with pure milk and not water, so the second extreme value will be taken as 0. The amount of water in the final solution is 1/2 of the total.

By alligation, we have,



Therefore, the ratio of the non replaced and replaced part is again 1/2: 1/8 or 4: 1 which means that $1/5^{th}$ of the original solution should be replaced.

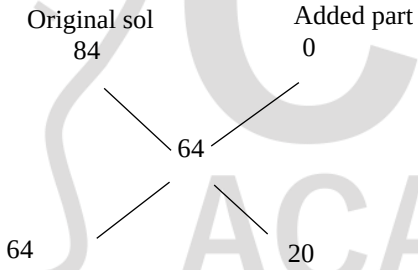
So, from the above two examples, it is now clear that we can solve the questions in either way i.e., by using either of the liquids of the solutions. Now onwards we will use only one liquid to get the answer.

Note: In the questions where some part of the solution is replaced, the total parts of the original solution will be the sum of replaced part and the non replaced part. E.g., in the above example, the ratio came out to be 4: 1. So the total parts of the original solution = 4 + 1 = 5.

Ex. **32 litres of milk and water solution contains 84% milk. How much water should be added to this solution so that the concentration of milk becomes 64%.**

Sol: Concentration of milk in the first solution is 84% and in the second is 0% (As pure water is added)

By rule of alligation,

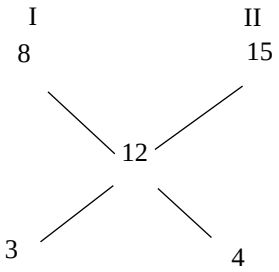


Therefore, the required ratio is 64: 30 or 32: 10. It means that if original part is 32 litre, then 10 litre water should be added.

Ex. **Ramesh invested Rs 14000 in two schemes at simple interest for one year. In the first scheme, he received 8% return and on the second he got 15% return. If he received a total of Rs 1680 as overall return, then find the amount invested in the two schemes.**

Sol: Overall return = 1680 × 100/14000 = 12%.

Now, using the alligation rule,



Hence, the amounts invested in the two schemes is in ratio 3: 4.

Therefore, amount invested in the first scheme = 3 / 7 × 14000 = Rs 6000

And the amount invested in the second scheme = 4/7 × 14000 = Rs 8000

Successive Replacement of Dilution:

Let us take an example to understand the successive replacement. If a vessel has x litre milk and y litre of milk is removed from it and replaced with y litre pure water, then the milk left in the vessel after the replacement is (x – y) litre and the total milk water solution in the vessel is again x litre (x – y + y). So, after the replacement, the proportion of the milk in the final solution is:

$\frac{Quantity\ of\ milk}{Quantity\ of\ milk\ and\ water\ solution} = \frac{x-y}{x}$. If the above process is repeated 'n' times then

Quantity of milk after n^{th} replacement = $x(1 - y/x)^n$.

Ex. **8 litres out of 80 litres of pure milk are replaced with an equal quantity of water. Again 8 litre of the resultant solution is replaced with an equal quantity of water. Find the final amount of the milk after the second iteration.**

Sol: We have, Quantity of milk after n^{th} replacement = $x(1 - y/x)^n$.

⇒ Quantity of milk after 2^{nd} replacement = $80 \times (1 - 8/80)^2 = 800 \times (9/10)^2 = 80 \times 81/100 = 64.8$ litre

Alternatively: We can say that 8/80 is 1/10. So, every time, $1/10^{th}$ of the milk is taken out and $9/10^{th}$ of the milk is left. Hence, final amount of milk = $80 \times 9/10 \times 9/10 = 64.8$ litre.

Ex. **A vessel is completely filled with petrol. 10 litre is drawn from this vessel and replaced with kerosine. 10 litre of this solution is again drawn from the vessel and replaced with kerosine. After the second iteration, the ratio of petrol and kerosine in the vessel is 49: 32. Find the capacity of the vessel.**

Sol: Let the total volume is x litre. As the final solution has ratio of petrol and kerosine as 49: 32, so petrol in the final solution = 49x/81.

Also, petrol after 2 iterations = $x(1 - 10/x)^2$

⇒ $49x/81 = x(1 - 10/x)^2$

⇒$7/9 = 1 - 10/x$ ⇒ $10/x = 1 - 7/9$ ⇒ $10/x = 2/9$ ⇒ $x = 10 \times 9 / 2 = 45$ litre.

Alternatively: In such questions $\frac{Quantity\ of\ original\ liquid\ left}{Total\ solution}$ is a perfect square or perfect cube or perfect fourth power and so on if the iterations are done twice, thrice, four times and so on respectively.

Now, in the above example, $\frac{Quantity\ of\ original\ liquid\ left}{Total\ solution}$ should be a perfect square as the iteration was done twice. The given ratio is 49: 32, so $\frac{Quantity\ of\ original\ liquid\ left}{Total\ solution} = \frac{49}{81} = \left(\frac{7}{9}\right)^2$.

It means that after every iteration, $7/9^{th}$ of the original liquid i.e., petrol is left. It means that every time, $2/9^{th}$ of the petrol was withdrawn. So, if the total volume is x litre, therefore, $2x/9 = 10$ ⇒ $x = 45$ litre.

Let us take another example.

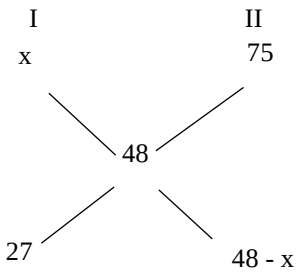
Ex. **A vessel is completely filled with milk. 15 litre milk is drawn from this vessel and replaced with water. 10 litre of this solution is again drawn from the vessel and replaced with water. This process is repeated once more. The final ratio of milk and water is 27: 37. Find the capacity of the vessel.**

Sol: Here, the process is repeated thrice, therefore, the ratio $\frac{Quantity\ of\ original\ liquid\ left}{Total\ solution}$ should be a perfect cube which is 27/64 or $(3/4)^3$. It means that every time $1/4^{th}$ of the total solution is withdrawn.

So, if the total volume is x, then 1/4 of x = 15 ⇒ x = 60 litre.

Ex. **When 18 litres of milk and water solution is mixed with 20 litres of milk and water solution having 75% percent of milk, the resultant mixture has 48% milk. Find the percentage of milk in the first solution.**

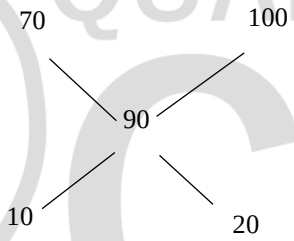
Sol: Let the original mixture has x% milk. The ratio of the two mixtures is 18: 20 or 9: 10. Using the rule of alligation, we have,



Now $\frac{27}{48-x} = \frac{9}{10} \Rightarrow 270 = 432 - 9x \Rightarrow 9x = 162 \Rightarrow x = 18$.

Ex. **How many litres of milk needs to be added to 15 litres of water and milk solution having 70% milk to result in a solution having 90% milk?**

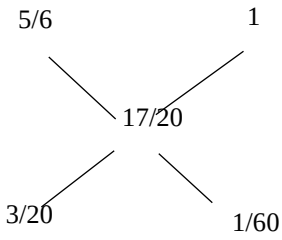
Sol: Here, we are adding pure milk i.e., 100% milk. Therefore, by alligation



Therefore, the two solutions should be mixed in ratio 10: 20 or 1: 2. As the first is 15 litres, so the second should be double of that i.e., 30 litres.

Ex. **A milk-man mixes water equal to 20% of the quantity of milk he has, to the milk. Suspecting that his customers will notice the dilution, he decides to add pure milk to the mixture such that the ratio of milk and water increases to 17: 3. If he had added 12 litres of water, how much litres of milk will he need to add.**

Sol: Initially the ratio of the milk and water is 5: 1. The ratio of milk and water in the final mixture is 17: 3. Let us, use the rule of alligation on milk.



The ratio of the original mixture and the added part is 3/20: 1/60 or 9: 1.

Now if he added 12 litres of water, then original milk was 60 litre and the total mixture was 12 + 60 = 72 litre. So, the amount of milk needs to add is 1/9 × 72 = 8 litre.

Ex. **A solution of milk and water having 20% milk is mixed with another solution having 50% milk in the ratio 2: 5. If 21 litre of this mixture is taken and mixed**

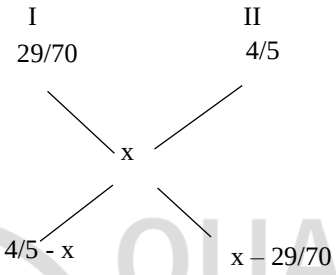
with 9 litre of another milk and water solution having 80% milk, find the percentage of milk in the final mixture.

Sol: Let the two original mixtures taken in the ratio 2: 5 are 20 litre and 50 litre respectively.

Amount of milk and water in the first mixture is 4 litre and 16 litre respectively. Amount of milk and water in the second mixture is 25 litre each. So, in the final solution of first step, the amount of milk is 29 litre and amount of water is 41 litre. Therefore, the ratio of milk and water is 29: 41.

Milk in this mixture is 29/70 of total and the milk in second solution is 80% or 4/5 of the total. Let the amount of milk in the final solution is x litre.

Using alligation,



Given that $\frac{4/5 - x}{x - 29/70} = \frac{21}{9} \Rightarrow 36/5 - 9x = 21x - 609/70$

$\Rightarrow 30x = 609/70 + 36/5 \Rightarrow 30x = 1113/70 \Rightarrow x = 1113/2100 = 53\%$.

Ex. **Anil removes a fraction of pure milk and replaces it with water to avoid detection. He does so on two consecutive days. If the milk now accounts for only 64% of the solution, what fraction of solution did he remove daily?**

Sol: Here, the process is repeated twice, therefore, the ratio $\frac{Quantity\ of\ original\ liquid\ left}{Total\ solution}$ should be a perfect square which is 64/100 or 16/25 or $(4/5)^2$. It means that every time $1/5^{th}$ of the total solution is withdrawn.

Ex. **A cask is full of wine but it has a leak in the bottom. When one-fourth of the cask empties out because of the leak, the cask is replenished with water. Next when half of the cask has leaked out, it is again filled with water. Finally, when three-fourths of the cask leaks out, it is again filled with water. What is the percentage of wine in the cask now?**

Sol: Let the original volume of the wine is 100 litre.

Wine after 3 iterations = 100 × 3/4 × 1/2 × 1/4 = 9.375 litre. As the total volume remains same, so, the percentage of wine in the final solution is 9.375%.

Ex. **A petrol tank at a filling station has capacity of 400 litres. The attendant sells 40 litre of petrol from the tank to one customer and then replenishes it with kerosine oil. This process is repeated with six customers. What quantity of the pure petrol will the seventh customer get when he purchases 40 litres of petrol?**

Sol: Quantity of petrol remained after 6 replacements = $400\ (1 - 40/400)^6 = 400 \times 0.531$ = 212.4 litre. So, the seventh customer will get 212.4 × 1/10 = 21.2 litres of petrol.

Exercise - 5

1. A solution of 20 litre of wine and water contains 80% wine. How much water (in litre) must be added to this solution in order to make it a 40% wine solution?

1. 15 2. 20 3. 18 4. 25

2. In a mixture of 120 litre, the ratio of milk and water is 2: 3. If 30 litre of mixture is replaced by 18 litre of milk and 12 litre of water, then new ratio of milk to water is:

1. 7: 9 2. 9: 11 3. 2: 3 4. 11: 9

3. In what ratio should 30% Phenyl Alcohol should be mixed with 60% of Phenyl Alcohol solution so that the resultant solution has 50% Phenyl Alcohol in it?

1. 1: 2 2. 2: 1 3. 1: 3 4. 3: 1

4. Two liquids A and B are mixed in the ratio of 3: 2 respectively and the mixture is sold at Rs. 11 per litre at 10% profit. If the first liquid costs Rs. 2 per litre more than the second, the cost (rupees per litre) of first liquid is:

1. 10.8 2. 8.8 3. 18.8 4. 5

5. In a mixture of 105 litre, the ratio of milk and water is 8 : 7. If some amount of mixture is removed in which 14 litre was water and 5 litre of water is added afterwards into the remaining mixture, then the new ratio of milk to water is:

1. 19: 20 2. 4: 5 3. 1: 1 4. 2: 3

6. A milkman mixes 20 litre of water with 80 litre of milk. After selling one-fourth of this mixture, he adds water to replenish the quantity that he had sold. What is the current ratio of water to milk?

1. 2: 3 2. 1: 2 3. 1: 3 4. 3: 4

7. A can contains a solution of two liquids A and B in the ratio of 7: 5. When 9 litre of solution is drawn off from the solution and the can is filled with B, the proportion of A and B becomes 7: 9. How many litre of liquid A was there in the can initially?

1. 21 2. 15 3. 24 4. None of these

8. After the addition of 35 liters of water to a can of diluted milk, the concentration of milk in the can becomes 30%. Now, further 40 liters of water is added to the can and the concentration of milk in the can gets reduced by 10 percentage points. How many more liters of water must be added to the can now such that the concentration of milk in the can becomes 8%?

1. 150 2. 175 3. 160 4. 180

9. There are two alloys of copper and aluminium in the ratio 7: 2 and 7: 11 respectively. If equal quantities of the two alloys are melted to form a third alloy, then find the ratio of copper to aluminium in newly formed alloy.

1. 3: 4 2. 5: 3 3. 7: 3 4. 7: 5

10. An alloy of copper and zinc contains 82% copper. After 18 kg of zinc had been added to the alloy, the copper content gets reduced to 70%. How much copper and zinc does the new alloy contain?

1. Copper 88 kg, zinc 35 kg 2. Copper 86.1 kg, zinc 36.9 kg

3. Copper 80.5 kg, zinc 42.5 kg 4. Copper 70 kg, zinc 30 kg

11. The volume of a sugar solution in three different vessels form an arithmetic progression with a common difference of 5 kg. All the three sugar solutions have the same percentage concentration of sugar. The difference between the numerical values of the concentration (in percentage) of sugar in the sugar solution and the volume of sugar solution in the vessel having the least quantity is 8. The total

quantity of sugar in the three vessels is 5.4 litre. What are the quantities of sugar solution in the three vessels?

1. 5 kg, 10 kg and 15 kg
2. 6 kg, 11 kg and 16 kg
3. 7 kg, 12 kg and 17 kg
4. 8 kg, 13 kg and 18 kg

12. Three milkmen Ramu, Kamu and Namu had 5L, 10L and 15L of milk-water solution respectively. In the solution that Kamu and Namu had, the ratio of milk and water were 2 : 3 and 1 : 4 respectively. All of them sat together and mixed their respective solutions together. They sold the combined mixture at Rs. 20 per litre and Ramu got Rs. 2.5 for every litre of mixture sold. What was the milk-water ratio in the solution that Ramu had with him? (Assume water comes free of cost and they share the revenue earned in the ratio of milk content)

1. 1: 4
2. 2: 3
3. 1: 5
4. 3: 5

13. One-litre can is full with a mixture of water and alcohol, and alcohol content is 20% of the total mixture. If 10% of this liquid is taken out and pure alcohol is added to fill the can, what will be the percentage of alcohol in the new mixture?

1. 25%
2. 28%
3. 30%
4. 22%

14. A vessel contains 180 litre wine. 60 litre is taken out of the vessel every day and an equal volume of water is put in it. What volume (in litre) of wine remains in the vessel at the end of 3 days?

1. 160/3
2. 20/3
3. 80
4. 35

15. Some amount of acid is taken out of a 54-litre vessel full of acid and then an equal amount of water is added to it. The process is repeated once more. As a result, the vessel now contains 24 litre pure acid. How much of the acid was taken out initially (in litre)?

1. 20
2. 18
3. 25
4. 22

16. A solution contains 20 L of pure milk. 20% of this is taken out. Another 20% of the remaining solution is taken out and finally 20% of the solution left is taken out. The total amount of solution taken out is replaced with water. Find the ratio of milk to water in the final solution.

1. 1: 5
2. 12: 38
3. 488: 512
4. 512: 488

17. Two identical vessels are filled with alcohol. From the first vessel 'a' litre of the solution is taken out and replaced with 'a' litre of water. From the resulting mixture 'a' litre of the mixture is removed and again replaced with 'a' litre of water. Same operation is done in the second vessel the same number of times, but the amount of solution removed and replaced with water is '2a' litre. What fraction of the volume of the vessels is 'a' if the strength of alcohols in the two vessels finally is in the ratio 25: 16?

1. 1/4
2. 5/6
3. 2/3
4. 1/6

18. A beaker contains milk and water in the ratio 5: 3. If a person removes 6 litre of this mixture and replaces it with pure water, the ratio of milk to water becomes 1: 1. What is the amount (in litre) of milk present in beaker initially?

1. 18.75
2. 16.25
3. 20
4. 18

19. Product M is produced by mixing chemical X and chemical Y in the ratio of 5: 4. Chemical X is prepared by mixing two raw materials, A and B, in the ratio of 1: 3. Chemical Y is prepared by mixing raw materials, B and C, in the ratio of 2: 1. Then the final mixture is prepared by mixing 864 units of product M with water. If the concentration of the raw material B in the final mixture is 50%, how much water was added to product M?

1. 328 units
2. 368 units
3. 392 units
4. 616 units

20. A container has 80 litres of milk. From this container, 8 litre of milk was taken out and replaced by water. The process was further repeated once. How much milk (in litre) is there in the container now?

1. 64.8 2. 62.8 3. 62 4. 64

21. In what ratio should water and 84% milk solution be mixed to obtain 48% milk solution?

1. 4: 3 2. 9: 13 3. 2: 3 4. 3: 4

22. Vessel A has 20 litres of a mixture of milk and water having 75% milk. Vessel B has x litre of a mixture of milk and water having 60% milk. The contents of the vessels are mixed to form a mixture having 66% milk. Find x.

1. 35 2. 30 3. 25 4. 20

23. A vessel has 30 litres of a mixture of milk and water containing 80% milk. How many litres of water must be added to the vessel to reverse the ratio of milk and water?

1. 60 2. 75 3. 90 4. 120

24. From 90 litres of pure milk, 9 litres is withdrawn and replaced by water. 9 litres of the mixture is then withdrawn and replaced by water. Find the ratio of milk and water in the present mixture.

1. 19: 81 2. 19: 100 3. 81: 19 4. 81: 100

25. The students of three classes X, Y and Z wrote a test. The average marks of the students of X, Y and Z are 70, 81 and 90 respectively. The average marks of the students of X and Y is 76. The average marks of the students of Y and Z is 86. Find the average marks of the students of all the three classes to the nearest integer.

1. 80 2. 82 3. 78 4. 85

26. Two alloys A and B contain copper and zinc in the ratio 4: 9 and 5: 6 respectively. If equal weights of the two are melted together to form a third alloy, find the ratio of the weights of copper and zinc in the third alloy named C.

1. 109: 167 2. 113: 164 3. 109: 177 4. 107: 158

27. Two varieties of wheat are mixed in the proportion of 3: 4 and the mixture is sold at Rs 28 per kg at a profit of 40%. If the second variety of wheat costs Rs 3 more than the first variety of wheat, find the cost price per kg of the first variety of wheat.

1. Rs 128/7 2. Rs 120/7 3. Rs 141/7 4. Rs 149/7

28. A vessel has 10 ml of a solution of milk and water containing 20% milk. x ml of milk was added to the vessel to reverse this ratio. y ml of water was then added to the vessel to reverse the ratio once again. Find x + y.

1. 160 2. 165 3. 140 4. 150

29. In what proportion can three varieties of sugar priced at Rs 10 per kg, Rs 12 per kg and Rs 18 per kg, be mixed so that the price of the mixture is Rs 14 per kg?

1. 2: 2: 5 2. 2: 3: 4 3. 1: 3: 4 4. 3: 4: 5

30. There are two containers having mixtures of hydrochloric acid and water. In container 1, the ratio of hydrochloric acid and water is 1: 2 and in container 2 the ratio of hydrochloric acid and water is 4: 1. Find the amount of the mixture that should be taken from container 1 in order to make 28 litres of a mixture containing equal amount of water and hydrochloric acid. (in litres)

1. 20 2. 18 3. 25 4. 15

31. The ratio of alcohol and water in three mixtures of alcohol and water is 3: 2, 4: 1 and 7: 3. If equal quantities of the mixtures are drawn and mixed, the concentration of alcohol in the resulting mixture will be:

1. 65% 2. 70% 3. 75% 4. 80%

32. From a vessel containing only alcohol, six litres are drawn and replaced with water. six litres of the mixture is now taken out and replaced with water. The ratio of alcohol to water now is 9: 16. How many litres of alcohol was there initially?

1. 12 2. 18 3. 14 4. 15

33. A vessel contains a mixture of 100 litres of milk and water. The concentration of milk is 90%. 10% of the contents of the vessel are withdrawn and replaced with an equal amount of water. The minimum number of times that this procedure must be carried out such that the concentration of milk in the vessel is less than 66 2/3% is:

1. 2 2. 4 3. 3 4. 5

34. Two vessels P and Q were partially filled with milk. 80% of the contents of P were transferred to Q. 60% of the contents of Q were then transferred back to P. At this stage, the ratio of the quantities of milk in P and Q is 23: 12. Find the ratio of the initial volumes of milk is P and Q.

1. 5: 3 2. 10: 6 3. 15: 11 4. 5: 2

35. Rs 8000 is lent in two parts. A part is lent at 10% p.a. and the other part is lent at 20% p.a. Both parts are lent at simple interest for a year. The total interest realized from them is Rs 960. Find the difference of the parts (in Rs).

1. 4800 2. 2400 3. 1600 4. 6400

36. Vessels A and B contain mixtures of milk and water. The ratios of milk and water in A and B is 4: 5 and 3: 1 respectively. In what ratio should the contents of A and B be drawn and mixed, to obtain a mixture having milk and water in the ratio 3: 2?

1. 27: 28 2. 23: 24 3. 11: 12 4. 35: 36

37. Two varieties of tea costing Rs 280 per kg and Rs 180 per kg are mixed in certain ratio to form varieties P and Q. P and Q are mixed in the ratio 1: 2 to form variety R which is sold for Rs 288 per kg at 20% profit. If P costs Rs 200 per kg, in what ratio were the two varieties mixed to form Q?

1. 2/1 2. 5/2 3. 4/1 4. 8/3

38. A vessel is full of a mixture of milk and water, with 9% milk. Nine litres are withdrawn and then replaced with pure water. If the milk is now 6%, how much does the vessel hold (in litre)?

1. 27 2. 18 3. 36 4. 40

39. A vessel is filled to its capacity with pure milk. Nine litres are withdrawn from the vessel and replaced with an equal amount of water. Nine litre of the mixture is again withdrawn and then replaced with an equal amount of water. After these changes, the vessel contains 17.1 litres of milk less than it did initially. Find the capacity of the vessel. (In litre)

1. 80 2. 90 3. 100 4. 75

Exercise – 6

1. The average temperature of the town on the first four days of a month was 58 degrees. The average of the temperatures on the second, third, fourth and fifth days was 60 degrees. If the temperature of the first and the fifth days were in the ratio of 7: 8, then find the temperature (in degrees) on the fifth day.

1. 64 2. 62 3. 60 4. 58

2. If the number of innings when Sachin got out for a duck (scored zero runs) is excluded, his average increases from 63 to 66. If Sachin has played a total of 110 innings, find the number of innings when he was out for a duck. (Average is found by dividing the total number of runs scored by the number of innings played)

1. 2 2. 3 3. 5 4. 6

3. The average of a group of eight friends increases by 4 years when the youngest of them is not considered and decreases by 3 years when the oldest of them is not considered. Find the difference (in years) between the ages of oldest and the youngest of them.

1. 49 2. 52 3. 55 4. 56

4. A bowler's average is found by dividing the runs conceded by the wickets taken. Akram's career bowling average before today's match was 19.2. In today's match he took 7 wickets while conceding 84 runs and his bowling average after the match decreased by 0.2. How many wickets had Akram taken in his career before today's match?

1. 231 2. 238 3. 245 4. 252

5. In 'The New Age B-School', there are 15 faculty members. Whenever a faculty attains the age of 60 in a particular year, he retires at the end of the year and is replaced by a new recruit from the start of the new year. During the year 2005, the average age of the faculty members was 48 years and three faculty members reached 60 years that year. At the start of 2006, they were replaced with three new members whose average age was 36 years. Find the average of the ages (in years) of the faculty in the year 2006.

1. 41 2. 44 3. 45 4. 47

6. Three years ago, the average age of a husband, wife and their child was 27 years. Five years ago, the average age of the wife and child was 20 years. Find the present age (in years) of the husband.

1. 36 2. 40 3. 42 4. 45

7. Out of 9 persons, 8 persons spent Rs. 30 each for their meals. The ninth one spent Rs. 20 more than the average of expenditure of all the nine. Find the total money (in Rs) spent by all of them.

1. 52.5 2. 290 3. 292.5 4. 300.25

8. The average age of a class, including the teacher is 1.5 years more than the average age of the class when the teacher is not considered. If the teacher is 30 years older than the average age of the students, find the number of students in the class.

1. 27 2. 24 3. 20 4. 19

9. The sum of ten consecutive odd natural numbers is 180 more than the average of the numbers. Find the sum of the largest and the smallest of the ten numbers.

1. 40 2. 45 3. 60 4. 90

10. While entering the monthly sales data of the year 2005 in a computer file, a sales figure of three digits was entered wrongly by reversing the order of the digits e.g., instead of '*abc*', it was keyed in as '*cba*'. Find the maximum decrease in the average monthly sales of the year that could be possible because of this error.

1. 80 2. 78.5 3. 76.75 4. 74.25

11. A man purchased a cow and a calf for Rs.1300. He sold the calf at a profit of 20% and the cow at a profit of 25%. His total profit was 23%. Find the selling price of the cow.

1. Rs. 500 2. Rs. 520 3. Rs. 620 4. Rs. 780

12. A dishonest milkman purchased milk at Rs. 10/litre and mixed 5 litres of water in it. By selling the mixture at the rate of Rs 10/litre he earns a profit of 25%. Find the total amount of the mixture (in litres) that he sold.

1. 10 2. 15 3. 20 4. 25

13. In what ratio should water be mixed with soda costing Rs 12 per litre so as to make a profit of 25% by selling the diluted liquid at Rs. 13.75 per litre?

1. 1: 4 2. 1: 5 3. 2: 9 4. 1: 11

14. A solution having milk and water in the ratio 2: 3 is mixed with another milk and water solution. The resultant solution has milk and water in the ratio 5: 4. Find the ratio of milk and water in the second solution if the two solutions were mixed in equal quantities.

1. 3: 1 2. 5: 9 3. 21: 8 4. 32: 13

15. Volume of a solution, which has milk and water in the ratio 3: 2, is increased by 10% by adding pure milk. What is the ratio of milk and water in the resultant solution?

1. 7: 4 2. 3: 4 3. 7: 11 4. 11: 7

16. When 20% of a solution of milk and water is removed and replaced with water, the ratio of milk and water becomes 2: 3. Find the ratio of milk and water in the original solution.

1. 1: 1 2. 1: 2 3. 2: 1 4. 1: 3

17. How many litres should be removed out of 40 litres of solution of milk and water and be replaced with equal amount of water so that the proportion of milk drops from 80% to 75%.

1. 2 2. 2.5 3. 3 4. 3.5

18. 10% of a solution of milk and water is removed and as much water as the quantity removed, is added to the solution. Next 9.09…% of solution is removed and as much water as the quantity removed, is added to the solution. Again 8.33…% of solution is removed and as much water as the quantity removed, is added to the solution. If the solution now has equal amount of water and milk, find the ratio of milk and water in the original solution.

1. 1: 2 2. 2: 3 3. 2: 1 4. 3: 2

19. To 10 litres of a solution of milk and water, 2 litres of water is added and then 2 litres of solution is removed. The ratio of milk and water in the solution now is 2: 3. Find the ratio of milk and water in the original solution.

1. 3: 2 2. 3: 4 3. 5: 6 4. 12: 13

20. A milkman ordered his assistant to first remove 3 litres out of the 8 litres of a solution of milk and water and then add 3 litres of water to it. Thinking that it would not make any difference, the lab assistant first added 3 litres of water and then removed 3 litres of the solution. Find the ratio of the expected concentration of the milk to the actual concentration of the milk.

1. 1: 1 2. 21: 52 3. 55: 64 4. 605: 512

21. A milk-man starts with a 5 litre can full of milk to deliver milk to his customers. Each of his customers purchases 2 litre of milk from him. After delivering to each of the first two household, he adds 1 litre of water to the can. After giving milk to the

third household, instead of adding water, he adds 1 litre of milk and delivers the rest to the fourth and last of his customer. Find the percentage of milk that the last customer gets.

1. 50% 2. 60% 3. 65% 4. 75%

22. From a solution having milk and water in the ratio 2: 3, 15 litres of solution is drawn out and replaced with water. This operation is done a total of thrice. If the ratio of milk and water now is 25: 231, find the volume (in litres) of original solution.

1. 30 2. 40 3. 42 4. 45

23. To a solution having milk and water in the ratio 2: 3, 15 litres of water is added and then 15 litres of the solution is drawn out. This operation is done a total of thrice. If the ratio of milk and water now is 25: 231, find the volume (in litres) of original solution.

1. 25 2. 30 3. 40 4. 45

24. In what ratio must water be mixed with milk so as to make a profit of $16\frac{2}{3}\%$ on selling the mixture at a rate same as that of cost price of milk. Given that the milk-man mixes Bisleri instead of tap water and the cost of Bisleri is 20% of the cost of milk.

1. 7: 41 2. 1: 6 3. 6: 7 4. 5: 23

25. Three solutions, A, B and C are such that they have milk and water in the ratio 2: 3, 1: 1 and 3: 2. Solutions A and B are mixed in the ratio 2: 1 to result in solution D. Solutions B and C are mixed in the ratio 1: 2 to result in solution E. Now solution D and E are mixed in the ratio 1: 1. Find the ratio of milk and water in the new solution.

1. 1: 1 2. 1: 2 3. 2: 3 4. 3: 2

26. The ratio of the average age of the class without the teacher to the average age including the teacher is 8: 9. If the ratio of the teacher's age and the average age of the students is 3: 1, find the number of students.

1. 9 2. 10 3. 12 4. 15

27. The average percentage scored by Amit in the six subjects in the board exams is 83%. If the highest percentage scored in any subject is 91%, what could be the lowest percentage of marks scored in any subject?

1. 43% 2. 48% 3. 51% 4. 66%

28. The average of 10 integers (with none of them being equal to zero) is zero. If the average of the negative numbers among them is –5, find the average of the positive numbers among them.

1. 5 2. 10 3. 20 4. Cannot be determined

29. The International School has three divisions A, B and C in class X^{th}, each having same number of students. The average marks scored by students of A and B is 60%, by those of B and C is 75% and by those of A and C is 62%. Arrange the three divisions in increasing order of the average marks scored by the students of the three divisions.

1. C, B, A 2. B, C, A 3. C, A, B 4. A, B, C

30. In analyzing the performance of a batsman, three averages are found out. One is found by dividing the total runs scored by the number of innings in which the batsman is out. Second average is calculated in the above way but by excluding all innings when the batsman was out for a duck. The third average is found by dividing the total runs scored by the number of innings played. If Rahul's averages are 45, 54 and 36 respectively, find the ratio of the number of innings played, the number of innings he was not out and the number of innings when he got out for a duck.

1. 15:3:2 2. 15: 3:4 3. 12:4:1 4. 12:6:1

31. The average marks of all students who scored more than 70% marks is 74%. The average of all students who scored more than or equal to 60% is 68%. Of the students scoring more than or equal to 60%, at least what percent of students have scored between 60% and 70%?

1. 30% 2. 34.25% 3. 37.76% 4. 42.84%

32. 40 consecutive natural numbers are written. When one of the numbers is erased, the average of the remaining 39 numbers is 45. Find the number that is erased.

1. 25 2. 65

3. Both (1) and (2) are possible. 4. More than 2 but finite possible answers

33. Consecutive natural numbers starting from 1 were written on a black board. A student comes and erases three consecutive numbers of the series. The average of the numbers now is $21\frac{15}{19}$. Find the average of the erased numbers.

1. 9 2. 10 3. 11 4. 12

34. A person bought equal number of oranges and mangoes. He sold the oranges at a profit of 20% and the mangoes at a profit of 30%. If the ratio of the cost-price of one orange and one mango was 1 : 3, find the overall profit percentage made by him.

1. 27.5% 2. 27% 3. 26.3% 4. 25%

35. Two persons, A and B, ran a race. B ran part of the distance at a speed of 20 kmph and part of the distance at a speed of 30 kmph. A ran the whole distance at a speed of 25 kmph. Both of them reached the finishing line at the same time. Find the ratio of the distance run by B at speed of 20 kmph and the distance run at speed of 30 kmph.

1. 1: 1 2. 2: 3 3. 4: 5 4. 5: 6

36. Two alloys of iron have different percentages of iron in them. The first of them weighs 6 kgs and the second one weighs 12 kgs. One piece each of equal weight was cut off from both the alloys. The piece from first was alloyed with the remaining second alloy and the piece from second was alloyed with the remaining first alloy. As a result, the percentage of iron became same in both the alloys. What was the weight of each piece cut off?

1. 2.45 kg 2. 3 kg 3. 3.7 kg 4. 4 kg

37. A second-grade cotton fiber has 24% waste. Through a process of purification, waste could be separated, but not entirely, to result in a first-grade cotton fiber which has 10% waste. How many kg of first grade cotton be extracted from 27 kg of second grade cotton?

1. 2.7 2. 20.52 3. 22.8 4. 11.25

38. Dry grapes contain 10% water whereas fresh grapes contain 75% water. To obtain 15 kg of dry grapes, how many kg of fresh grapes are needed?

1. 54 2. 20 3. 150 4. 112.5

39. Two vessels of equal volumes are completely filled with milk and water solutions. The ratio of milk and water in first vessel is 2: 3 and that in second vessel is 4: 1. Next a certain fraction of solution is taken out from first vessel and a certain fraction (not necessarily same as that for first vessel) is taken out from second vessel and are then poured into the other vessels. The fractions removed are such that solution in neither of the vessel spills out and also the ratio of milk and water in first vessel becomes 1: 1. Find the ratio of milk and water in second vessel.

1. 2: 3 2. 2: 5 3. 3: 8 4. 7: 3

Directions for questions 40 & 41: A container is having 30 litres of pure milk. In the first operation 10 litres of it is flushed away and replaced with 20 litres of water.

Then 10 litres of the resulting solution is again flushed away and replaced with 20 litres of water. This process is repeated n times.

40. What is the volume of pure milk (in litres) in the container after 4 such operations?

1. 25 2. 20 3. 18 4. 10

41. What is the ratio of water to milk after 5 such operations?

1. 11: 2 2. 21: 7 3. 25: 3 4. 28: 5

Directions for questions 42 and 43: Two vessels, each of 10 litres, are filled completely, one with milk and other with water respectively. 1 litre of solution is drawn out from each of the vessel simultaneously and transferred to the other vessel. This operation is done thrice.

42. Find the ratio of the amount of water in vessel 1 to the amount of milk in vessel 2.

1. 1: 1 2. 2: 5 3. 6: 5 4. 4: 1

43. Find the ratio of milk and water in each of the two vessels.

1. 3: 1, 1: 3 2. 31: 215, 31: 215

3. 41: 9, 41: 9 4. 189: 61, 61: 189

44. 20% of solution having milk and water in the ratio 3: 1 is removed and replaced with solution having milk and water in the ratio 1: 3. Again 20% of the solution is removed and replaced with same earlier solution i.e., having milk and water in ratio 1: 3. This operation is done once more, i.e. the operation is done a total of thrice. Find the ratio of milk and water in the solution now.

1. 13: 7 2. 57: 47 3. 145: 141 4. 253: 247

45. Brass is an alloy of Copper and Zinc; Bronze is an alloy containing 80% of Copper, 4% of Zinc and 16% of Tin. A fused mass of Brass and Bronze is found to contain 74% of Copper, 16% of Zinc and 10% of Tin. Find the ratio of Copper to Zinc in the composition of Brass.

1. 3: 2 2. 3: 4 3. 9: 16 4. 16: 9

QUANTIFIERS CAT ACADEMY

Percentage

Percent literally means 'per cent', cent as in century. Thus, percent means 'per 100' and is represented with the symbol '%'.

This means that any data, a out of b, is proportionally reduced or increased so as to be 'out of 100'

E.g., A score of 80 marks out of 150 marks. To find the percentage of marks scored, we need to find the equivalent marks scored had the maximum marks been 100

80 marks out of 150 $\Rightarrow \frac{80}{150}$ marks out of 1 (Dividing by 150)

$\Rightarrow \frac{80}{150} \times 100$ marks out of 100. (Multiplying by 100)

Thus, 80 marks out of 150 is equivalent to a percent of $\frac{80}{150} \times 100$ i.e., 53.33%

E.g., In a class of 60 students, 45 of them are girls. To find the percent of girls in the class, we have to find the equivalent number of girls had the strength of the class been 100.

45 girls in a class of 60 $\Rightarrow \frac{45}{60}$ girls in a class of 1 (Dividing by 60)

$\Rightarrow \frac{45}{60} \times 100$ girls in a class of 100 (Multiplying by 100)

To convert the data 'a out of b' to a percentage ..... we need to do the following: $\frac{a}{b} \times 100$

One needs to be slightly careful of what comes in the denominator. It is always the data associated with 'of' – 80 marks out of 150 or 45 girls in a class of 60.

Ex. **50 is what percent of 80? Also find 80 as a percentage of 50.**

Sol: As seen in the above examples, the required answer is $\frac{50}{80} \times 100 = \frac{250}{4} = 62.5\%$

While finding 80 as a percentage of 50, we would need to do: $\frac{80}{50} \times 100 = 160\%$

Finding *x*% of *y*

A percent figure is always a percentage of something. Thus, when we say the attendance was 80%, we mean that 80% of the total students were present; when we say that Rohit scored 85%, we mean 85% of the maximum marks; when we say that a number increased by 20%, we mean 20% of itself; when we say the profit was 40%, we mean 40% of the cost; when we say the sales grew by 35%, we mean 35% of the earlier sales figure.

To find x % of y ..... we need to do $\frac{x}{100} \times y$

Ex. **In a poll, a total of 1,20,000 votes were polled of which 10% were invalid. A candidate received 60% of the valid votes. Find the number of votes polled by the candidate.**

Sol: The number of invalid votes = 10% of 1,20,000 = $\frac{10}{100} \times 1{,}20{,}000 = 12{,}000$.

Thus, the number of valid votes = 1,20,000 – 12,000 = 1,08,000.

The candidate received 60% of 1,08,000 i.e., $\frac{60}{100} \times 1{,}08{,}000 = 64{,}800$.

Ex. **When 40% of a number is subtracted from 100, the answer is 80% of the number. Find the number.**

Sol: If the number is n, then $100 - \frac{40}{100} \times n = \frac{80}{100} \times n$ i.e., $100 - 0.4n = 0.8n$ i.e. $1.2n = 100$ i.e. $n = 83.33$

Identifying the 'base' i.e., what the percentage is of:

It is important to correctly identify the given percentage is of what number or the percentage asked is of what number. This number, of which the percent is found/given will be called as 'base'.

Compare the following examples of similar situations, one straightforward and other very similar one but in which beginners commit errors. In the second example, the error is made in identifying the base incorrectly.

Ex. **What is 20% of 450?**

Sol: Here 20% of 450 is to be found, so 450 is your base of which 20% is to be found. Thus, the answer is $\frac{20}{100} \times 450 = 90$

Ex. **20% of what number is 450?**

Sol: Here you do not have to find 20% of 450. The number of which 20% is to be found is unknown. So, if we assume the number to be n, the working would go as follows: $\frac{20}{100} \times n = 450 \Rightarrow n = 2250$

In the first example above, the base is 450 and in the second example, the base is the unknown number to be found.

Ex. **What is the result when 100 is increased by 20 %?**

Sol: This should be a sitter. Here 100 is increased by 20 %, so we will find 20% of 100 and add it to 100. Thus, the answer is 120.

Ex. **A number increased by 20% becomes 100. Find the number.**

Sol: The error is committed because of the presence of 100 and in haste most students find 20% of 100 as 20. And since the number has to be less than 100, the number is found as 100 – 20 = 80.

Please notice that 20% is of the number and not of 100. 100 is the final result after an increase of 100. Thus, the correct approach should be assuming the number as n and then forming the equation $n + \frac{20}{100} \times n = 100 \Rightarrow \frac{6}{5}n = 100 \Rightarrow n = \frac{500}{6} = 83.333$

Ex. **Ram's salary is 90% of Shyam's salary. Shyam's salary is what percent of Ram's salary?**

Sol: R is 90% of S. Thus, $\frac{R}{S} \times 100 = 90 \Rightarrow \frac{R}{S} = \frac{9}{10}$

The question requires us to find S as a percent of R i.e., $\frac{S}{R} \times 100$. And the required answer will be $\frac{10}{9} \times 100 = \frac{1000}{9} = 111.11\%$

Alternately, Ram's salary is 90% that <u>of</u> Shyam's salary. So, assuming Shyam's salary as 100, we can get Ram's salary as 90% of 100 i.e., 90. Now the question is asking us to find 100 as a percent of 90 i.e., the base now is 90. Thus, the answer is $\frac{100}{90} \times 100 = \frac{1000}{9} = 111.11\%$

Ex. **An alloy consists of 80% Bronze and rest Brass. Bronze and Brass, themselves are alloys with Bronze having 60% Copper and the rest Zinc whereas Brass has Copper and Zinc in equal proportions. What percentage of the alloy of Bronze and Brass is Zinc?**

Sol: Let's assume the alloy of Bronze and Brass as 100 gm. Thus, it will have 80 gm of Bronze and 20 gm of Brass. Let's find the amount of zinc in both. In 80 gm of Bronze, amount of Zinc is 40%. This 40% is of the amount of Bronze i.e., of 80 gm. Quantity of Zinc in Bronze = $\frac{40}{100} \times 80 = 32$ gm. Similarly, quantity of Zinc in Brass = $\frac{50}{100} \times 20 =$ 10 gm. Thus, the total quantity of Zinc in the alloy is 32 + 10 = 42 gm. Now we have to find the percentage of Zinc in the alloy. The alloy is 100 gm and amount of Zinc in it is 42 gm. Thus, the percentage of Zinc is 42%.

Ex. **Raj scored 50% of the maximum marks in an exam and yet failed by 12 marks. Had he scored 10% more than what he scored, he would have just got passing marks. Find the maximum marks of the exam.**

Sol: Let the maximum marks of the paper be x. Thus, Raj scored $\frac{50}{100} \times x = \frac{x}{2}$. Now had he scored 10% more than what he scored means we have to calculate 10% of $\frac{x}{2}$ and this will be equal to 12 marks because he just managed to pass with these additional marks. Thus, we have $\frac{10}{100} \times \frac{x}{2} = 12 \Rightarrow x = 240$.

Alternatively, if you wanted to avoid equations, 10% of the marks he scored is equal to 12. Thus, he scored 120 marks. And this is 50% of the maximum marks. Thus, maximum marks = 240.

Ex. **When 30 is subtracted from a number, the result is 80% of the number. Find the number.**

1. 150 **2. 120** **3. 100** **4. 80**

Sol: Let the number is x.

We have, x – 30 = 80x/100

⇒ x – 80x/100 = 30 ⇒ 20x/100 = 30 ⇒ x/5 = 30 ⇒ x = 150.

Ex. **Mohan spends 40% of his income on rent, 20% on food, 10% on entertainment and saves the rest. If he saves a net amount of Rs. 4380, find his income.**

1. 21,900 **2. 17,800** **3. 15,200** **4. 14,600**

Sol: Here every expenditure is given on the original income.

Let the original income is Rs x. Now, he spent 40% on rent, 20% on food and 10% on entertainment. So, he has spent 70% of his income. Hence, he is left with 30% of the income.

Now, given that $\frac{30}{100} \times x = 4380 \Rightarrow$ x = Rs 14600.

Ex. **The prices of chocolates dropped by 15% and thus Rashi was able to save Rs. 1.35 on each chocolate. Find the new price of one chocolate.**

1. Rs. 7.65 **2. Rs. 8.65** **3. Rs. 9** **4. Rs. 9.35**

Sol: Let the original price of the chocolate is Rs x.

As the price is dropped by 15% and she saves Rs 1.35

$\Rightarrow \frac{15}{100} \times x = 1.35 \Rightarrow$ x = 9.

The new price is 85 × 9/100 = Rs 7.65.

Ex. **In a mixture of milk and water, milk accounts for 75%. Find the amount of the mixture (in litre) if it contains 28 litres of water.**

1. 21 **2. 56** **3. 84** **4. 112**

Sol: As the milk is 75% of the total mixture so the water is 25% of the mixture.

Let the total mixture is x litre

⇒ 25x/100 = 28 ⇒ x = 112.

Ex. **A milk-man mixes milk and water in the ratio 5: 3. Find the percentage of milk in the resulting mixture.**

1. 60% **2. 62.5%** **3. 67.5%** **4. 75%**

Sol: The ratio of milk and water is 5: 3. So, if milk is 5 litres, then water is 3 litres i.e., total mixture is 8 litres.

Required percentage = $\frac{5}{8} \times 100 = 62.5\%$

Ex. **When 1 kg of goods is kept on an electronic weighing scale, the scale shows a reading of 950 gm. Find the percentage error in the measurement.**

1. 11.11 **2. 5.55%** **3. 5%** **4. 5.26%**

Sol: Actual error is 1000 – 950 = 50 gm.

As the actual weight should be 1000 gm, so, percentage error = $\frac{50}{1000} \times 100 = 5\%$.

Ex. **An unscrupulous trader has rigged his electronic weighing scale such that it shows a reading of 1 kg when 950 gm of goods are kept on it. Find the percentage error in the measurement.**

1. 11.11 **2. 5.55%** **3. 5%** **4. 5.26%**

Sol: Actual error is 1000 – 950 = 50 gm.

As the actual weight should be 950 gm, so, percentage error = $\frac{50}{950} \times 100 = 5.26\%$.

Ex. ***A*'s salary is 80% of *B*'s salary. Find what percent of *A*'s salary is *B*'s salary.**

1. 120% **2. 125%** **3. 140%** **4. 150%**

Sol: Let the salary of B is Rs 100, then, salary of A is Rs 80

Required %age = $\frac{100}{80} \times 100 = 125\%$.

Ex. **To pass an exam of 250 marks, a candidate should obtain 40% marks. Raj scores 10% more than the passing marks. Find the percent of marks scored by Raj.**

1. 44% **2. 50%** **3. 54%** **4. 60%**

Sol: Here, the passing marks = 40/100 × 250 = 100

Marks obtained by Raj = 100 × 1.1 = 110.

Required %age = $\frac{110}{250} \times 100 = 44\%$.

Ex. **In a country 55.55% of the population are males and rest females. Of the males 60% are literate and of the females, 40% are literate. Find the percentage of the population that are literate.**

1. 50% **2. 51.11%** **3. 55.55%** **4. 45.45%**

Sol: As 55.55% = 5/9, so, let the total population is 90.

Number of males = 55.55% of 90 = 5/9 × 90 = 50

Number of females = 90 – 50 = 40

Number of literate males = 60% of 50 = 30

Number of literate females = 40% of 40 = 16

Total number of literate people = 30 + 16 = 46.

Required %age = = $\frac{46}{90} \times 100 = 51.11\%$.

Percentage-fraction equivalence:

In the course of solving questions on percentages, we would very often have to find the percentages from expressions like $\frac{5}{6} \times 100$, $\frac{10}{11} \times 100$, $\frac{5}{8} \times 100$ or we would be finding 37.5% or 22.22% or 18.18% of a certain number. These could be tedious calculations and unnecessarily wear out a student. On the contrary these are very easy if one has already learnt a few percentage-fraction equivalence.

E.g., $\frac{2}{11}$ as a fraction is equivalent to 18.18%. This means that $\frac{2}{11} \times 100 = 18.18\%$ and 18.18% of n is same as $\frac{2}{11} \times n$.

E.g., Knowing that $37.5\% \approx \frac{3}{8}^{th}$, the next time we have to find $\frac{3}{8} \times 100$, one can write the answer in a jiffy as 37.5%.

This will also help us in finding 37.5% of say 72. Rather than perform the tedious calculation $\frac{37.5}{100} \times 72$, one could just orally think of the expression as $\frac{3}{8} \times 72$ to arrive at 27 as the answer.

Fraction-Percentage

Throughout this book, quite often while finding a percent, the 100 will be ignored and use would be made only of the expression, $\frac{3}{8} = 37.5\%$.

Needless to say, $\frac{3}{8}$ is actually equal to 0.375 and will be 37.5 only when expressed as a percentage i.e., $\frac{3}{8} \times 100 = 37.5\%$. But as stated, the 100 would be ignored quite often.

The following is a comprehensive list of the percentage-fraction equivalence that you would need to memorise. Groups that can be memorized together are given in one row.

$\frac{1}{2} = 50\%,\ \frac{1}{4} = 25\%,\ \frac{1}{8} = 12.5\%,\ \frac{1}{16} = 6.25\%$

$\frac{1}{3} = 33.33\%,\ \frac{2}{3} = 66.66\%$

$\frac{1}{4} = 25\%,\ \frac{3}{4} = 75\%$

$\frac{1}{5} = 20\%,\ \frac{2}{5} = 40\%,\ \frac{3}{5} = 60\%,\ \frac{4}{5} = 80\%$

$\frac{1}{6} = 16.66\%,\ \frac{5}{6} = 83.66\%$

$\frac{1}{7} = 14.28\%,\ \frac{2}{7} = 28.56\%,$

$\frac{1}{8} = 12.5\%,\ \frac{3}{8} = 37.5\%,\ \frac{5}{8} = 62.5\%,\ \frac{7}{8} = 87.5\%$

$\frac{1}{9} = 11.11\%,\ \frac{2}{9} = 22.22\%,\ \frac{4}{9} = 44.44\%,\ \ldots\ldots, \frac{7}{9} = 77.77\%$

$\frac{1}{10} = 10\%$

$\frac{1}{11} = 9.0909\%,\ \frac{2}{11} = 18.1818\%,\ \frac{3}{11} = 27.2727\%,\ \ldots\ldots,$

$\frac{1}{12} = 8.33\%, \frac{1}{13} = 7.69\%, \frac{1}{14} = 7.14\%\ \frac{1}{15} = 6.66\%,\ \frac{1}{16} = 6.25\%$

The Multiplying Factor

The concept of multiplying factor is the crux of this chapter. It will be useful to us in any scenario where percentages are involved – profit loss discount, simple & compound interest, growth rates in DI. So, make sure you comprehend this very well.

Multiplying factor is a fraction, $\frac{a}{b}$, by which one will multiply the 'base' to find a percent of it or to find a number that is some percent more/less than it. We will denote the multiplying factor as f throughout this book, f to suggest it is a fraction.

Thus, the above states that to find a percent of n or a number that is some percent more or less than n, we would do the calculation $n \times f$.

E.g., To find 9.0909% of n, we multiply n with $\frac{1}{11}$ i.e., do the calculation $\frac{1}{11} \times n$

To find 40% of n, we will do the multiplication $\frac{2}{5} \times n$

Percentage increase

Consider that a number n is increased by 22.22%. Find the expression for the value after the increase.

By now it would be evident that 22.22% is that of the number itself i.e. the increase is $\frac{2}{9} \times n$. This is just the increase, we have to find the final value after the increase. Thus, this increase will be added back to the number i.e., the final value is $n + \frac{2}{9} \times n$.

The process of finding 22.22% of n i.e., $\frac{2}{9}$ of n and then adding it back to n can be compressed into one step by considering our answer in the form $n \times \left(1 + \frac{2}{9}\right)$ i.e., $n \times \frac{11}{9}$. Thus, increasing n by 22.22% is same as multiplying it by $\frac{11}{9}$. Thus, we say that $\frac{11}{9}$ is the multiplying factor' corresponding to an 'increase of 22.22%'.

Distinction between increase and final value

Say n is increased by 10% .....

10% of n or $\frac{1}{10} \times n$ is just the INCREASE in the value

The final value is n + 10% × n i.e., $\frac{11}{10} \times n$

While the above is very easy, it is just a cautionary note asking you to notice carefully if one has to find '10% of n' or '10% more than n'

Check if you understand finding the multiplying factors for a percentage increase as explained below:

Increase of 16.66% is equivalent to multiplying base by $\left(1 + \frac{1}{6}\right)$ i.e., $\frac{7}{6}$

Increase of 37.5% is equivalent to multiplying base by $\left(1 + \frac{3}{8}\right)$ i.e., $\frac{11}{8}$

Increase of 125% is equivalent to multiplying base by $\left(1 + \frac{5}{4}\right)$ i.e., $\frac{9}{4}$

Percent Increase ⇒ f is more than 1

In the case of a percent increase, the multiplying fraction, f will be more than 1. And this should obviously be so since only when a number is multiplied with a number greater than 1, would it increase. The 1 in the bracket $\left(1 + \frac{a}{b}\right)$ refers to the original number and the $+\frac{a}{b}$ refers to the increase.

With the above logic …

$n \times f$, where f is greater than 1, will be a percentage increase. The amount by which the fraction is more than 1 will be the value of the percentage increase i.e. $(f - 1)$ will be equivalent to the percent increase, remember 1 refers to the original value and it should be subtracted from the final value to find the increase.

As explained in the above box, one should be able to work in the reverse way also.

$n \times \frac{8}{7}$ is same as n being increased by $\frac{8}{7} - 1 = \frac{1}{7}$ i.e., 14.28%

$n \times \frac{11}{9}$ is same as n being increased by $\frac{11}{9} - 1 = \frac{2}{9}$ i.e., 22.22%

Percentage Decrease

Consider we have to decrease n by 20%.

In this case the decrease will be $\frac{1}{5} \times n$ and since it is a decrease, it will have to be subtracted from n. Thus, final value = $n - \frac{1}{5} \times n$ i.e., $n \times \left(1 - \frac{1}{5}\right)$ i.e. $n \times \frac{4}{5}$

By the same logic ……

…… decrease of 11.11% is equivalent to multiplying base by $\left(1 - \frac{1}{9}\right)$ i.e., $\frac{8}{9}$

…… decrease of 27.27% is equivalent to multiplying base by $\left(1 - \frac{3}{11}\right)$ i.e., $\frac{8}{11}$

Percent Decrease ⇒ f is less than 1

In the case of a percent decrease, the multiplying fraction f will be less than 1. And this should obviously be so since only when a number is multiplied with a number lesser than 1, would it decrease. The 1 in the bracket $\left(1 - \frac{a}{b}\right)$ refers to the original number and the $-\frac{a}{b}$ refers to the decrease.

$n \times f$, where f is less than 1, will be a percentage decrease. The amount by which the fraction is less than 1 will be the value of the percentage decrease i.e. $(1 - f)$ will be equivalent to the percent increase, remember 1 refers to the original value and thus the final value has to be subtracted from it to find the decrease.

As explained in the above box, one should be able to work in the reverse way also.

$n \times \frac{7}{8}$ is same as n being decreased by $1 - \frac{7}{8} = \frac{1}{8}$ i.e., 12.5%

$n \times \frac{2}{5}$ is same as n being decreased by $1 - \frac{2}{5} = \frac{3}{5}$ i.e., 60%

Multiplying factors as decimals instead of fractions

In certain easy cases like 10 % increase or decrease, 20 % increase or decrease, one should also have the flexibility to use decimals instead of fractions i.e.

A 20 % increase should not only be thought as multiplying by $\frac{6}{5}$ but could also be thought as multiplying by 1.2, which is numerically exactly the same.

Similarly, a 10 % decrease could also translate to a multiplying decimal of 0.9.

If multiplying by 0.9 or 1.2 is easier, use them else use the fractions. But be flexible enough and realize that they are the same.

Application of Multiplying Factor

The following text is a collection of solved examples involving standard questions about percentage changes especially when numeric values of underlying variables are not given.

The most typical example of such a question is "If x is 20% more than y, y is what percent less than x?". There are two equally easy ways to handle questions of this type as explained ……

Method 1: Using multiplying factors

In the data 'x is 20% more than y', the base is y and thus, $x = \frac{6}{5} \times y$.

Whereas in the question phrase 'y is what percent less than x', the base is x. Thus, rearranging the above relation so that the base, x, is multiplied by a fraction, $y = \frac{5}{6} \times x$. This is equivalent to a $\left(1 - \frac{5}{6}\right)$ i.e., $\frac{1}{6}$ i.e., 16.66% decrease.

Method 2: Using numbers

Since the data given has the base as y, we can assume y = 100 and find x = 120.

Now the question requires us to find by what percent is 100 less than 120. And the answer will be $\frac{20}{120} \times 100$ i.e., $\frac{1}{6}$ i.e., 16.66%

Ex. **If A is 37.5% less than B, by what percent is B more than A?**

Sol: Method 1: Using multiplying factors

$A = \frac{5}{8} \times B \Rightarrow B = \frac{8}{5} \times A$ i.e., B is $\left(\frac{8}{5} - 1\right)$ i.e., $\frac{3}{5}$ i.e. 60% more than A

Method 2: Assuming numbers

Since we have to find 37.5% i.e., $\frac{3^{th}}{8}$ of a number, let B = 8. Thus, A is 3 less than B i.e. A = 5.

We have to find 8 is what percent more than 5 i.e., $\frac{3}{5}$ or 60%

Please note that in this example we did not assume B as 100 as that would lead to more calculations. Instead, we used B = 8 since we had to find $\frac{3^{th}}{8}$ of B and this value of B would have made this calculation very easy.

The same working as above would can also be used in standard questions of the type, "If the length of a rectangle is increased by 12.5%, by what percent should the breadth of the rectangle be decreased so that the area remains same" The solution is exactly similar to the working in above examples …

Method 1: Using multiplying factors. If the original length and breadth is denoted as l and b respectively, equating the areas before increase and after increase,

$L \times B = \left(\frac{9}{8}\right) L \times (f)B$, we are using f as the multiplying factor for the change in breadth since we do not know the value of it.

From the above we get $f = \frac{8}{9}$ and this is equivalent to a decrease (less than 1) of

$1 - \frac{8}{9}$ i.e., $\frac{1}{9}$ or 11.11%

Method 2: Using numbers

If length was assumed as 100, the increased length would have been a not so friendly number, 112.5. Since length is increased by 12.5% i.e., by $\frac{1^{th}}{8}$, assume length as 8. Thus, new length = 9.

To maintain the area, a smart work could be to assume the values of the original breadth as 9 and new one as 8 (i.e., original area = 8 × 9 and new area = 9 × 8). Thus, when breadth changes from 9 to 8, the percentage decrease is $\frac{1}{9}$ i.e., 11.11%

The area (or the product) need not always be constant as seen in the next example. Also, the following example will depict that assuming numbers need not always be the best method and one should make an attempt to master the approach of using multiplying factors.

Ex. **The height and base of a triangle are changed such that even though the base is decreased by 10%, the area of the triangle increases by 10%. Find the percentage change in the height of the triangle.**

Sol: Method 1: Using Multiplying factors.

Final Area = $\frac{11}{10}\times$ Original Area

i.e., $\frac{1}{2} \times (f \times H) \times \left(\frac{9}{10} \times B\right) = \frac{11}{10} \times \left(\frac{1}{2} \times H \times B\right) \Rightarrow f = \frac{11}{9}$.

Thus, the height is increased by $\left(\frac{11}{9} - 1\right)$ i.e., $\frac{2}{9}$ i.e., 22.22%

Method 2: Assuming numbers

Since area increases by 10%, let's assume area as 10 and since base decreases by 10%, let's assume base = 10. Thus, $\frac{1}{2} \times 10 \times h = 10 \Rightarrow h = 2$, where h is the original height.

New area will be 11 and new base will be 9 and hence $\frac{1}{2} \times 9 \times h_1 = 11 \Rightarrow h_1 = \frac{22}{9}$, where h_1 is the new height.

Thus, height increases from 2 to $\frac{22}{9}$ i.e., an increase percent of $\frac{\frac{22}{9}-2}{2} = \frac{2}{9}$ i.e. 22.22%.

Relations of the type C = A × B

In the above question we have Area = Length × Breadth. There are many relations of this type and the context in questions like above could be any of these relations. The most commonly seen relation in questions on percentages is

Revenue (Or Expenditure) = Price × Quantity

Another very common application of above is in Data Interpretation, where,

Sales = Market Share × Market

Ex. **If prices of sugar decrease by 6.25%, by what percent can a household increase its consumption of sugar in the same expenditure?**

Sol: Method 1: Multiplying factor

If P and Q refer to the original price and quantity, we have

$$P \times Q = \left(\frac{15}{16}\right) P \times (f) Q \Rightarrow f = \frac{16}{15}$$

Thus, the new quantity will be $\frac{16}{15}$ times the earlier quantity i.e., a percent increase of $\frac{1}{15}$ i.e., 6.66%

Method 2: Assuming numbers

Let price increase decrease from 16 to 15 (because it is a decrease of $1/16^{th}$ and hence these values are chosen)

For product of price × quantity to remain same, we can smartly assume the quantity to increase from 15 to 16 i.e., an increase of 1/15 i.e., 6.66%

Ex. **When prices of milk went up by 18.18%, a household decreased the consumption of milk by 8.33%. By what percent did the expenditure of the family change?**

Sol: As explained above, with such percentages, assuming numbers do not always result in easier calculation. Hence this question is being done only by multiplying factors.

If P and Q refer to the original price and quantity, then

New expenditure = $\left(\frac{13}{11}\right) P \times \left(\frac{11}{12}\right) Q = \frac{13}{12} \times (P \times Q)$

i.e., new expenditure is 13/12 times earlier expenditure. Thus, the expenditure increased by 1/12 i.e., 8.33%.

Ex. **Since prices of mangoes went up by 20%, a housewife was able to purchase 4 mangoes less in Rs. 60. Find the original price of each mango.**

Sol: Though this question seems to be different from earlier, one should notice that the expenditure is the same, before and after increase.

Price have become 6/5 times of itself. Thus, quantity will be come 5/6 times of earlier. This is $1/6^{th}$ less than earlier quantity and is given as 4 mangoes less. Hence originally the housewife was purchasing 24 mangoes in Rs. 60 i.e., a price of 60/24 i.e., 5/2 i.e., Rs. 2.5 for each.

If the question asked increased price, the housewife could now purchase 24 – 4 = 20 mangoes in Rs. 60 i.e., a price of Rs. 3 each.

If ratio is your strength area you could also think of the above as:

Since prices increased from, say 5 to 6, quantity can be assumed to be in the ratio 6: 5. And we know that the difference in the quantity is 4 and hence the housewife was purchasing 24 earlier and now 20, both in Rs. 60. Thus original price of a mango will be 60/24 i.e., 5/2 i.e. Rs. 2.5.

Successive Percentage Changes

The population of a city was 80 lacs in year 2000. In the years 2001, 2002 and 2003 the population of the city increased by 20%, 16.66% and 10% respectively. What is the population of the city at the end of 2003.

In this question, there are three successive percentage increases. We say they are "successive" because for each percentage increase, the base is the value after the previous increase. In the year 2001, population grew to 80 × 1.2 = 96 lacs. Now in 2002, the 16.66% increase is over this new value 96 lacs and not over 80 lacs. Thus, population at the end of 2002 is $96 \times \frac{7}{6}$ = 102 lacs and the population at end of 2003 is $102 \times \frac{8}{7}$ = 128 lacs.

All the above can be combined into the single expression: $80 \times \frac{6}{5} \times \frac{7}{6} \times \frac{8}{7}$ and without finding the populations in the intermediate years one could have found it directly for the last year as $80 \times \frac{8}{5}$ = 16 × 8 = 128.

Ex. **The population of a city increased by 25%, 18.18% and 10% in three consecutive censuses but decreased by 9.09% in the fourth census. If the population after the fourth census was 65 lacs, find the population before the first census.**

Sol: Here one would realise the importance of multiplying factors. Since the population of the base year is not given, it would be cumbersome to work unless one can use the following:

$x \times \frac{5}{4} \times \frac{13}{11} \times \frac{11}{10} \times \frac{10}{11} = 65 \Rightarrow x \times \frac{65}{44} = 64$ giving x = 44 lacs.

Formula for two successive percent changes

The equivalent percentage change of two percentage changes a % and b % is

$a + b + \frac{a \times b}{100}$%.

While it is recommended to use the concept of multiplying factors, as explained in the examples, in certain cases where percent changes are very easy values like 10 % and 20 %, use of the formula can be resorted to.

The formula works well enough for percentage increase and also decrease if signs are taken correctly e.g., a 10 % increase and a 20 % increase will be a net percent of $10 + 20 + \frac{10 \times 20}{100}$= 32 % whereas a 10 % increase and a 20 % decrease will be a net percent of $10 - 20 - \frac{10 \times 20}{100}$= –12% i.e., 12% decrease.

Two successive percent decrease of 10 % and 20 % cannot be thought of as

$10 + 20 + \frac{10 \times 20}{100}$= 32 % and then appending a minus sign because of the decreases i.e., a –32 %. This is wrong. The correct way is to assume any percent decrease as negative and work as $-10 - 20 + \frac{(-10) \times (-20)}{100}$= –28 %. Notice how the third term becomes positive because of two negatives being multiplied together.

Ex. **What is the single percentage change equivalent to two percentage decreases of 10%.**

Sol: Using multiplying factors, $\frac{9}{10} \times \frac{9}{10} = \frac{81}{100}$ i.e., a decrease of $\frac{19}{100}$ or decrease of 19%.

Using the formula, the equivalent percentage change is $-10 - 10 + \frac{(-10)\times(-10)}{100} = -19\%$.

Ex. **If the length of a rectangle increases by 9.09%, by what percent should the breadth of the rectangle decrease to maintain the area of the rectangle?**

1. 8.33% **2. 9.09%** **3. 10%** **4. 11.11%**

Sol: Let the original length and breadth is L and B respectively. Let the new breadth is b.

Now, 12L/11 × b = LB ⇒ b = 11B/12.

Therefore, the new breadth is 11/12 times the original breadth.

Hence, the breadth should be reduced by 1/12 or 8.33%.

Ex. **If the price of the entrance ticket of a circus is decreased by 6.25%, by what percent should the number of viewers increase so as to earn as much as that earned before the decrease in price?**

1. 5% **2. 5.555** **3. 6.25%** **4. 6.66%**

Sol: Let the original number of viewers is x and the price of the ticket is Rs y. Let the number of viewers after the price reduction is X.

Revenue = x × y

As the revenue remained unchanged, so x × y = X × 15y/16 ⇒ X = 16x/15

Therefore, the new viewers increased by 1/15 or 6.66%.

Ex. **If both the length and the breadth of a rectangle increases by 15%, by what percent does the area of the rectangle increase?**

1. 15% **2. 30.25%** **3. 32.25%** **4. 22.25%**

Sol: Let the original length and breadth of the rectangle is L and B respectively. When the length and breadth are increased by 15%, the new length become 1.15L and new breadth becomes 1.15B. Therefore, the new area = 1.15L × 1.15B = 1.3225LB.

Hence, the area is increased by 0.3225 or 0.3225×100 = 32.25%

Alternatively, we can also use $a + b + \frac{a\times b}{100}\%$ formula. Here the length and breadth are both increasing by 15%, so take a = 15 and b = 15.

Required %age = 15 + 15 + 15×15/100 = 32.25%.

Ex. **While preparing the price tag, instead of increasing the prices by 12.5%, a shopkeeper absentmindedly decreased the prices by 12.5%. Find the percentage difference in the price on the tag compared to what should have been the price tag.**

1. 25% **2. 22.22%** **3. 1.5625%** **4. 6.25%**

Sol: Let the original price is Rs 100.

As the shopkeeper wants to increase the price by 12.5%, the increased price should be Rs 112.5.

But he decreased the price by 12.5%. So, the new price became Rs 87.5.

Difference in the prices = 112.5 – 87.5 = Rs 25

Required %age = $\frac{25}{112.5} \times 100 = 22.22\%$.

Ex. **In spite of an increase in prices by 20%, Gopal managed to let his expenditures increase by just 10%. Find the percentage change in the quantity consumed.**

1. 10.2% **2. 10%** **3. 9.09%** **4. 8.33%**

Sol: Let the initial price and consumption is P and C respectively. Let the final consumption is c.

We have expenditure = P × C

Now, the price is increased by 20% and expenditure increased by 10%.

So, 1.1 × P × C = 1.2 × P × c ⇒ c = 11C/12

Therefore, new consumption is 11/12 times the original consumption i.e., it decreased by 1/12 or 8.33%.

Ex. **Since the prices of mangoes increased by 6.66%, the wholesaler was able to purchase 18 mangoes less in his budget of Rs 864. What is the increased price of each mango?**

1. Rs. 1.5 **2. Rs. 2.5** **3. Rs. 3** **4. Rs. 3.2**

Sol: Price has increased by 6.66% or by 1/15, so it become 16/15 times of itself. Thus, quantity will be come 15/16 times of earlier. This is $1/16^{th}$ less than earlier quantity and is given as 18 mangoes less. Hence originally the wholesaler was purchasing 18 × 16 = 288 mangoes in Rs. 864 i.e., a price of 864/288 = Rs 3.

The new price = 3 × 16/15 = Rs 3.2.

Ex. **A large watermelon weighs 20 kg with 96% of its weight being water. It is allowed to stand in the sun for a while and some of the water evaporates so that now only 95% of it is water. Its reduced weight will be?**

1. 16 kg **2. 19.2 kg** **3. 18 kg** **4. 19.8 kg**

Sol: Initial weight of watermelon = 20kg

As 96% is water ⇒ 4% is pulp. So, pulp = 4% of 20 = 0.8 kg

After evaporation, let the weight of watermelon is x kg. As water now forms 95% of the total weight, so remaining 5% will be the pulp which is 0.8 kg

⇒ 5% of x = 0.8 ⇒ x = 0.8/0.05 = 16 kg.

Exercise 7

1. If all the sides of a cuboid are increased by 20%, then by what percentage does its volume increase?

1. 20% 2. 44% 3. 60% 4. 72.8%

2. Gun powder contains 75% nitre, 10% of sulphur and the rest is charcoal by weight. If quantity of gunpowder is 24 kg, then what is the weight of charcoal?

1. 3.6 kg 2. 4.5 kg 3. 5 kg 4. 5.5 kg

3. In an election between two candidates, the candidate who gets 30% of the votes polled is defeated by 15,000 votes. The number of votes polled in favour of the winning candidate is:

1. 11,250 2. 15,000 3. 26,250 4. 37,500

4. If sides of a square are increased by 30%, then what is the percentage increase in its area?

1. 9% 2. 30% 3. 60% 4. 69%

5. A vendor sells 30% of his fruits and throws away 40% of the rest of the fruits. Next day he sells 50% of the remainder fruits and throws away the rest. What percent of the fruits does the vendor throw?

1. 51% 2. 49% 3. 63% 4. 72%

6. If the price of a bicycle is increased by 9.09%, then by 8.33% and then by 7.7%, the price becomes Rs. 1,274. What is the original price of the bicycle?

1. Rs. 999 2. Rs. 1,001 3. Rs. 1,100 4. Rs. 1,150

7. Vaibhav was writing the CAT exam. There were four passages of equal length (in words) and they had 5, 8, 8 and 6 questions respectively. Vaibhav can answer exactly 12 questions in the time he takes to read any one of the four passages. Assume that his rate of reading and answering question remains the same across all four passages. By what rate should Vaibhav increase his reading speed in order to cutdown his total time spent by 10%?

1. 20% 2. 18.5% 3. 25% 4. 9.8%

8. The price per unit of an article decreases by 4%, and the consumption (in units) decreases by 8%. The expenditure would decrease by:

1. 12% 2. 12.32% 3. 11.68% 4. 4.32%

9. In a class, 70% of the students are boys and 14% of the students play cricket. If 10% of boys play cricket, then what percentage of the cricket players are boys?

1. 80% 2. 70% 3. 60% 4. 50%

10. In a stockpile of products produced by three machines M1, M2 and M3, 40% and 30% were manufactured by M1 and M2 and the rest by M3. 3% of the products manufactured by M1 are defective, 1% of the products by M2 are defective, while 95% of the products of M3 are not defective. What is the percentage of defective products in the stockpile?

1. 3% 2. 5% 3. 30% 4. 32.7%

11. ABC & Company produces nuts for FGH Ltd. The rejection rate for the nuts produced was 4% for the first quarter of the month and 8% for the next quarter of the month. The monthly rejection rate was maintained at 7%. Thus, ratio of first quarter to second quarter production was:

1. 1: 3 2. 2: 1 3. 1: 1 4. 1: 2

12. A candidate scores 39% of the maximum marks and fails by 58 marks, while another candidate who scores 55% of the maximum marks, gets 22 marks more than minimum required marks to pass the examination. Find maximum marks of examination.

1. 450 2. 650 3. 500 4. 550

13. Two lumps having mixture of gold and silver together weighs 20 kg. One lump contains 75% gold and 31.25 gm silver per kg. Another lump contains 85% gold and 30 gm silver per kg. Total quantity of silver in two lumps is 617.5 gm. If two lumps are melted to form one, then percentage of gold in it will be:

1. 75% 2. 87% 3. 15.6% 4. 78%

14. A and B are two friends, each having at least a rupee. If A gives B a sum of Rs. 20, then A has 40% of the amount with B. If B gives A, a sum of Rs. 40, then B will have 40% of the amount with A. What is the amount (in Rs.) with A initially?

1. 60 2. 40 3. 50 4. 70

15. A college has raised 75% of the amount it needs for a new building by receiving an average donation of Rs. 600 from people already solicited. People already solicited represents 60% of people, the college will ask for donations. If college is to raise exactly the amount needed for the new building, what should be the average donation (in Rs.) from remaining people to be solicited?

1. 325 2. 375 3. 400 4. 300

16. In a certain town, people are either married or unmarried. Similarly, either they work or they don't work. 37% of the total population was married and 76% of the married people were working. Similarly, 54% of the unmarried people were working. If the total number of people who work was 1,24,280, then the total population of the town was:

1. 90,000 2. 2,00,000 3. 1,80,000 4. 2,16,000

17. There is an alloy (A) of silver and copper. A certain weight of this alloy is mixed with 15 kg of pure silver and is melted. The new alloy (B) contains 90% of silver. If alloy (A) is mixed with 10kg of a 90% silver alloy, and is melted to form new alloy (C), then (C) is found to contain 84% silver. Find the percentage of silver in (A).

1. 80% 2. 90% 3. 75% 4. 85%

Exercise 8

1. Ashok secured 70% of the votes polled in an election and was elected by a majority of 168 votes. All the votes polled were valid. Find the number of votes polled if there were only two contestants.

1. 490 2. 420 3. 350 4. 560

2. Ravi's salary before he got an increment was 20% of the total income of his family. His increment was one-fourth of his salary after the increment. What percentage of the total income of his family is his new salary?

1. 16.67% 2. 20% 3. 33.33% 4. 25%

3. The price of a TV is decreased by 20%. By what percent must it be increased to bring it back to the original price?

1. 25% 2. 20% 3. 16.67% 4. 15%

4. The price of an article is decreased by 20% and then increased by 20%. Find the net percentage change in the price.

1. 0% 2. 4% increase 3. 4% decrease 4. None of these

5. There are three natural numbers. The first and second are less than the third by 40% and 50% respectively. What percentage of the second number is the first number?

1. 125% 2. 133 1/3% 3. 116 2/3% 4. 120%

6. A student secures 38% of the total marks in an exam and gets 18 marks more than the pass mark. A second student secures 27% of the total marks in the same exam and fails by 37 marks. What is the pass mark as a percentage of the total marks?

1. 34.4% 2. 31.5% 3. 38.6% 4. None of these

7. Due to inflation the total cost of monthly household items has gone up by 20%, but the salary of the family increased by only 10%. Initially, the family used to spend 20% of the salary on household items. What percentage of the present salary should the family spend to buy the same quantities of household items?

1. 10% 2. 20 2/11% 3. 22 2/11% 4. 21 9/11%

8. In 2021, the transportation cost of an item was 30% of its manufacturing cost. From 2021 to 2022, its manufacturing cost went up by 20%. Its transportation cost in 2022 was 33.33% of its manufacturing cost in that year. By what percent should the total cost in 2022 be reduced to bring it back to that in 2021?

1. 17.25% 2. 23.07% 3. 16.53% 4. 18.75%

9. In a town, the population of males decreased by 25% from 2021 to 2022. The population of females increased by 20% in this period. If females formed 44.44% of the population in 2022, what percentage of the population in 2021 were males?

1. 50% 2. 66 2/3% 3. 80% 4. 75%

10. The length of a rectangle is increased by 20% and its breadth is increased by 10%. Which of the following is a possible value of the percentage increase in its perimeter?

1. 13% 2. 14% 3. 12% 4. 16%

11. The price of 1 bat is equal to the price of 2 pairs of pads. The price of 1 pair of pads is equal to the price of 3 pairs of gloves. The price of 1 pair of gloves is equal to the price of 4 balls. Find the price of a ball as a percentage of the price of a bat.

1. 4.166 % 2. 8.33 % 3. 12.5 % 4. 16.33 %

12. A building worth Rs. 5,00,000 was constructed on a land worth Rs. 3,00,000 Co-incidentally that year, the value of land appreciated by x% and the value of the building depreciated by the same x%. Another co-incidence was that at the end of

the year, the value of the land was same as the value of the building. Find the value of x.

1. 16.66% 2. 20% 3. 25% 4. 33.33%

13. At the start of a year, the land and the building a company owned was together valued at Rs. 10,00,000. In the year the value of land appreciates by 20%, whereas the value of a building depreciates by 10%. If at the end of the year, the land and building was together valued at Rs. 10,20,000, find the value of the land alone at the start of the year.

1. Rs. 7,00,000 2. Rs. 6,00,000 3. Rs. 5,00,000 4. Rs. 4,00,000

14. My salary in 2005 was Rs. X. In 2006, it increased by p%. In 2007, it decreased by q%. Then in 2008 it decreased by p%. In 2009 it increased by q%. If now the salary was Rs. Y, which of the following statement is true?

1. Y < X for any values of p and q. 2. Y > X for any values of p and q.

3. Y = X for any values of p and q 4. Y = X if and only if $p = q$

15. During a heating process, the surface area of a sphere increases by 44%. Find the percentage change in the sphere's radius and volume

1. 11%, 48% 2. 20%, 72.8% 3. 22%, 66% 4. 25%, 62.8%

16. The height of a triangle increases by 14.28%. By what percentage should its corresponding base change so that area of the triangle increases by 60%?

1. 10% 2. 25% 3. 28.56% 4. 40%

17. The production of coal in 2006 is 10% more than the production of coal in 2005. If the share of Lignite, a type of coal, in the total coal production was 25% and 20% in the years 2005 and 2006, by what percentage has the production of lignite changed in 2006 compared to the year 2005?

1. 5% 2. – 5% 3. 12% 4. – 12%

18. A godown maintains inventory of two products A and B. While product A accounts for 40% of the number of products stored, it accounts for only 25% of the total value of goods stored. If product A is priced at Rs. 5000 each, find the price of each unit of product B.

1. Rs. 12,000 2. Rs. 7,500 3. Rs. 8,000 4. Rs. 10,000

19. In a Gallop Poll conducted among all voters, 60% of the voters claimed that they would vote for candidate A and the rest claimed that they would vote for candidate B. But on polling day x% of each candidate's supporters changed their mind and voted for the other candidate, which resulted in both candidates securing equal votes. Find the value of x.

1. 5% 2. 10% 3. 40% 4. 50%

20. The mileage (km per liter) of a Maruti car is 20% more than the mileage of an Ikon. Also, the consumption (liter per km) of an Ikon is 20% more than the consumption of a Santro. By what percentage is it costlier/cheaper to drive a Santro than a Maruti? (Assume distance travelled is same)

1. 4% cheaper 2. 4% costlier 3. 5% cheaper 4. Both costs the same

21. Since the prices of petrol increased by 10%, I changed my car to a more efficient model which gave 20% more mileage per litre of petrol. If I continue to travel the same distance, by what percent does my expenditure change?

1. Increases by 32% 2. Decreases by 12%

3. Increases by 9.09% 4. Decreases by 8.33%

22. In the production of a watch, labour charges account for 25% of the cost and the rest of the cost is material cost. If the cost of material increases and now it accounts

for 80% of the total cost of the watch, find the percentage increase in the cost of material.

1. 16.66% 2. 20% 3. 25% 4. 33.33%

23. The engine of a car accounts for 20% of the car's cost. The cost of the engine increases by x% and consequently the cost of the car also increases by the same amount as the increase in engine's cost. Now the cost of the engine accounts for 25% of the car's cost. Find the value of x%.

1. 20% 2. 25% 3. 33.33% 4. 40%

24. CET is an objective type paper of 200 questions with 1 mark for each correctly answered question and no negative marks for wrong answers or questions left un-attempted. If Anuj had attempted 60% more questions than he did and had made 20% fewer errors than he did, his score would have increased by 100%. What percent of the questions attempted by Anuj are correct?

1. 25% 2. 33.33% 3. 40% 4. 66.66%

25. McCain and Obama contest an election. At the election 10% of the people on the voting list did not vote and 60 votes recorded were rejected as illegal. The margin by which Obama won was 308 and it was found he had been supported by 47% of the whole number on the voters list. What was the number of legal votes recorded?

1. 2606 2. 5580 3. 4580 4. 5520

QUANTIFIERS CAT ACADEMY

Profit & Loss

Basic terms of Profit & Loss:

The topic Profit, Loss and Discount is a straightforward application of Percentages. Following are some basic terms which you must know before the start of the topic.

Cost Price: The trader would either be purchasing the good from else-where or manufacturing it. In either of the cases, he would be incurring a cost. This is called his Cost Price (CP, in short)

Selling Price: In most of the problems there would be a trader who sells goods to customer. The price at which the trader sells the goods is called the Selling Price (SP, in short).

To find if the trade has resulted in a profit or loss, we would always consider the difference (SP – CP).

If the SP is greater than the CP, as usually is the case, the difference is positive and we say that the trader has earned a Profit.

If the SP is less than the CP, the difference is negative and we say the trader has incurred a Loss.

Profit or Loss = (SP – CP), if positive, it's Profit, if negative it's Loss

Profit & Loss percentage:

Whenever we have to compare two products, the absolute value of the profit is not of much use. A product may be resulting in a huge profit, say of Rs. 1000, as compared to another product which results in a profit of only Rs. 100. While the first product may seem attractive because of the higher absolute value of profit, but if the first product costs Rs. 10,000 and the second product costs only Rs. 200, the scenario changes. Having spent Rs. 10,000 and making a profit of only 1,000 ($1/10^{th}$ of the investment) is not as lucrative as spending only Rs. 200 and making a profit of Rs. 100 (1/2 of the investment). Thus, to compare two products, a better measure would be $\frac{\text{Profit}}{\text{Cost}}$. This figure is usually expressed in percentage terms i.e., profit per Rs. 100 of cost. This is termed as Profit Percentage. Thus, Profit Percentage = $\frac{\text{Profit}}{\text{Cost}} \times 100\%$.

The same expression is used in the case of Loss Percentage as well. It's just that Profit in that case will be negative and the profit percentage will also be negative.

Profit is always expressed as a percentage of CP. Thus, to find the Profit Percentage, the CP (and NOT the SP) will be the base.

Ex. **Find the profit percentage if the CP is Rs. 450 and the SP is Rs. 500.**

Sol: Here the profit = SP – CP = 500 – 450 = Rs 50.

Profit Percentage = $\frac{50}{450} \times 100$ i.e., $\frac{1}{9} \times 100$ i.e., 11.11%

Ex. **A trader purchases a good at Rs. 500 and sold it at Rs. 450. Find his loss percentage.**

Sol: In this case the loss is = 500 – 400 = Rs 50

Loss percentage = $\frac{50}{500} \times 100 = \frac{1}{10} \times 100 = 10\%$ loss.

Ex. **Amit sells his watch for Rs. 200 for a profit of 20%. At what price did Amit purchase the watch?**

Sol: Here note that 20% is of the CP which is not known and it is NOT of 200.

Thus CP + 20% of CP = 200

This is same as a 20% increase in CP results in 200. Thus, using percentage increase result learnt in the earlier chapter, we have $\frac{6}{5} \times \text{CP} = 200 \Rightarrow \text{CP} = \frac{1000}{6} = 166.66$.

Ex. **Ramesh sells his watch for Rs. 100 and makes a loss of Rs. 10. Find the loss percentage.**

Sol: In this question 10% loss is wrong answer as 10 is found as a percentage of 100, which is the SP and not CP.

CP = 100 + 10 = 110

Thus, Loss percentage = $\frac{10}{110} \times 100$ i.e., $\frac{1}{11}$ i.e. 9.0909%

Some standard Problems on Profit and Loss:

Ex. **A manufacturer sells to the wholesaler at a profit of 20%. The wholesaler sells to the dealer at a profit of 12.5%. The dealer sells to a retailer at a profit of 10% and the retailer to the customer at a profit of 6.66%. If the cost of manufacturing the product for the manufacturer was Rs. 625, find the price paid by the customer.**

Sol: All the profit percentages given are nothing by successive percentage increases on the cost of manufacturing. Thus, the final price paid by the customer is $625 \times \frac{6}{5} \times \frac{9}{8} \times \frac{11}{10} \times \frac{16}{15} = 990$.

Ex. **A manufacturer sells to the wholesaler at a profit of 16.66%. The wholesaler sells to the dealer at a profit of 9.09%. The dealer sells to a retailer at a profit of 10% and the retailer to the customer at a profit of 14.28%. If the customer paid Rs. 468 for the article, find the cost of manufacturing the product incurred by the manufacturer.**

Sol: In this question the final price is given and the base price, cost of manufacturing is not known. Thus, if the cost of manufacturing is assumed to be x, we get

$x \times \frac{7}{6} \times \frac{12}{11} \times \frac{13}{12} \times \frac{8}{7} = 468 \Rightarrow x \times \frac{13 \times 4}{3 \times 11} = 468$ i.e., $x = 297$.

Ex. **If the selling price of 10 articles is equal to the cost price of 9 articles, find the profit or loss percentage.**

Sol: Method 1:

Let the SP of 10 articles = CP of 9 articles = Rs. k

SP of 1 article = Rs. $\frac{k}{10}$ and CP of 1 article = Rs. $\frac{k}{9}$

Using, $f = \frac{\text{SP}}{\text{CP}}$, we find the multiplying factor corresponding to the profit percentage as $\frac{9}{10}$ i.e., a loss of $\frac{1}{10}$ i.e., 10% loss.

Method 2:

To avoid all work involving fractions, a simple technique is to make use of LCM as follows:

Let the SP of 10 articles = CP of 9 articles = Rs. 10 × 9 say

SP of 1 article = Rs. 9 and CP of 1 article = Rs. 10

Thus, profit percentage = $-\frac{1}{10}$ i.e., 10% loss.

Ex. **If the cost price of 16 articles is equal to the selling price of 15 articles, find the profit or loss percentage.**

Sol: By the use of the LCM technique learnt above, CP of 1 article = Rs. 15 and SP of 1 article = Rs. 16. Thus profit percentage = 1/15 i.e., 6.66%.

Ex. **By selling 12 articles, Naveen managed to make a profit equal to the selling price of 3 articles. Find the profit or loss percentage.**

Sol: Here, SP of 12 articles – CP of 12 articles = SP of 3 articles

SP of 9 articles = CP of 12 articles

Using the LCM approach, SP of 1 article = Rs. 12 and CP of 1 article = Rs. 9 and profit percentage is 3/9 i.e., 1/3 or 33.33%.

Alternately, $f = \frac{\frac{k}{9}}{\frac{k}{12}} = \frac{4}{3}$ i.e., a profit of 33.33%.

Ex. **A person buys 12 oranges in a rupee. How many should he sell in a rupee so that he earns a profit of 20%.**

Sol: CP of 1 orange = Rs. $\frac{1}{12}$

SP of 1 orange= Rs. $\frac{1}{12} \times \frac{6}{5} = \frac{1}{10}$

Thus 10 oranges have to be sold in 1 Re.

Ex. **A person buys 12 oranges for 5 Rs. How many oranges should he sell for Rs. 4 such that he makes a profit of 6.66%**

Sol: CP of 1 orange = Rs. $\frac{5}{12}$

Profit of 6.66% will result in a multiplying factor of 16/15

Thus, SP of 1 orange = Rs. $\frac{5}{12} \times \frac{16}{15} = \frac{4}{9}$ and in Rs 4 he should sell $\frac{4}{\frac{4}{9}}$= 9 oranges.

Note: If Two items are sold for same price, one at profit of x % and other at loss of x % then in such cases there would always be a loss of $\frac{x^2}{100}$ %.

However, make sure that the selling price is same and that that profit percentage and loss percentage are numerically equivalent. Compare the question with e.g., 19, when the cost price was same. In this case the overall there is no profit no loss. Compare it with the approach used in next e.g., 21, when the profit and loss percent are different.

Ex. **Rohan sells two commodities for Rs. 19,8000 each. He sells one at a profit of 10 % and sells the other at a loss of 10 %. Find his overall profit or loss percent and also the amount of profit or loss.**

Sol: As the SP is same in both the cases, so the loss percent is = $\frac{10^2}{100}$ = 1%.

To calculate the amount of loss, we would have to find the CP of the articles. Thus,

$CP_1 \times \frac{11}{10} = 19{,}800 \Rightarrow CP_1 = 18{,}000$ and $CP_2 \times \frac{9}{10} = 19{,}800 \Rightarrow CP_2 = 22{,}000$

Thus, total cost = Rs. 18,000 + Rs. 22,000 = Rs. 40,000

Total earnings = Rs. 19,800 + Rs. 19,800 = Rs 39,600.

Thus loss = Rs. 400.

Alternately, after finding the costs, loss = 1% of total cost = 1% of 40,000 = Rs. 400.

Ex. **Rohan sells two commodities for Rs. 12,000 each. He sells one for a profit of 20% and other at a loss of 11.11%. Find his overall profit or loss percentage and also the amount of profit or loss.**

Sol: While the selling price of the two commodities in this question is the same, but the profit % and loss % on the two articles is not the same numeric percentage and hence we cannot use the short-cut learnt in earlier question. In this case, we would have to work out the costs and only then find the profit/loss percentage.

$CP_1 \times \frac{6}{5} = 12{,}000 \Rightarrow CP_1 = 10{,}000$ and $CP_2 \times \frac{8}{9} = 12{,}000 \Rightarrow CP_2 = 13{,}500$

Thus, total cost = Rs. 23,500 and total earnings = Rs. 24,000.

Thus, profit percentage = 500/23,500 i.e., 1/47 or $\frac{1}{47} \times 100$ i.e., approximately 2.1%.

Mark up and Discount:

Since customers haggle for a discount, the usual practice is that shopkeepers prepare the price tag that is higher than the acceptable price at which the shop-keeper is willing to sell at. Then a discount is offered on the Marked Price (MP), also called List Price and the article is sold at a Selling Price lower than the Marked Price. The profit is still calculated based on the final Selling Price and the Cost Price. The associated terms are defined below:

Marked Price:

This is the price that is marked on the price tag

Markup:

The percentage by which the MP is higher than the CP. Thus, mark-up percentage is a percentage of the Cost Price.

Mark-up should be viewed as just a percentage increase is. Thus, for a particular mark-up percentage, there would be an associated multiplying factor, say f_m and we would have the relation CP $\times f_m$ = MP

Discount:

Discount is the amount by which the MP is lowered while selling due to sales promotion or due to bargaining. It is usually expressed as a percentage and is a percentage of the Marked Price.

Discount should be viewed as just a percentage decrease is. Thus, for a particular discount percentage, there would be an associated multiplying factor, say f_d and we would have the relation MP $\times f_d$ = SP

Ex. **A trader marks-up his goods by 30% and then offers a discount of 10%. Find the net profit or loss percentage that he makes.**

Sol: Profit percentage = $30 - 10 - \frac{30 \times 10}{100}$ = 17% we have

Ex. **A trader marks-up his goods by 27.27% and then offers a discount of 8.33%. Find his profit percentage.**

Sol: Since the numbers are not very amenable in the formula, working on multiplying factors,

$f_p = \frac{14}{11} \times \frac{11}{12} = \frac{7}{6}$. Thus, the profit percentage is 1/6 i.e., 16.66%

Rigged balances and faulty weights:

Unscrupulous traders, in addition to selling at a price higher than his cost price (which is not why he is unscrupulous), also cheats on the volume by using a rigged scale or weight. So, if a trader says that he is selling the articles at cost price but is using some faulty weight then the percentage profit in that case will be $\frac{Error}{Quantity\ given} \times 100$.

Ex. **A trader claims that he sells the sugar at cost price but he weighs 900 gm instead of 1 kg. Find his profit percent.**

Sol: Here the error = 1000 – 900 = 100 gm and quantity given = 900 gm.

%age profit = $\frac{Error}{Quantity\ given} \times 100 = \frac{100}{900} \times 100 = 11.11\%$.

Ex. **A trader sells at a mark-up of 10% and at the same time uses a 1kg that weighs only 900 gm. Find his profit percentage.**

Sol: This question is also similar to the earlier question in the sense that the customer pays for 1 kg but receives only 900 gm of goods. Additionally, there is also a mark-up that the trader applies i.e., his rate of selling is 10% more than his rate of purchase. Thus, if he buys goods at the rate of Rs. x per gm, he sells goods at the rate of Rs. $1.1 \times x$ per gm. Thus, earnings of the trader = 1000 gm × $1.1x$ = $1100x$, and cost of the trader = 900 gm × x = $900x$.

Thus, profit percentage = 200/900 i.e., 2/9 or 22.22%.

Ex. **In summers, all metallic meter scales expand by 10%. Find the profit that a cloth trader makes in summers if he uses a metallic meter scale to measure cloth and if he sells at a mark-up of 10%.**

Sol: A meter scale is 100 cm in length in its true state. In summers it expands and will measure 110 cm but it is still a meter scale i.e., its readings will not change and thus the price charged for will only be for 1 mt i.e. 100 cm. Thus, if the rate at which the cloth trader purchases his goods is Rs. x per cm, earnings of the cloth trader = 100 cm × $1.1x$ = $110x$, and cost to the cloth trader = 110 cm × x = $110x$.

Thus, it's the case of no profit no loss.

Ex. **A milk-man mixes water equal to 16.66% of the milk that he has. Further he sells at a mark-up of 20%. Find his profit percentage, if water is available free of cost.**

Sol: Let's assume that the milk man has 6 litres of milk. Thus, his cost is limited to just 6 litres of milk. But then he mixes 1 litre of water to it and thus he would be earning by selling 7 litres of the diluted milk. If the rate at which he purchases milk is Rs. x per litre, earnings of the milk-man = 7 litres × $1.2x$ = Rs. $8.4x$, and cost to the milk-man = 6 litres × x = Rs. $6x$.

Thus, his profit percentage = 2.4/6 i.e., 2/5 or 40%.

Ex. **If the cost price is 25% less than the selling price, find the profit percentage.**

1. 25% **2. 27.27%** **3. 32.5%** **4. 33.33%**

Sol: Let the selling price is Rs 100. Cost price is 25% less than the selling price, so, cost price is Rs 75.

Here profit = 100 – 75 = Rs 25.

Profit percentage = 25 × 100/75 = 33.33%.

Ex. **An article sold at a certain price result in a loss of 7% whereas when it is sold at Rs. 100 more, it results in a 13% profit. Find the cost price of the article.**

1. Rs. 400 **2. Rs. 500** **3. Rs. 600** **4. Rs. 800**

Sol: Let the cost price of the article is Rs x.

In the first case there is a loss of 7%, so, the selling price = 0.93x

In the second case there is a profit of 13%, so, the selling price = 1.13x

It is given that the difference between the two prices is Rs 100

$\Rightarrow 1.13x - 0.93x = 100 \Rightarrow 0.2x = 100 \Rightarrow x$ = Rs 500.

Alternatively, we can say that, to change from loss of 7%, to a profit of 13%, there should be an increase of 20% in the price of the article as both profit and loss are calculated on same cost price. This cost price is given to be equal to Rs 100.

So, we can say that 20% of CP = 100 $\Rightarrow$ CP = 100/0.2 = 500.

Ex. **An article sold at Rs. 423 results in a loss of 10%. What should be the selling price to result in a profit of 10%?**

1. Rs. 443 **2. Rs. 500** **3. Rs. 507.6** **4. Rs. 517**

Sol: Let the Cp is Rs x.

In the first case the loss is 10%, so, the SP = 0.9x

$\Rightarrow 0.9x = 423 \Rightarrow x = Rs\ 470$.

In order to gain 10%, the SP = 470 × 1.1 = Rs 517.

Ex. **A man gains 10% when he sells the article at a certain price. Find his profit percentage if he increases the selling price by 50%.**

1. 15% **2. 60%** **3. 64%** **4. 65%**

Sol: Let the cost price is Rs 100.

When he is gaining 10%, the SP = 100 × 1.1 = Rs 110

Now, he increased the selling price by 50%, which means the new SP = 110 × 1.5 = Rs 165.

Profit = 165 – 100 = Rs 65. So, the profit percentage = 65%.

Ex. **When the cost of an article increases by Rs. 340, a trader increases his selling price by 10%. Because of these changes his profit percentage decreases from 20% to 15%. Find the cost price of the article after the increase.**

1. Rs 3600 **2. Rs 2300** **3. Rs 3940** **4. Rs 2640**

Sol: Let the cost price in the first case is Rs x and selling price is Rs y.

Profit is 20%, so $y - x = 1/5 \times x \Rightarrow y = 6x/5$ ..... (i)

In the second case, CP = x + 340 and SP = 1.1y. Profit in this case is 15%.

$1.1y - (x + 340) = 15 \times (x + 340)/100$

$\Rightarrow 1.1y = 115\ (x + 340)/100$

$\Rightarrow 110y = 115\ (x + 340)$

$\Rightarrow 110 \times 6x/5 = 115\ (x + 340)$

$\Rightarrow 132x = 115x + 39100 \Rightarrow 17x = 39100 \Rightarrow x = 2300$.

Therefore, the new cost price = 2300 + 340 = Rs 2640.

Ex. **On selling 20 meters of cloth, a merchant realizes that he has made a loss equal to the selling price of 3 meter of cloth. Find his loss in percentage terms.**

1. 15.12% **2. 13.04%** **3. 16.66%** **4. 20%**

Sol: Let the SP of 1 meter cloth is Re 1.

SP of 20 meters of cloth = Rs 20

Loss = Rs 3

CP of 20 meters of cloth = Rs 20 + 3 = Rs 23

Loss percentage = 3 × 100/23 = 13.04%.

Alternatively, CP of 20 meter – SP of 20 meter = SP of 3 meter

CP of 20 meter = SP of 23 meter

Let CP of 1 meter = Rs 23 ⇒ SP of 1 meter = Rs 20

Loss = 23 – 20 = Rs 3

Loss percentage = 3 × 100/23 = 13.04%.

Ex. **The cost price and selling price is such that the cost price of 36 articles is equal to the selling price of *x* articles. Find the value of *x* that will result in a profit of 12.5%**

1. 40 **2. 36** **3. 32** **4. 27**

Sol: Cost price of 36 articles = SP of x articles.

Let the cost price of one article = Rs 8. As the profit is 12.5%, so SP of one article = Rs 9.

Now, CP of 36 articles = 36 × 8 = Rs 288

⇒ 9x = 288 ⇒ x = 32.

Ex. **A trader sells 10 articles for a rupee and manages a profit of 20%. How many articles did he purchase in a Rupee?**

1. 9 **2. 10** **3. 11** **4. 12**

Sol: SP of 10 articles = Re 1

As the profit is 20%, co 1.2CP = 1 ⇒ CP of 10 articles = Rs 10/12 or Rs 5/6

In Rs 5/6, he bought 10 articles

So, in Re 1, he bought 10 × 6/5 = 12 articles.

QUANTIFIERS CAT ACADEMY

Exercise 9

1. Rehman buys a few apples at 15 for a rupee and the same number of apples at 20 for a rupee. He mixes the two lots and sells them at 35 for 2 rupees. What is his gain or loss percentage?

1. 3.62% loss 2. 2.04% profit 3. No profit, no loss 4. 2.04% loss

2. The cost of 4 apples equals the selling price of 3 apples. Find the profit/loss percentage.

1. 33.33% 2. 28% 3. 25% 4. 42.84%

3. The profit made by selling 5 m of a cloth equals the selling price of 2 m of that cloth. Find the profit percentage made.

1. 25% 2. 75% 3. 33.33% 4. 66.67%

4. A trader promised his customers to sell at cost price. But he cheats his customers by giving 100 gm less for every kg that he sells. Find his profit percentage.

1. 33.33% 2. 11.11% 3. 22.22% 4. 25%

5. A sold a chair to B at 10% profit. B sold it to C at 20% profit. If C bought it for Rs 660, Find the price (in Rs) at which A bought the chair.

1. 550 2. 450 3. 500 4. 600

6. Kiran sold a table at 20% loss. If he sold it at 10% profit, he would have earned Rs 300 more. Find the cost price (in Rs) of Kiran.

1. 1000 2. 1200 3. 800 4. 1500

7. Ashok bought 15 kg of cashew nuts for Rs 540. He was forced to sell them at a loss equal to the amount he would have realized by selling 3 kg. Find his selling price (in Rs) per kg.

1. 25 2. 28 3. 30 4. 32

8. Shyam purchased some goods for Rs 1800. He sold one-third of the goods purchased at 20% loss. Find the profit percentage at which the rest of the goods must be sold to realize an overall profit of 20%.

1. 25% 2. 30% 3. 40% 4. 50%

9. Balu sold two TV sets, one at 10% profit and the other at 10% loss. Find his overall profit/loss percentage if he sold both the sets at the same price.

1. 1% profit 2. 1% loss 3. 2% profit 4. No profit No loss

10. Sachin gave a discount of 20% on the marked price of his watch and then sold it. He made a profit of 25%. By what percentage did he mark the watch above its cost price?

1. 56.25% 2. 42.84% 3. 60% 4. 33.33%

11. Two articles are bought at the same price. One is sold at 20% profit and the other is sold at 10% loss. Find the overall profit/loss percentage.

1. 10% profit 2. 10% loss 3. 5% loss 4. 5% profit

12. A shopkeeper sells an item for Rs 60 at a profit of 20%. At what price (in Rs) should he sell it to gain 30%?

1. 63 2. 65 3. 68 4. 70

13. The profit made on selling 5 m of a cloth equals the cost price of 2 m of that cloth. Find the profit percentage in selling each metre of the cloth.

1. 66 2/3% 2. 50% 3. 40% 4. 28 4/7%

14. The cost price of 80 articles is Rs 12.50 per article. Twenty of them were sold for Rs 18 each. At what price (in Rs) should each of the remaining articles be sold so as to get an overall profit of Rs 4.50 per article?

1. 15 2. 16.67 3. 17.33 4. 18

15. Govind marked an article 25% above its cost price and allowed a discount of 30%. Find his loss percentage.

1. 10% 2. 15% 3. 7.5% 4. 12.5%

16. A shopkeeper bought an article for Rs 360. The profit made by the shopkeeper after selling it after a 11 1/9% discount is Rs 40. Find the marked price (in Rs) of the article.

1. 400 2. 450 3. 500 4. 475

17. A bicycle was sold at a loss of 18%. Had it been sold for Rs 990 more, there would have been a profit of 15%. At what price (in Rs) should the bicycle be sold to make a profit of 10%?

1. 3000 2. 3300 3. 3200 4. 3500

18. The cost of production of a motorbike which is sold at 20% profit went up by 40%. What should be the percentage increase in the selling price to maintain the profit percentage the same even at the new cost of production?

1. 40% 2. 35% 3. 50% 4. 45%

19. A man bought 100 mangoes at a certain price, with the intention of selling each at a profit of 25%. But 20 mangoes got spoilt. If he sold the rest at the intended price, what was his profit or loss percentage?

1. 0% 2. 6.66% profit 3. 6.25% loss 4. 12.5% profit

20. A shopkeeper always weighs 20% less than the correct weight. One day, he weighed 20% more than the quantity usually weighed by him. If the profit on the correct weight is 20%, what is the effective percentage of profit in this transaction?

1. 10% 2. 25% 3. 20% 4. 15%

21. A dealer bought 50 television sets at Rs 10000 each. For every set purchased from him, he gave one set free. The loss made by him is equal to the selling price of 15 sets. What is the selling price of each set, that is bought?

1. Rs 10000 2. Rs 15000 3. Rs 12500 4. Rs 20000

22. To manufacture a product X, a company needs raw materials P, Q and R and others. The cost price of P is 10% of the total cost price of X. The cost price of each of Q and R is 20% of the total cost of X. The cost of P is increased by 20%. The costs of each of Q and R is increased by 10%. The total cost is increased by 15%. Find the percentage increase in the cost of the others.

1. 12% 2. 15% 3. 16% 4. 18%

23. Rakesh bought some chocolates at 10 for Rs 1.50 and an equal number at 20 paise each. He sold them at a rate of 20 for Rs 4 and made a profit of Rs 10. How many chocolates did he purchase?

1. 100 2. 200 3. 300 4. 400

24. Kanchan has bought 50 articles. He sells 20% of the articles and makes a profit of Rs 1200, which is also equal to the cost of 5 articles. If the selling price for all 50 articles is the same, what is the value of the remaining articles at the selling price?

1. 12400 2. 14400 3. 10000 4. 12500

25. Ajay sold an article at 20% profit to Balu. Balu sold it at 30% profit to Chetan. Dinesh sold a similar article at 20% loss to David. David sold it at 30% loss to Edward. The sum of the price that Chetan and Edward paid for their respective articles is Rs 28000 more than what Ajay paid. If Ajay and Dinesh bought the article for the same price, Find the sum of the prices paid by Ajay and Dinesh (in rupees) for their respective articles.

1. 50000 2. 40000 3. 45000 4. 52000

26. If the discount and profit percentage are both 20% by what percent is the marked price above the cost price?

1. 20% 2. 30% 3. 40% 4. 50%

27. A car dealer sold a car at a discount of Rs 100000. Even after the discount, he made a profit of 15%. What is the marked price of the car, if the marked price is 25% more than the cost price?

1. Rs 1000000 2. Rs 1115000 3. Rs 2500000 4. Rs 1250000

28. Feroze marks up an article by 30% and sells it at a discount of 20% to Sohail. Sohail marks up the price of the article to a certain amount which happens to be 20% more than Feroze's cost price. What is the maximum discount Sohail can offer without going into loss?

1. 30% 2. 20% 3. 16 2/3% 4. 13 1/3%

29. Girish marked an article at 60% above his cost price. What is the maximum number of successive discounts, each being 10%, he can give such that he does not make a loss?

1. 2 2. 3 3. 4 4. 5

30. When the selling price of an article is increased by 20% and the cost remains the same, the profit increases by 50%. Find the profit percentage at which the item was being sold originally.

1. 25% 2. 33.33% 3. 40% 4. 66.66%

31. Since the cost price of an article increased by Rs.120, the selling price was increased by Rs.160. If the profit percentage after the changes is 33.33%, find the profit percentage the trader was making before the changes.

1. 33.33% 2. 28% 3. 25% 4. 20%

32. Even though the cost price of an article increases by Rs. 50, a trader manages to increase the selling price by only Rs. 40. If his profits reduce by 12.5%, find the amount of profit he is making now.

1. Rs. 70 2. Rs. 80 3. Rs. 75 4. Rs. 90

33. Following an increase of 20% in the cost price, the selling price had to be increased by Rs. 32 to maintain the profit percentage. Find the new selling price.

1. Rs. 192 2. Rs. 182 3. Rs. 162 4. Rs. 160

34. When the cost price of an article increases by Rs. 50, the selling price has to be increased by Rs. 80 to maintain the profit percentage. Find the margin percentage at which the article is being sold.

1. 60% 2. 40% 3. 37.5% 4. 33.33%

35. Neeta bargains and manages to get the price of a dozen of oranges reduced by Re. 1 and thus gets 1 dozen more oranges in Rs. 240 than what she would have got. Find the original price of a dozen oranges.

1. Rs. 15 2. Rs. 16 3. Rs. 20 4. Rs. 24

36. The engine of a car accounts for 20% of the car's cost. The cost of the engine increases by 10%. By what percentage should the selling price of the car be increased so that the profit percentage is maintained?

1. 2% 2. 10% 3. 20% 4. 30%

37. A car is being sold at a margin of 10%. The cost of the car increases by Rs. 45,000. What should be the increase in the selling price of the car so that the profit percentage is maintained?

1. Rs. 45,000 2. Rs. 45,454 3. Rs. 49,500 4. Rs. 50,000

38. Which of the following is a better deal for a customer:

I. Two successive discounts of 10% and 20%

II. A single discount of 30%

III. Three successive discounts of 10% each

IV. Two successive discounts of 15% each

1. I 2. II 3. III 4. IV

39. Buy x shirts, get y shirts free. Find the least values of x and y such that the offer is equivalent to a discount of 22.22%. (Assuming that all shirts cost the same.)

1. $x = 10, y = 2$ 2. $x = 9, y = 2$ 3. $x = 7, y = 2$ 4. $x = 8, y = 1$

40. Two traders A and B offer 12.5% and 8.33% discount respectively. If their selling price are in the ratio 7: 11, find the ratio of their marked price.

1. 11: 7 2. 4: 3 3. 3: 2 4. 2: 3

41. Because a discount of Rs. 30 was given, the profit was lower by 37.5%. Find the list price of the article, if its cost price was Rs. 450.

1. Rs. 510 2. Rs. 530 3. Rs. 560 4. Rs. 590

42. A retailer purchases goods at a discount of 25% on the MRP. What is the maximum discount that the retailer can offer to a customer on the MRP if he wishes to make a profit of 10%?

1. 12.5% 2. 15% 3. 16.66% 4. 17.5%

43. A retailer purchases his goods from a wholesaler. The list price of the wholesaler is Rs. 2400, but the retailer gets a discount of 12% on the list price. At what price should the retailer list his goods if he desires a profit of 9.09% after giving a discount of 4%?

1. Rs. 2500 2. Rs. 2750 3. Rs. 2250 4. Rs. 2400

44. A shopkeeper sells goods by marking the price 20% more than his rate of purchase. But since his balance is faulty, he makes a profit of 12.5%. What would the balance read if a weight of 500 gm is placed on it?

1. 460 gm 2. 462.5 gm 3. 468.75 gm 4. 472.5 gm

45. The spices at the Food Shop contain 10% impurities and are sold at cost price. The Mega Shop owner uses a balance that reads 90 gm for every 100 gm and sells at a

markup of 1%. The Family Shop owner adds 5% impurities and sells at a mark-up of 5%. Among these shops, where should a customer shop at?

1. Food shop
2. Mega shop
3. Family shop
4. Food Shop or Family Shop

QUANTIFIERS CAT ACADEMY

Exercise 10

1. A trader marked the price of his goods 20% more than that of the cost price. He then sells 1/4 of his stock at a discount of 10%, and half of the stock at the marked price, and the rest at a discount of 25%. Find his gain percentage.

1. 18% 2. 9.5% 3. 12% 4. 2.25%

2. The aggregate cost of 2 apples, 3 mangoes and 4 oranges is Rs. 6 more than the aggregate cost 1 apple, 2 mangoes and 2 oranges, and the aggregate cost of 1 apple, 2 mangoes and 1 orange is Rs. 8 less than the aggregate cost of 3 apples, 3 mangoes and 5 oranges. If the aggregate list price of 3 apples, 1 mango and 6 oranges is Rs. 12, then find how much percent discount one should give on the sale of 3 apples, 1 mango and 6 oranges such that there is a net profit of 5%?

1. 10% 2. 7.5% 3. 15% 4. 12.5%

3. Mr Alphonso buys 2 lots of mangoes. The first lot costs him Rs. 30 per dozen and the second lot Rs. 50 per dozen. He sold all the mangoes at Rs. 40 per dozen. Find his profit or loss percentage if he spent the same amount of money on each of the lots.

1. 6.66% loss 2. 6.67% profit 3. 5% profit 4. 5% loss

4. A retail bookseller buys books at 58% of the list price. He expects to earn a minimum of 20% net profit on his selling price. What is the maximum discount that he can offer to his customer?

1. 31.6% 2. 25.5% 3. 27.5% 4. 30%

5. If a shopkeeper offers a discount of 20% on the list price of a washing machine, then he makes a profit of 12%. What is the percentage profit or loss, if he sells at a discount of 25% on the list price?

1. 0.6% loss 2. 0.5% profit 3. 4.25% loss 4. 5% profit

6. By selling 6 dozen oranges, a person incurs a loss which is equal to selling price of 1 dozen oranges. Find his loss percentage?

1. 14.28% loss 2. 12.25% loss 3. 16.66% loss 4. 14.66% loss

7. A television set listed at Rs. 3,200 is sold to a retailer at successive discounts of 25% and 15%. What is the final amount paid by retailer of that television set?

1. Rs. 2,700 2. Rs. 2,040 3. Rs. 2,584 4. None of these

8. Two horses were sold for Rs. 12,000 each, one at a loss of 20% and the other at a gain of 20%. The entire transaction resulted in:

1. no loss no gain 2. loss of Rs. 1,000 3. gain of Rs. 1,000 4. None of these

9. A man sells an article at a profit of 20%. If he had bought it at 20% less and sold it for Rs. 5 less, he would have gained 25%. Find the cost price (in Rs.) of the article.

1. 30 2. 28 3. 35 4. 25

10. If cost price of 4 articles is equal to selling price of 5 articles, then find the profit/loss percentage.

1. 20% loss 2. 25% profit 3. 25% loss 4. None of these

11. The cost of manufacturing a commodity increased by 20%. A trader who did not revise the selling price noted that there was a drop of Rs.20 in his profit. What was the original cost price (in Rs.)?

1. 120 2. 100 3. 150 4. 75

12. A shopkeeper marks an item for sale at Rs. 850 and then gives a discount of 20% on it. The item cost the shopkeeper Rs. 500 when he purchased the same from the wholesale. What is the percentage profit earned by the shopkeeper?

1. 36% 2. 32% 3. 30% 4. 25%

13. The Santabil shop offers that get a T-shirt free on the purchase of every three T-shirts. But they mark-up the price of the T-shirts by 60%. If a customer buys three T-shirts, then what is the profit or loss percentage for the store?

1. 25% 2. 22% 3. 30% 4. 20%

14. Given that the cost price of 10 oranges is equal to the cost price of 1 kg of apples and the cost price of 12 apples is equal to the cost price of 1 kg of oranges. If the selling price of 15 oranges is equal to the selling price of 1 kg of apples, then the selling price of 1 kg of oranges is equal to selling price of (Assume that all the apples are identical and this holds true for the oranges as well.)

1. 8 apples 2. 9 apples 3. 10 apples 4. 12 apples

15. On giving 3 pencils free with every 5 pens bought, a shopkeeper makes a profit of 20% and on giving 6 pencils free with every 2 pens bought, he suffers a loss of 25%. Find the approximate profit percent made by the shopkeeper when he gives 4 pencils free with every 6 pens bought. (Assume that the pencils are identical and the same applies to the pens.)

1. 18% 2. 20% 3. 24% 4. 16%

16. A merchant has 100 kg of sugar, a part of which he sells at 7% profit and the rest at 17% profit. He gains 10% on the whole. How much is sold at 17% profit?

1. 70 kg 2. 50 kg 3. 35 kg 4. 30 kg

17. A company had been selling its pianos at a discount of 20% on the marked price of Rs. 325. To increase its sales, it decided to allow an additional discount so that a piano could be sold for Rs. 234. What was the second discount allowed?

1. 7% 2. 8% 3. 9% 4. 10%

18. A merchant uses a weight of 0.85 kg in place of 1 kg to sell potatoes. Let his actual % profit or loss to be 'm' when he pretends to sell his articles at 5% loss and 'n' when he pretends to sell his articles at 5% gain. Find the value of (m + n).

1. 400/17 2. 500/17 3. 600/17 4. 800/17

19. I bought 5 pens, 7 pencils and 4 erasers. Rajan bought 6 pens, 8 erasers and 14 pencils for an amount which was 50% more than what I had paid. What percentage of the total amount spent by me was spent on the pens?

1. 37.5% 2. 56.5% 3. 50% 4. 62.5%

20. A machine produces articles at the rate of 50 units per hour. The articles are sold for Rs. 100 each and the cost of production is Rs. 40 each. However, 20% of the articles have some defect and thus cannot be sold. The production rate of machine can be increased, but every increase of x units per hour would increase the production cost by 2x% and the number of defectives would become (20 + 1.5x)%. What is the maximum increase in production that can be undertaken without incurring losses?

1. 14 units 2. 15 units 3. 16 units 4. 17 units

21. A shopkeeper sold two items at the same selling price. He made a loss on one item and made a profit on the other. The profit and loss percentage on the two items are 10 % and 20 %, not necessarily in that order. Find the net profit percentage for the shopkeeper given that he made a profit in the entire transaction.

1. 3% 2. 3.16% 3. 3.56% 4. 2.86%

22. A book was sold for a certain sum and there was a loss of 20%. Had it been sold for Rs. 3 more, then there would have been a profit of 30%. If it was sold for Rs. 6.60, then the profit/loss percentage would have been

1. Profit 10% 2. Profit 8.33% 3. Loss 8.33% 4. Loss 10%

23. The different heads on which expenditure incurred in manufacturing a product, are material, man power and overheads cost. These costs are in the ratio 9: 5: 11 respectively. If the profit made on the product is 20% of the total cost, what is the ratio of the man power cost to profit?

1. 3: 5 2. 1: 1 3. 1: 7 4. 7: 11

24. The spring balance of a trader weighed 800 gm for 1 kg. What is the net result, if he claims to sell at cost price?

1. 25% loss 2. 25% profit 3. 20% loss 4. 20% profit

25. A shopkeeper goes to the wholesale market to buy his merchandise. The wholesaler has a weighing balance that reads 1,100 g for a kilogram. The shopkeeper sells the entire purchase to a customer after marking up his by 10%. What is his net profit or loss percentage?

1. 20% profit 2. 1.09% profit 3. 1.09% loss 4. No profit, no loss

26. A dishonest dealer claims to sell a product at its cost price. He uses a counterfeit weight which is 20% less than the real weight. Further greed overtakes him and he adds 20% impurities to the product. Find the net profit percentage of the dealer?

1. 44% 2. 40% 3. 50% 4. 56.25%

Simple and Compound Interest

Rate of interest, Principal and Time

When we part with our money to keep it in the bank, the bank pays us an 'interest' to do so. Similarly, when we borrow money, we have to pay an 'interest'. The interest received or paid depends on three factors:

Principal (P): The amount of money that is loaned out or is borrowed is called the principal. Interest would obviously depend on the principal. Higher is the amount, more is the interest and lower the amount, lesser is the interest.

Time Period (*t*): Again, it should be obvious that if the money is loaned or borrowed for a longer time period, more interest will be charged and for a shorter time period, lesser will be the interest.

Rate of Interest (*r*): There is a third factor which determines the Interest. This is the Rate of Interest, expressed as a percentage. This is a pre-determined amount that the giver and the receiver agree to and it specifies the amount of interest charged per Rs. 100 of the principal for a specified time period. Thus, if the rate of interest agreed is 8% per annum, it means that on every Rs. 100 of the principal amount, the interest will Rs. 8 per year.

Further, there are two ways in which an Interest can be charged. For each of the year the money is borrowed/loaned, Interest could be charged just on the principal amount loaned/borrowed (as done in Simple Interest) or for each of the year the interest could be charged on the amount outstanding i.e., principal amount and the accrued interest of the previous years (as done in Compound Interest). These two methods are discussed in details in the following sections.

Simple Interest

In this method the interest charged per year is calculated only on the principal. What this means is that if Rs. 1000 will be loaned/borrowed, the interest will be charged only on Rs. 1000 even in successive years.

Thus, if the rate is r% per annum, then Simple Interest per year = r% of P and thus we get the following formula for *SI*, which most of us are already acquainted with.

$$\text{SI} = \frac{P \times r \times t}{100}$$

Amount returnable = Principal + SI

Ex. **If A deposits Rs. 20,000 in a bank for 3 years at a rate of 10%, what is the simple interest he will get at the end of the period?**

Sol: Here P = 20,000, r = 10%, t = 3 years

Hence SI = $\frac{20{,}000 \times 10 \times 3}{100}$ = Rs. 6,000

Ex. **Sagar borrows Rs. 50,000 from a bank for 5 years. What is the rate of simple interest charged by the bank if after 5 years Sagar had to pay Rs. 66,000 to the bank?**

Sol: Here 66,000 is the total outstanding amount which is Principal + SI. Since principal is given as Rs. 50,000, we get SI = 66,000 – 50,000 = Rs. 16,000

Using the above formula, $16{,}000 = \frac{50{,}000 \times r \times 5}{100} \Rightarrow r = \frac{32}{5}$ = 6.4%

Ex. **After how many years would an amount double itself at 15% rate of interest?**

Sol: We don't have the principal here; we can denote it by P. If the amount doubles, then obviously the SI would be equal to the principal.

So, we have $P = \frac{P \times 15 \times t}{100} \Rightarrow t = \frac{100}{15}$ = 6.667 years

Ex. **The rate of interest of a bank is 10%. Akash was taking a loan of a huge amount for one year so the bank agreed to give him the loan at a rate of 8%. They reasoned that even with 8% they will get twice the interest that they would have got had they given out a loan of Rs. 40,00,000 at 10% for a year. What is the amount of Akash's loan?**

Sol: If P is the loan taken by Akash, writing the relation comparing the two simple interests,

$\frac{P\times 8\times 1}{100} = 2\times\frac{40,00,000\times 10\times 1}{100} \Rightarrow$ P = 1,00,00,000

Compound Interest:

In the case of Compound Interest, after a fixed time intervals (again pre-determined and agreed to by the giver and receiver) the interest amount is calculated on the principal and this interest amount is added to the principal to determine the current outstanding amount. Then during the next time interval, Interest is charged on the current outstanding amount and not on the principal. Again, this interest is added to find the outstanding amount. This process is continued for successive time intervals.

Explanation of the above using a numeric example:

Consider a loan of Rs. 1000 is taken with compound interest being charged at a rate of 10% p.a. Read the following table row-wise.

Year	Amount outstanding at start of year	Interest charged this year	Amount outstanding at end of year
1st year	1,000	100	1,100
2nd year	1,100	110	1,210
3rd year	1210	121	1,331
4th year	1,331	133.1	1,464.1

The formula in case of CI is: $A = P\left(1+\frac{r}{100}\right)^n$, where r is the agreed rate for a predefined interval of time and n is the number of time periods the loan is taken or amount is deposited for.

This formula gives us the final outstanding amount and NOT the Compound Interest.

Compound Interest = Amount – Principal

Ex. **A bank charges a rate of interest of 10% compounded annually. What is the total amount to be paid on a loan of Rs. 36000 for 2 yrs?**

Sol: Using the above given formula, $A = 36,000\left(1+\frac{10}{100}\right)^2 = 36,000\times\frac{121}{100}$ = Rs. 43,560

Ex. **A man takes a loan of Rs. 1,00,000 for two years at compound interest. If he has to return Rs. 1,10,250, find the rate of interest charged.**

Sol: Using the formula we have 1,10,250 = $1,00,000\left(1+\frac{r}{100}\right)^2$

Doing this calculation and finding the square root is going to be a cumbersome process. The better alternative is to make an educated guess and then confirm this by squaring as follows:

The man pays an interest of Rs. 10,250 on a principal of 1,00,000. Thus, the interest paid in two years is a little over 10%. A good guess would be that the interest rate

per annum is 5%. Assuming it is 5% and finding the square of 1.05 we see that 1.05^2 = 1.1025. Thus, we can confirm that the interest rate was indeed 5%.

Ex. **Manu lends a sum of money to his friend at interest such that the amount triples after 5 years when compounded annually. In how many years would the amount payable back become 9 times the sum loaned?**

Sol: Here Amount becomes thrice the principal.

So, $3P = P\left(1+\frac{r}{100}\right)^5 \Rightarrow \left(1+\frac{r}{100}\right)^5 = 3$

The question requires us to find n when $9P = P\left(1+\frac{r}{100}\right)^n$

i.e., when $\left(1+\frac{r}{100}\right)^n = 9$.

Denoting $\left(1+\frac{r}{100}\right)$ as f, we have $f^5 = 3$ and we want to find $f^n = 9$. Obviously since 9 is square of 3, we will have $f^{10} = 9$. Thus, the amount will become 9 times the principal in 10 years.

Alternate method: In the case of compound interest, it is a fresh beginning after every compounding. Thus after 5 years, when the principal triples, it is as good as a fresh beginning. In the next 5 years this amount, 3P, will again triple i.e., will become 9P. Thus, in a total of 10 years, the amount will become 9 times.

Ex. **A man loans out Rs. 50,000 at a rate of 8% simple interest for 2 years and another Rs. 50,000 at the same rate compounded annually for 2 years. Is there any difference in the two amounts he gets back after 2 years?**

Sol: Lets first calculate the total amount in the case of Simple Interest

SI = $\frac{50{,}000\times8\times2}{100}$ = Rs. 8000

Now, let's calculate the total amount in the case of Compound Interest

A = 50,000 × 1.08^2 = 50,000 × 1.1664 = Rs. 58,320

This is the amount and not the CI. The CI = 58,320 – 50,000 = Rs. 8,320

Hence there is a difference of Rs. 320 in the two amounts he gets back

Non-Annual Compounding

The formula for non-annual compounding remains the same, $A = P\left(1+\frac{r}{100}\right)^n$. The only difference being that here r will be the rate of interest per 'compounding period'. If in a question, it is given that compounding happens half yearly but r is given as the rate per annum, then we will need to divide r by 2 to get half yearly rate of interest. Similarly for quarterly compounding, if r is given as the rate per annum, then we will need to divide r by 4 to get quarterly rate of interest.

Also remember that 't' here is the number of compounding periods. So, if the interest is compounded half yearly, and number of years, n is given, the number of time periods will be equal to 2 × n.

Ex. **If interest is accrued semiannually at 10% per annum, what will be the total amount at the end of 2 years if principal is Rs. 15000.**

Sol: Since the interest is compounded semiannually and rate of interest is per annum, the semiannual rate of interest will be 10/2 = 5%. Also, t will be 2 × 2 = 4.

So, A = $15{,}000\left(1+\frac{5}{100}\right)^4 = 15{,}000\times\left(\frac{21}{20}\right)^4$ = Rs. 18,233

Ex. **If a bank offers two schemes (i) Annual compounding at 11% (ii) Semi-annual compounding at 10%, which of the two is a better scheme for depositors who want to deposit their money for two years?**

Sol: Let's assume principal to 100.

Case (i) In case of annual compounding $r = 11\%$, $t = 2$

So, $A = 100 \times 1.1^2 = 121$.

Case (ii) In case of Semi-annual compounding $r = 10/2 = 5\%$, $t = 4$

So, $A = 100 \times 1.05^4 = 100 \times 1.1025^2$.

One should not calculate and find the square. Remember we just need to compare and it is obvious that 1.1025^2 will surely be greater than 1.1^2.

Hence Case (ii) of semi-annual compounding is better for the depositors.

It should be obvious that people seeking loans would prefer Case (i) since they will need to pay less interest at the end of the year.

Note: Population, Appreciation and Depreciation are generally calculated at compound rate of interest unless otherwise stated. In most of the other cases unless otherwise stated, we will assume simple rate of interest. Unless otherwise stated, compound interest will be compounded annually.

Ex. **What rate per annum of simple interest would yield the same amount as that got at compound interest rate of 25% p.a. when the same principal is kept for three years in both the cases.**

Sol: Using formula, the amount at 25% compound rate after 3 years is $P \times 1.25^3$. Finding 1.25^3 will be slightly difficult.

Alternately, in compound interest the total percentage increase over the three years will be the net effect of 25% & 25% & 25% increase. 25% & 25% increase is equivalent to 56.25% increase.

Thus, the net increase is $25 + 56.25 + \frac{25 \times 56.25}{100} = 81.25 + 14.0625 = 95.3125\%$.

For the same percentage increase over three years in the case of simple interest, since each year we get the same percentage increase, the rate should be $\frac{95.3125}{3} = 31.7708$ per year.

Ex. **The population of a city grows at a rate of 12.5% per annum. If in 2006 its population is 7,29,000, what was its population in 2003?**

Sol: A 12.5% increase corresponds to a multiplying factor of $\frac{9}{8}$.

Thus, 7,29,000 = $x \times \frac{9}{8} \times \frac{9}{8} \times \frac{9}{8}$, where x is the population in year 2003. Since $9^3 = 729$ and $8^3 = 512$, we can find $x = 5,12,000$

This approach is better than finding 1.125^3

Ex. **If the ratio of CI in the 7th and 8th year is 10: 11, find the rate of interest being offered.**

Sol: Rather than use the formula for the ratio, the CI in the two years can be assumed as $10k$ and $11k$. Needless to say the extra k interest earned is on $10k$ and hence the rate of interest is $1/10$ i.e., 10%.

Ex. **Vijay took part of Rs. 10,000 loan at 4% and the rest at 6%. If he pays a total interest of Rs. 900 in two years, find the amount taken on loan at 4%. The interest rate charged by the bank is Simple Interest.**

1. Rs 8000 **2. Rs 7500** **3. Rs 7000** **4. Rs 6000**

Sol: Let the amount taken on 4% is Rs x and amount taken on 6% is Rs 10000 – x.

Simple interest for 2 years is Rs 900, so for one year, it will be Rs 450.

Given that $4x/100 + 6(10000 - x)/100 = 450$

$\Rightarrow 4x + 60000 - 6x = 45000 \Rightarrow 2x = 15000 \Rightarrow x = $ Rs 7500.

Ex. **The underworld don Chhota Pappu loans money to people at simple interest. He charges a certain rate of interest for the first year. Next year he doubles the initial rate of interest on the amount. Third year he triples the initial rate of interest and so on... A man took an amount of Rs. 9000 and after 3 years paid back an amount of Rs. 15000 back. What was the rate of interest in the first year?**

1. 9% **2. 9.09%** **3. 10%** **4. 11.11%**

Sol: Let the rate of interest in the first year was r%. In the second year it will be 2r% and in the third year it will be 3r%. As the type of interest is simple, so the total interest paid is 6r%.

Here, Principal = Rs 9000 and Amount = Rs 15000.

SI = 15000 – 9000 = Rs 6000

So $6r/100 \times 9000 = 6000 \Rightarrow r = 11.11\%$

Ex. **I needed Rs. 1,20,000 to buy a Plasma TV and hence I borrowed Rs. 75,000 from Vani and the rest from Vivek. Vani and Vivek charge me a rate of interest such that the interest amount payable to both of them is the same. If in all I re-pay them a total of Rs. 1,50,000 at the end of 2 years, what is the rate of interest charged per annum by the two?**

Sol: Amount borrowed from Vani is Rs 75000 and from Vivek is Rs 45000. Let the rates of interest charged by Vani and Vivek is x% and y% respectively.

Interest paid = 150000 – 120000 = Rs 30000 for two years

Therefore, the interest for one year is Rs 15000

Given that x% of 75000 = y% of 45000 $\Rightarrow x = 3y/5$ .....(i)

Also, x% of 75000 + y% of 45000 = 15000

$\Rightarrow 3y/5 \times 75000 + 45000y = 1500000$ (Using (i))

$\Rightarrow 90000y = 1500000 \Rightarrow y = 16.66\%$

So, $x = 16.66 \times 3/5 = 10\%$

Ex. **At a certain rate of simple interest, a principal becomes three times in 15 years. In how many years will the principal amount become nine times?**

1. 45 years **2. 30 years** **3. 60 years** **4. 75 years**

Sol: Let the rate percent is r%. Now the principal becomes 3 times, it means, the increase is of 200% in principal.

$\Rightarrow 15 \times r = 200 \Rightarrow r = 200/15 = 40/3\%$

When the principal becomes 9 times, it will increase by 800%.

Let the time required is 't' years

So, $40/3 \times t = 800 \Rightarrow t = 60$ years.

Ex. **The population of a city grows at a rate of 5% per annum. If in 2006 its population is 18,52,200, what was its population in 2004?**

1. 12,60,000 **2. 13,60,000** **3. 15,60,000** **4. 16,80,000**

Sol: Let the population in 2004 is x.

We have $x \times 1.05 \times 1.05 = 1852200 \Rightarrow 1.1025x = 1852200 \Rightarrow x = 1680000$

Ex. **I bought an Astra two years back. Its value depreciated by 9% every year. If at present its value is Rs. 9,10,910, at what cost had I bought it?**

1. 1091910 **2. 1091900** **3. 1000000** **4. 1100000**

Sol: Let the cost price is Rs x.

We have x × 0.91 × 0.91 = 910910 ⇒ x × 0.8281 = 910910 ⇒ x = 1100000

Ex. **I deposited an amount of Rs. 20,000 at a 10% p.a. rate of interest compounded quarterly. What is the approximate amount due to me at the end of one year?**

1. Rs 22,000 **2. Rs 22,025** **3. Rs 22,050** **4. Rs 22,075**

Sol: Here the interest is compounded quarterly. So effective rate of interest is 10/4 = 2.5%.

Amount = $20000 \times (1 + 2.5/100)^4 = 20000 \times (1.025)^4$ = Rs 22076.25

Ex. **The CI earned in the 7th year is Rs. 500. If the rate of interest is 15%, find the compound interest earned in the 8th year.**

1. Rs 500 **2. Rs 525** **3. Rs 550** **4. Rs 575**

Sol: CI earned in the 7th year is Rs 500 and in the 8th year Rs 500 will be obtained along with the interest on Rs 500. The rate of interest is 15%.

So, one year interest on Rs 500 = 15% of 500 = Rs 75.

Therefore, the total interest for the 8th year = 500 + 75 = Rs 575.

Ex. **The compound interest earned in the 3rd and 4th year is Rs. 450 and Rs. 500. Find the rate of interest.**

1. 9.09% **2. 10%** **3. 11.11%** **4. 12.5%**

Sol: Here the extra Rs 50 in the interest of the 4th year is the interest earned on Rs 450 (the interest of 3rd year). Therefore, the rate of interest = 50/450 × 100 = 11.11%.

Ex. **At a compound interest rate of 10%, the compound interest earned in the 8th year is Rs. 484. Find the compound interest earned in the 6th year.**

1. Rs 360 **2. Rs 400** **3. Rs 440** **4. Rs 480**

Sol: If the compound interest in the 6th year is Rs x, then in the 7th year the interest will be x plus interest on Rs x i.e., x + 10% of x = 1.1x

In the 8th year, the interest will be 1.1x + 10% of 1.1x = 1.21x which is equal to Rs 484.

⇒ 1.21x = 484 ⇒ x = Rs 400.

Ex. **If the rate of compound interest is 12.5%, find the ratio of compound interest earned in the 24th year and that earned in the 25th year.**

1. 8: 9 **2. 9: 8** **3. 24: 25** **4. 192: 225**

Sol: If the interest in 24th year is Rs 100, then in the 25th year, it will be Rs 112.5

Therefore, the required ratio is 100: 112.5 or 8: 9.

Ex. **The difference between the compound interest and simple interest on a certain sum at 10% per annum for 2 years is Rs. 631. Find the sum.**

1. Rs 6,310 **2. Rs 63,100** **3. Rs 6,31,000** **4. 63,10,000**

Sol: Let the Sum is Rs 100. As the rate of interest is 10%, so the SI for 2 years is 20% of 100 or Rs 20.

Compound interest for 2 years is 100 × 1.1 × 1.1 – 100 = Rs 21

The difference between CI and SI is 21 – 20 = Re 1

If the difference is Re 1, the sum is Rs 100

If difference is Rs 631, the sum is Rs 63100.

Ex. **The difference between the compound interest and simple interest accrued on an amount of Rs. 18,000 in 2 years was Rs. 405. Find the rate of interest, if it is same in the case of simple and compound interest.**

1. 10% **2. 12.5%** **3. 15%** **4. 17.5%**

Sol: In case of two years, the difference between CI and SI = P × $(R/100)^2$, where, P is the principal and R is the rate percent.

Therefore, 405 = 18000 × $R^2/10000$

⇒ 405 = R^2 × 1.8 ⇒ R^2 = 225 ⇒ R = 15.

Ex. **I kept Rs. 20,000 at 5% rate of simple interest for two years. Find the difference in interest earned if I had kept the same amount for same years at same rate but at compound interest.**

1. Rs 1 **2. Rs 2** **3. Rs 5** **5. Rs 50**

Sol: In case of two years, the difference between CI and SI = P × $(R/100)^2$, where, P is the principal and R is the rate percent.

Therefore, difference = 20000 × 25/10000 = Rs 50.

QUANTIFIERS CAT ACADEMY

Exercise 11

1. A sum of Rs 4000 becomes Rs 4500 in 2 years under simple interest. In how many years will Rs 5000 become Rs 5625 under simple interest at the same rate of interest?

1. 2 2. 2.5 3. 3 4. 3.5

2. Find the value that Rs 1000 would amount to under compound interest at 20% p.a., interest being compounded annually in 3 years.

1. 1624 2. 1728 3. 1788 4. 1688

3. Find the sum that would amount to Rs 6600 under simple interest in 4 years at 8% p.a.

1. Rs 4000 2. Rs 4500 3. Rs 5000 4. Rs 6000

4. If a sum triples in 4 years under simple interest, find the time (in years) that it would take to become 5 times itself at the same rate of interest.

1. 6 2. 7 3. 9 4. 8

5. A sum triples in 4 years under compound interest at a certain rate of interest, interest being compounded annually. Find the time (in years) it would take to become 9 times itself.

1. 8 2. 9 3. 10 4. 12

6. If Rs 4000 is lent at 10% p.a, interest being compounded annually, find the interest (in Rs) for the fourth year.

1. 522.8 2. 554.4 3. 520.2 4. 532.4

7. The interest on a sum is compounded every 3 months. If the rate of interest is 40% p.a., find the effective rate of interest per annum.

1. 40% 2. 38.81% 3. 46.41% 4. 36.55%

8. A sum amounts to Rs 28800 in two years and to Rs 34560 in three years under compound interest, interest being compounded annually. Find the sum (in Rs).

1. 25000 2. 20000 3. 28000 4. 15000

9. The compound interest on a certain sum for the 2nd year and the 3rd year are Rs 3300 and Rs 3630 respectively. Find the sum (in Rs).

1. 25000 2. 30000 3. 28000 4. 20000

10. Sanjay borrowed Rs 15000 at 20% p.a. under compound interest, interest being compound annually. He repaid Rs 12000 at the end of the first year. Find the amount (in Rs) he must repay at the end of the second year to clear the loan.

1. 7000 2. 7200 3. 7500 4. 6800

11. Praveen borrowed Rs 26400 at 20% p.a. under compound interest, interest being compounded annually. If he has to repay this in two equal annual instalments, find the value (in Rs) of each instalment.

1. 17280 2. 17600 3. 16880 4. 16800

12. The simple interest for the second year on a certain sum at a certain rate of interest is Rs 1000. Find the sum of the interests (in Rs) accrued on it for the 6^{th}, 7^{th} and 8^{th} years.

1. 1000 2. 2000 3. 3000 4. 4000

13. What sum will fetch a simple interest of Rs 25,410 in five and a half years at 6% p.a. rate of interest?

1. Rs 55,000 2. Rs 88,000 3. Rs 77,000 4. Rs 70,000

14. A sum yields Rs 54 more interest when lent at 37% p.a. simple interest than when lent at 33% p.a. simple interest in six years. Find the sum.

1. Rs 225 2. Rs 250 3. Rs 450 4. Rs 255

15. In how many years will a sum of money become sixteen times itself at 30% p.a. simple interest?

1. 25 2. 40 3. 30 4. 50

16. A sum becomes 84% more in seven years when invested at simple interest. Find the annual rate of interest.

1. 28% 2. 24% 3. 12% 4. 18%

17. A sum of money becomes ten times itself at simple interest. If the time period (in years) is numerically equal to the rate of interest, find the annual rate of interest.

1. 25% 2. 20% 3. 30% 4. 90%

18. Bhavan invested two equal amounts at simple interest. He invested one amount at 15% p.a. and the other at 20% p.a. The total interest earned at the end of a year was Rs 1400. Find the total amount invested (in Rs).

1. 6000 2. 8000 3. 7000 4. 10000

19. A sum under compound interest, interest being compounded annually amounts to Rs 6000 in two years and Rs 7200 in three years. Find the rate of interest p.a.

1. 10% 2. 20% 3. 18% 4. 15%

20. The compound interest on a sum for the third year is Rs 2420, interest being compounded annually. The interest on it for the fourth year is Rs 2662. Find the rate of interest.

1. 12% 2. 10% 3. 15% 4. 8%

21. A sum of money becomes four times itself in eight years at compound interest. In how many years will the same sum become sixteen times itself?

1. 64 2. 32 3. 44 4. 16

22. A sum becomes 2.197 times of itself in three years at compound interest. Find the annual rate of interest.

1. 20% 2. 30% 3. 15% 4. 25%

23. A sum becomes 33.1% more in three years when invested under compound interest, interest being compounded annually. Find the annual rate of interest.

1. 10.5% 2. 11.03% 3. 10% 4. 16.55%

24. Which of the following schemes of computing interest yields the maximum interest for a year?

1. Interest compounded monthly at 2% per month.

2. Interest compounded per quarter 6% per quarter.

3. Interest compounded every six months at 12% for every 6 months.

4. Interest compounded every year at 24% p.a.

25. Given that a principal amounts to Rs 10080 at 10% p.a. simple interest after two years what is the compound interest for two years on this principal at the same rate?

1. Rs 10164 2. Rs 1764 3. Rs 1640 4. Rs 3764

26. Prashanth borrows a sum at compound interest and it amounts to Rs 1,38,240 at the end of three years and to Rs 1,99,065.60 at the end of five years compounded annually. What is the sum borrowed?

1. Rs 76,000 2. Rs 80,000 3. Rs 90,000 4. Rs 96,000

27. Rs 10000 was lent at compound interest, interest being compounded annually for 3 years. The annual rates of interest for the first, second and third years were 10%, 20% and 30% respectively. If it was instead, lent at 20% p.a. simple interest for the same time, how much more / less interest (in Rs) would be realized?

1. 530 more 2. 530 less 3. 1160 more 4. 1160 less

28. The compound interest on a certain sum for the third and the fourth years is Rs 1815 and Rs 1996.50 respectively. What is the annual rate of interest?

1. 5% 2. 8% 3. 10% 4. 12%

29. Prakash invested a certain amount in a six-year fixed deposit scheme, interest being compounded annually. The interests accrued on this deposit for the fourth and fifth years, respectively, are Rs 1331 are Rs 1464.10. If George deposited Rs 12000 in the same scheme, how much interest (in Rs) would be accrued on this deposit for the first two years?

1. 1320 2. 1452 3. 2520 4. 1440

30. Mr. Patel borrows Rs 10000 at 20% p.a. for five years at simple interest. But, from the fourth year onwards, on the entire amount due at the end of three years, the lender begins to charge 20% p.a. interest compounded annually. What is the amount repaid by Mr. Patel after five years from the beginning?

1. Rs 15342 2. Rs 18432 3. Rs 17324 4. Rs 23040

31. Krishna takes a loan of Rs 8000 at simple interest. After four years he takes an additional loan of Rs 14440. From that point, compound interest at 10% per annum is calculated on the total amount repayable on the first loan as well as the second loan. He repays a total of Rs 30250 after two more years to clear the entire loan amount. What is the annual rate of simple interest?

1. 8% 2. 10% 3. 12% 4. 9%

32. Prasad invests Rs 34000, part of it in scheme A at 20% p.a. and the rest of it in scheme B at 60% p.a., both at compound interest. If, after two years, the amount in scheme A is nine times that in scheme B, how much was invested (in Rs) in scheme B?

1. 2500 2. 3000 3. 2200 4. 2000

33. A sum was lent for a year; another sum was lent for 2 years and another sum was lent for 3 years. Each sum was lent at 5% p.a. compound interest. If each sum amounted to the same value, the ratio of the first, second and third sums is:

1. 400: 420: 441 2. 20: 21: 22 3. 22: 21: 20 4. 441: 420: 400

34. Suhaas borrowed a certain amount at 28% p.a. compound interest and repays it in one year. Bhanu borrows a certain amount at a certain interest rate under simple interest and returns it after four years. If the amounts repaid by Suhaas and Bhanu are the same and that is equal to Rs 38400 and the sum of their principals borrowed is Rs 54000, at what rate (p.a.) Bhanu paid the interest?

1. 10% 2. 12% 3. 15% 4. 18%

35. The difference between the amounts to be repaid by a man at the end of two years, at 20% per annum compounded annually and half yearly, is Rs 1084.50. What is the principal (in Rs)?

1. 40000 2. 45000 3. 42000 4. 50000

QUANTIFIERS CAT ACADEMY

Exercise 12

1. Rs 2.4 crores invested at a certain rate of compound interest becomes Rs 19.2 crores in 18 years. The value that Rs 4 crores invested at the same rate of compound interest would amount to in 24 years is (in Rs crores).

1. 60 2. 62 3. 64 4. 50

2. The compound interest and the simple interest on a certain sum for two years are Rs 1230 and Rs 1200 respectively. The rate of interest is same for both compound interest and simple interest. What is the principal?

1. Rs 10000 2. Rs 12000 3. Rs 12500 4. Rs 16000

3. The difference between the compound and simple interests on a sum for three years at 20% p.a. is Rs 3200. Find the sum.

1. Rs 20000 2. Rs 10000 3. Rs 15000 4. Rs 25000

4. Sourabh borrows Rs 2500000 at 12% compound interest from a bank and invests in shares. The investment gives him a return of 20% per annum and he repays Rs 500000 at the end of first year. How much does he make for himself after paying all the outstanding amount at the end of the second year?

1. Rs 424000 2. Rs 356241.50 3. Rs 525000 4. 484241.80

5. Karthik borrows Rs 24000 from a bank at 12% p.a. simple interest. He repays 20% of the principal at the end of every year. What is the simple interest that has accrued for the first four years (in Rs)?

1. 7486 2. 8064 3. 8144 4. 7864

6. What annual instalment will discharge a debt of Rs 1450 due after five years at 8% p.a. simple interest?

1. Rs 320 2. Rs 450 3. Rs 250 4. Rs 400

7. A man took a loan of Rs 100000 at 8% per annum compound interest. He repays Rs 10000 per annum. What is the amount due from him at the beginning of the third year?

1. Rs 91917 2. Rs 81917 3. Rs 93970 4. Rs 95840

8. The difference between the compound interests on a certain sum for the second and the third years at 5% p.a. is Rs 42. Find the sum (in Rs).

1. 14000 2. 15000 3. 16000 4. 20000

9. A sum was borrowed at 20% p.a. compound interest. It was repaid in three annual instalments with each instalment being paid at the end of a year. The first, second and third instalments were Rs 1200, Rs 1152 and Rs 2592 respectively. Find the sum borrowed (in Rs).

1. 3000 2. 3400 3. 3500 4. 3300

10. A sum of money invested at simple interest amounts to Rs 2832 at the end of three years and Rs 3120 at the end of five years. Find the principal.

1. Rs 2400 2. Rs 2000 3. Rs 3500 4. 2500

11. If a sum was Rs 10000 more it would fetch Rs 4000 extra as simple interest, if it was lent at a certain rate of interest for two years. Find the annual rate of interest.

1. 5% 2. 10% 3. 20% 4. 25%

12. If a sum invested at simple interest doubles itself in ten years, how long will it take to become five times itself?

1. 20 2. 40 3. 25 4. 30

13. A sum was invested at simple interest. At the end of four years, the total interest was equal to the sum. At the end of five years the total interest was Rs 12500. Find the interest on the sum at the end of three years (in Rs).

1. 6000 2. 7500 3. 9000 4. 4500

14. Rohan lent a total of Rs 1,00,000 to two people. He lent a certain amount to P at 10% p.a. simple interest and lent the remaining amount to Q at 20% p.a. simple interest. He lent each amount for two years and received Rs 32000 as the total interest. Find the sum he lent to P (in Rs).

1. 30000 2. 45000 3. 40000 4. 35000

15. Rajesh, a money lender, lends in the following manner. For the first year, he charges 2% p.a., for the second year, he charges 4% p.a. for the third year, he charges 6% p.a. and so on. If he lends a sum in this way at simple interest, find the least integral number of years in which it will fetch an interest at least equal to itself.

1. 9 2. 10 3. 11 4. 12

16. Ajay borrowed Rs 20,000 from Balu at 10% p.a. simple interest and twice that amount from Chetan at 20% p.a. simple interest. He then added an amount of his own and lent the entire sum at 18% p.a. simple interest. All the deals were for a year. At the end of the year, he earned Rs 4400. Find the amount that he added (in Rs).

1. 15000 2. 26000 3. 24000 4. 20000

17. A sum was invested under compound interest; interest being compounded annually. It fetches Rs 14400 as interest in the second year and Rs 17280 as interest in the third year. Find the annual rate of interest.

1. 10% 2. 15% 3. 20% 4. 25%

18. The interest on a sum under compound interest when interest is compounded annually for the sixth year and the seventh year are Rs 1,256 and Rs 1,413 respectively. Find the rate of interest.

1. 10% 2. 15% 3. 7.5% 4. 12.5%

19. A sum was lent at 20% p.a. compound interest; interest being compounded annually. The interest for the first year was Rs 4000. Find the total interest for the second and third years (in Rs).

1. 10520 2. 11450 3. 10840 4. 10560

20. A sum when lent at a certain rate compounded annually amounts to Rs 1200 in two years and when lent at twice the previous rate amounts to Rs 1323 in the same time. What is the rate p.a. at which it was initially lent?

1. 4% 2. 4 3/7% 3. 5 5/19% 4. 6 1/4%

21. Srikar saves Rs 20000 at the beginning of each year in a savings bank account that pays 5% p.a. interest being compounded annually. If, at the beginning of the third year, instead of depositing Rs 20000, he withdraws Rs 10000, how much would be the total savings of the man at the end of three years?

1. Rs 28124.24 2. Rs 29324.20 3. Rs 31349.75 4. Rs 34702.50

22. Anwar borrowed Rs 72000 at 20% p.a. compound interest, interest being compounded annually. He repaid Rs x at the end of the first year. He repaid Rs 57600 at the end of the second year and thereby cleared the loan. Find x.

1. 36500 2. 38400 3. 38200 4. 33200

23. A sum was divided into two equal parts. One part was lent at 20% p.a. simple interest. The other part was lent at 20% p.a. compound interest, interest being compounded annually. The difference in the interests fetched by the parts in the second year is Rs 400. Find the difference in the interests fetched by the parts in the fourth year (in Rs).

1. 1414 2. 1442 3. 1456 4. 1484

24. The value of a machine depreciates at the rate of 10% p.a. every year. What was the value (in Rs) of the machine two years ago, if its present value is Rs 24,300?

1. 25000 2. 28000 3. 30000 4. 22000

25. Buffet invested a certain amount in 3 different schemes viz. Gold, Silver and Platinum. The three schemes respectively offer an interest rate of 10%, 12% and 15%. If the total interest accrued in one year was Rs. 3200 and the amount invested in the Platinum scheme was 150% of the amount invested in the Gold scheme and 240% of the amount invested in the Silver scheme, what was the amount invested in the Silver scheme?

1. Rs 8400 2. Rs 7500 3. Rs 6400 4. Rs 5000

26. If the ratio of SI and CI earned on a certain sum at a certain rate of interest in the first two years is $\frac{51}{50}$ and the difference between the SI and CI earned in the first two years is 16, what is the sum of SI and CI earned in the second year ?

1. Rs. 636 2. Rs. 724 3. Rs. 808 4. Rs. 816

27. What is the difference in the interest earned on a sum of Rs. 5000, over one year, at 8% per annum if kept at simple interest and if kept at semi-annual compounding?

1. Rs. 0 2. Rs. 8 3. Rs. 400 4. Rs. 440

28. At a compound interest rate of 8%, the compound interest earned in the 20^{th} year is Rs. 2160, what is the compound interest earned in the 17^{th} year?

1. Rs. 1810 2. Rs. 1525 3. Rs. 1500 4. Rs. 1250

29. A sum of money compounded annually becomes Rs. 625 in 2 years and Rs. 675 in 3 years. The rate of interest per annum is:

1. 7% 2. 10% 3. 5% 4. 8%

30. Difference between compound interest and simple interest for 2 years on a sum of money is Rs. 160. If total simple interest is Rs. 2880, then find rate of interest.

1. 5 5/9% 2. 12 1/2% 3. 11 1/9% 4. 9%

31. The compound interest on a certain sum of money for 2 years at 10% per annum is Rs. 420. The simple interest on the same sum at the same rate and for the same time will be:

1. Rs. 350 2. Rs. 375 3. Rs. 400 4. None of these

32. Narayan and Murthy invested some money at 6% and 7% per annum respectively of SI. At the end of 2 years, they found that together they received Rs. 354 as interest. One-fourth of Narayan's initial investment is equal to one-fifth of the money invested by Murthy. Find the total money invested.

1. Rs. 2,746.5 2. Rs. 2,600 3. Rs. 2,700 4. Rs. 2,880

33. A certain amount of money compounded annually at r% becomes Rs. 1,440 and Rs. 1,728 after 2 and 3 years respectively. Find the value of r.

1. 5% 2. 10% 3. 15% 4. 20%

34. Rs. 2,189 is divided into three parts such that their amounts after 1, 2 and 3 years respectively may be equal. If the rate of simple interest being 4% p.a. in all cases, the smallest part is:

1. Rs. 389 2. Rs. 756 3. Rs. 703 4. Rs. 352

35. Rohit decides to invest a certain amount of money in savings account of a bank that pays 20% compound interest per annum for a period of two years. What is the sum of money that Rohit should invest so that he receives Rs. 200 at the end of first year, and Rs. 400 at the end of second year?

1. Rs 500 2. Rs 1400/3 3. Rs 4000/9 4. None of these

36. An amount of Rs. 7,000 was divided into two equal parts. The first part was deposited in a bank at simple interest rate of 8% per annum for three years. The second part was deposited in another bank at the rate of 10% per annum, compounded annually, for 2 years. What is the difference (in Rs) in the interests earned from the two amounts?

1. 105 2. 110 3. 100 4. 115

37. Equal sums of money are invested for 2 years at 10% compound interest - one gets compounded annually and the other half yearly. Find the difference (in Rs.) between the amounts at the end of 2 years if the sum invested was Rs. 5,000.

1. Rs. 26.50 2. Rs. 27.50 3. Rs. 25.50 4. Rs. 24.50

38. A man borrows Rs 6000 at 5% interest, on reducing balance, at the start of the year. If he repays Rs 1200 at the end of each year, find the amount of loan outstanding, in Rs. at the beginning of the third year.

1. 3162.75 2. 4125 3. 4155 4. 5100

39. A cell phone is available for Rs. 39000 cash or Rs. 6000 cash down payment followed by 4 equal monthly instalments at 24% p.a. compound interest, compounded monthly. Find the value of each instalment.

1. 12000 2. 15320 3. 9640 4. None of these

Chain Rule and Time & Work

Chain Rule

A certain type of problems involving joint variations are more commonly called problems on chain rule. They are an easier lot and also very easily identifiable as being questions on chain rule. Rather than use any theory to solve them, we will use pure logic to approach such problems and the logic is as explained in the following examples.

Let's start with a simple question.

If 10 weavers weave 10 carpets in 10 days, how many carpets will 1 weaver weave in 1 day?

If you have got your answer as 1 carpet, make sure you read the following very carefully, as it is not the correct answer.

10 weaver, 10 carpets, 10 days is one instance of the variables involved.

Had there been just 1 weaver i.e., the number of weavers reduce to $\frac{1}{10}$, obviously the number of carpets weaved would also reduce (proportionally)

Thus... 1 weaver weaves 1 carpet in 10 days.

If the number of days is reduced, from earlier 10 days to just 1 day now, obviously the 1 weaver would have weaved lesser carpet, to be exact $\frac{1}{10}$ of the carpet.

Thus, 1 weaver would weave $\frac{1}{10}^{th}$ carpet in 1 day.

Let's take another example:

Ex. **4 carpenters make 20 chairs in 5 days. How many chairs will 8 carpenters make in 10 days?**

Sol: 4 carpenters make 20 chairs in 5 days. So, 8 carpenters will make double the number of chairs in 5 days (same time as earlier) i.e., $20 \times \frac{8}{4} = 40$chairs.

When you are accounting for the change in the number of chairs made because the number of carpenters has increased, you have to keep all the other factors (days available, etc.) constant. Their effect will be included when that particular variable is taken as the focus.

Next since the number of days has also increased from 5 to 10 i.e., doubled, these 8 carpenters who earlier made 40 chairs in 5 days will make double the number of chairs in 10 days i.e., $40 \times \frac{10}{5} = 80$ chairs.

Ex. **6 labourers can build a wall in 10 days. How many labourers of double the efficiency, are needed to build 3 such walls in 5 days?**

Sol: To build 1 wall in 10 days, 6 laborers are required, so to build 3 walls in 10 days, laborers will be required.

If the number of days in which the work has to be completed, decreases, the number of labourers required will increase. So when the days in which the work has to be completed reduce from 10 to 5, the men needed would increase proportionally i.e., labourers would now be needed.

But since the new labourers have twice the efficiency, only half of them will be needed i.e., 18 laborers.

Taking an example, let's establish a method of doing these kind of questions:

Ex. **If 6 painters can paint 18 walls in 2 weeks, how many walls of twice the area can 16 painters paint in three weeks?**

Sol: We are supposed to find the number of walls. Earlier 14 walls were painted. So start by writing '18 × ' on your sheet.

Now the number of painters has increased from 6 to 16. Thus, more walls would be painted. Which of the ratio, $\frac{16}{6}$ or $\frac{6}{16}$, when multiplied with 18 will lead to more number of walls than 18? Obviously only when you multiply by a number greater than 1, will the number of walls increase, so it has to be $\frac{16}{6}$.

Hence since painters have increased from 6 to 15, the number of walls painted will be $18 \times \frac{16}{6}$. Don't calculate it as yet.

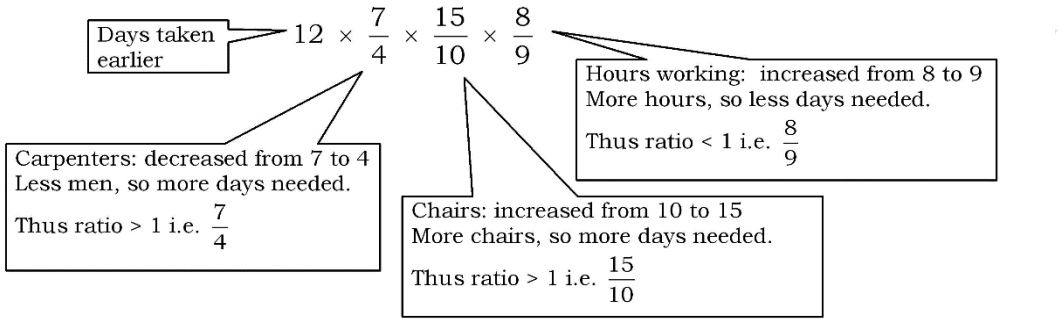
Next since the walls are twice the size, the number of walls painted will be less, to be precise will be $\frac{1}{2}$ of earlier i.e., $18 \times \frac{16}{6} \times \frac{1}{2}$.

Lastly since the number of weeks has increase from 2 to 3, one will be able to paint more walls. 'More' suggests a ratio greater than 1 i.e., $\frac{3}{2}$. Thus, the number of walls that can be painted is $18 \times \frac{16}{6} \times \frac{1}{2} \times \frac{3}{2}$ i.e., 36 walls.

Ex. If 8 carpenters build 10 chairs in 12 days working 9 hours a day, how many days will it take for 4 carpenters to build 15 chairs if they work for 8 hours a day?

Sol: In the above question we need to find the number of days. So start with the number of days and then taking each of the other changing variable one at a time, just spend a thought whether the number of days will increase or decrease because of the change. E.g., if number of carpenters decrease, number of days required to build the same number of chairs will be more. Also if number of chairs to be made increases, the number of days required to make them will also be more.

Accordingly multiply the number of days with the ratio of the variables that are changing i.e., number of carpenters and number of chairs here. Multiply with a ratio greater than 1 if the number of days has to increase and with the ratio less than 1 if the number of days has to decrease.



Thus, number of days needed = $12 \times \frac{7}{4} \times \frac{15}{10} \times \frac{8}{9} = 28$.

Concept of man-days

Another way of approaching such problems is by quantifying the work as man-days or man-hours. E.g., If 8 carpenters build 10 chairs in 12 days working 9 hours a day, then the work i.e. 10 chairs can be quantified as involving 8 men × 12 days × 9 hr/day i.e. 8 × 12 × 9 man-hours.

Needless to add, the work and required man-hours are directly proportional (if work doubles, the man-hours needed will double). Thus, if m, d, h and w refer to the number of men, days needed, hours per day and work respectively, then we can also use

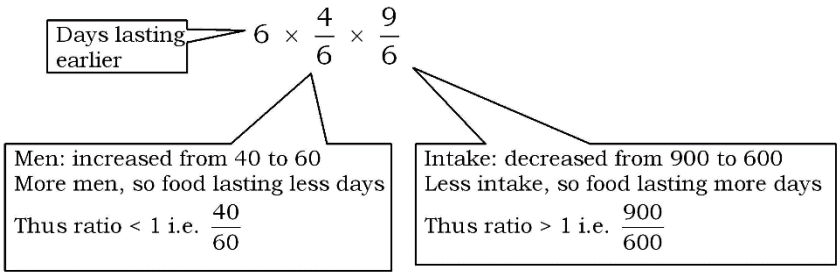
$$\frac{m_1 \times d_1 \times h_1}{w_1} = \frac{m_2 \times d_2 \times h_2}{w_2}$$

This relation obtained is a result of joint variation. The individual relations $w \propto m$; $w \propto d$; $w \propto h$ have all be combined into one joint variation i.e., $w \propto m \times d \times h$ or $\frac{w}{m \times d \times h}$= constant.

Use of this formulaic approach makes the solution less thought intensive, but then if other variables are introduced in the question, you will have to think where will the new variable appear - in the numerator or denominator.

Ex. **A garrison has enough food to sustain its 40 men for 6 months if each man consumes 900 gm of food per day. A reinforcement of 20 men join the garrison. For how long will the food last now, if each man now consumes only 600 gm of food per day?**

Sol: Approach 1: Using proportionality directly



Thus, the food will again last for 6 months.

Approach 2: Using man-days

One could also quantify the amount of food (equivalent of work in this example) in terms of man-days.

There is 'enough food to sustain its 40 men for 6 months if each man consumes 900 gm of food per day' means that the food available is equivalent of 40 × 6 × 900 man-month-grams

Now when 20 more men join in, the number of men becomes 60 and since each eat 600 gm food each day, the available quantity of food will last for $\frac{40\times6\times900 \text{ man-month-grams}}{60\times600 \text{ man-grams}}$ = 6 months.

Time & Work:

Almost every entrance test has questions based on time and work and these questions are one of the easier ones to solve. So it will pay rich dividends to do this topic thoroughly.

Questions on work usually fall in one of the following categories:

1. Relation between days taken by individuals to complete a given work independently and to complete while working simultaneously or alternately.

2. Teams of men, women, children and time taken by the teams to complete a work independently or working simultaneously.

There are very few innovative questions that can be based on this topic and thus most questions are the standard types. Also, the approach to solve questions is very standard one. There are two approaches, which are exactly the same but seem to be different – approach using the per day's work and approach using LCM. The two approaches are best explained through an example.

Basics

If A takes x days to do a piece of work, then each day he would be doing $\frac{1}{x}^{th}$ of the work.

Conversely, if in one day f fraction of work is done, then complete work will be done in $\frac{1}{f}$ days.

Ex. **If *A* does a work in 10 days and *B* does the same work individually in 12 days, in how many days will the work be completed if they work simultaneously?**

Sol: Approach 1: Per day's work

Since A completes the entire work in 10 days, A does $\frac{1}{10}^{th}$ of the work in 1 day.

Since B completes the entire work in 12 days, B does $\frac{1}{12}^{th}$ of the work in 1 day.

Working simultaneously, they do $\frac{1}{10}+\frac{1}{12}=\frac{11}{60}^{th}$ of the work in 1 day

Thus, total days taken by both working simultaneously = $\frac{60}{11}$ days

Approach 2: LCM

Let the amount of work be 60 units (LCM of 10 and 12)

Since A does 60 units in 10 days, he does 6 units every day

Since B does 60 units in 12 days, he does 5 units every day.

Working simultaneously, they do 6 + 5 = 11 units each day.

Thus, to complete 60 units of work, they will take $\frac{60}{11}$ days.

The two approaches are absolutely identical, it is just that in the earlier approach the work was assumed as 1 unit instead of 60 units.

The above example illustrates all the theory that you will have to use. All other matter in a question is building a story around the above.

Ex. **If *A* does a work in 10 days and *B* does the same work individually in 12 days. *A* starts working alone and after 3 days is joined by *B*. After another 1 days, *A* quits. How many more days will be taken by *B* to complete the work?**

Sol: Approach 1: Per day's work

A is effectively working for 4 day and thus will do $\frac{4}{10}$ i.e., $\frac{2}{5}^{th}$ of the work.

B has to do $\frac{3}{5}^{th}$ of the work and he will take a total of $\frac{\frac{3}{5}}{\frac{1}{12}}=\frac{36}{5}$ = 7.2 days for it. He has already put in 1 day of work. So, he will take another 6.2 days

You should also be familiar with the equation: $\frac{4}{10}+\frac{d}{12}=1$, where d is the days B has worked. The right-hand side of the equation is 1 because the entire work is done and, in this approach, the entire work is assumed as 1. This equation will also give us d = 7.2

Approach 2: LCM

Let amount of work be 60 units.

Thus, A does 6 units per day and B does 5 units per day.

A works for total of 4 days and competes 24 units. *B* does work for 1 day and does 5 units. Thus 29 units of the work is already done and 31 units are remaining which will be done at the rate of 5 units per day. Thus, $\frac{31}{5}$= 6.2 more days will be taken.

Concept of Negative Work, Pipes and Cisterns

While in the above examples, both players were working to get the work done, it is also quite possible that one of them might be working towards 'breaking' the work i.e., doing negative work. Such type of problems do appear in the above contexts also, but are more popular in questions called 'Pipes and Cisterns'. These questions are exactly the same as above where the work to be done is to fill a cistern (anything that collects water e.g., tank, bucket, etc.) using inlet pipes. In case of cistern, a leak at the bottom of the tank or an outlet pipe denotes negative work.

Ex. **An inlet pipe can fill in an empty cistern in 30 minutes whereas a leak in the bottom of the cistern can empty a filled tank in 40 minutes. Find the time taken to fill the cistern when both the inlet pipe and the leak are on.**

Sol: Part of cistern that is filled each minute = $\frac{1}{30}-\frac{1}{40}=\frac{1}{120}$. Thus, entire cistern is filled in 120 minutes.

Ex. **Two inlet pipes can individually fill a cistern in 10 minutes and 20 minutes respectively. However, it took them 8 minutes to fill the cistern working simultaneously, because of a leak at the bottom of the cistern. In how many minutes would the leak empty the entire filled cistern if the inlet pipes are closed?**

Sol: Since cistern is filled in 8 minutes, each minute $\frac{1}{8}^{th}$ of it is getting filled. Thus, if the leak can empty the filled cistern in *x* minutes, we have $\frac{1}{10}+\frac{1}{20}-\frac{1}{x}=\frac{1}{8}\Rightarrow\frac{1}{x}=\frac{4+2-5}{40}=\frac{1}{40}$. Thus, the leak can empty the cistern in 40 minutes.

Ex. **If 4 boys or 3 men can complete a piece of work in 10 days how many days will it take for 4 men and 3 boys to finish the work?**

Sol: Approach 1: Converting to one denomination only.

The team of 4 boys is equivalent to team of 3 men, since both these team takes the same number of days. Thus, 1 boy is equivalent to $\frac{3}{4}$ men. And hence 3 boys will be equivalent to $\frac{9}{4}$ men.

Now the combination of 4 men and 3 boys can be thought of as $4+\frac{9}{4}=\frac{25}{4}$ men.

3 men complete the work in 10 days. Hence 25/4 men can complete the work in $\underbrace{\frac{3}{\frac{25}{4}}}_{\substack{\text{more men}\Rightarrow\\ \text{less days}}}\times 10=\frac{3\times4\times10}{25}=\frac{24}{5}$days.

Approach 2: Treating them as two individuals working simultaneously.

If 4 boys can finish the work in 10 days, then 3 boys can finish it in $\underbrace{\frac{4}{3}}_{\substack{\text{less boys}\Rightarrow\\ \text{more days}}}\times 10=$

$\frac{40}{3}$ days. Thus, 3 boys do $\frac{3}{40}^{th}$ of the work in 1 day

If 3 men can finish the work in 10 days, then 4 men can finish it in $\underbrace{\frac{3}{4}}_{\substack{\text{more men}\Rightarrow\\ \text{less days}}}\times 10=$

$\frac{15}{2}$ days. Thus, 4 men do $\frac{2}{15}^{th}$ of the work in 1 day.

Thus, work done by 3 boys and 4 men in 1 day = $\frac{3}{40}+\frac{2}{15}$ i.e., $\frac{25}{120}$ i.e. $\frac{5}{24}^{th}$ of the work.

Therefore, days taken to complete the work = $\frac{24}{5}$ days.

Ex. **1 man and 2 women can complete a work in 10 days. 2 men and 3 women can complete the work in 6 days. Find the number of days needed by 4 men and 4 women to complete the work.**

Sol: Approach 1: Treating them as two individuals working simultaneously

If 1 man, working alone, takes m days to finish the entire work and 1 woman, working alone, takes w days to finish the entire work, then,

$\frac{1}{m}+\frac{2}{w}=\frac{1}{10}$ ..... (i)

$\frac{2}{m}+\frac{3}{w}=\frac{1}{6}$ ..... (ii)

2 × (i) – (ii) $\frac{1}{w}=\frac{1}{5}-\frac{1}{6}=\frac{1}{30}$.

Substituting 1/w in (i), $\frac{1}{m}=\frac{1}{10}-\frac{1}{15}=\frac{1}{30}$

Work done by 4 men and 4 women in 1 day will be $\frac{4}{m}+\frac{4}{w}=\frac{4}{15}^{th}$ of the work. Thus, the number of days taken is $\frac{15}{4}$days.

Approach 2: Converting to one denomination only

1 man and 2 women can complete the work in 10 days. Hence if work has to be completed in 1 day, we would need 10 such teams i.e., 10 men and 20 women.

2 men and 3 women can complete the work in 6 days. Hence if work has to be completed in 1 day, we would need 6 such teams i.e. 12 men and 18 women.

Thus (10 men & 20 women) is equivalent to (12 men & 18 women) i.e., 2 women = 2 men i.e., one woman is equivalent to one man.

Thus, 3 persons can complete the work in 10 days and hence 8 individuals can complete the work in $\underbrace{\frac{3}{8}}_{\substack{\text{more men} \Rightarrow \\ \text{less days}}} \times 10 = \frac{15}{4}$days.

Ex. **If 17 labourers can dig a ditch 20 m long in 18 days, working 8 hours a day, how many more labourers should be engaged to dig a similar ditch 39 m long in maximum of 6 days, each labourer working 9 hours per day.**

1. 89 **2. 84** **3. 78** **4. 72**

Sol: Using $m_1 \times d_1 \times h_1 \times w_2 = m_2 \times d_2 \times h_2 \times w_1$, we get

$17 \times 18 \times 8 \times 39 = x \times 6 \times 9 \times 20 \Rightarrow x = 88.4$

Now, as the work has to be completed in maximum 6 days, so the number of labourers cannot be less than 88.4 or it cannot be 88. So, the minimum number of labourers required is 89. The additional labourers = 89 – 17 = 72.

Ex. **If 5 engines consume 6 metric tonnes of coal when each is running for 9 hours a day, how many metric tonnes of coal will be needed for 8 engines, each running 10 hours a day, given that 3 engines of the former type consume as much as 4 engines of the latter type.**

1. 8 **2. 9** **3. 12** **4. 12.5**

Sol: Here the 3 engines of the first type = 4 engines of second type

⇒ 6 engines of the first type = 8 engines of second type ..... (i)

Let the coal required by second type of engines is x metric tonnes.

We have, $5 \times 9 \times x = 8 \times 10 \times 6 \Rightarrow 5 \times 9 \times x = 6 \times 10 \times 6$ (Using (i)) $\Rightarrow x = 8$

Ex. **The electricity bill of an office varies directly with the usage of the air conditioners. If 8 air conditioners of 1 ton each are used for 8 hours every day, the monthly bill is Rs. 5000. What will the approximate bill amount be, if 12 air conditioners of 1.5 tons each are used for 10 hours every day?**

1. 8000 **2. 9000** **3. 10000** **5. 14000**

Sol: Let the monthly bill is Rs x.

We have, $8 \times 1 \times 8 \times x = 12 \times 1.5 \times 10 \times 5000$

$\Rightarrow x = \text{Rs } 14062.5 \approx \text{Rs } 14000$

Ex. **A garrison has sufficient food for 75 soldiers for a period of 90 days. After 10 days, one-third of the soldiers leave. After another 10 days, 5 soldiers return. From this day on, how many days will the food last?**

1. 80 **2. 90** **3. 100** **4. 110**

Sol: Let the required days be x

We have $75 \times 90 = 75 \times 10 + 50 \times 10 + 55 \times x$

$\Rightarrow 6000 - 500 = 55x \Rightarrow 55x = 5500 \Rightarrow x = 100.$

Ex. **A contractor undertakes to do a job in 20 days so he employs 36 laborers. After 5 days he realizes that $3/11^{th}$ of the work is over. To finish the work exactly on time, how many more laborers does he need to employ or how many can he afford to fire?**

1. Hire 4 **2. Hire 32** **3. Fire 4** **4. Hire 32**

Sol: Here we have

Men	Days	Work
36	5	3/11
x	15	8/11

Now, using $M_1 \times D_1 \times W_2 = M_2 \times D_2 \times W_1$, we get,

$36 \times 5 \times 8/11 = x \times 15 \times 3/11 \Rightarrow x = 32.$

Therefore, the remaining work can be completed by 32 workers in the remaining days. Hence, he can fire 4 workers.

Ex. ***A* tank, which usually takes 5 hours to be filled, takes 6 hours to fill because of a leak. Find the time (in hours) in which the leak can empty the tank if it is filled and the inlet pipe is shut.**

1. 30 **2. 27.5** **3. 25** **4. 22.5**

Sol: Let the capacity of the tank is 30 litres.

Pipe can fill 6 litres/hour. As actual time taken is 6 hours, so, the actual rate of filling was 5 litres/hour. Therefore, the leak has drained 1 litre water per hour. Hence, it can drain the whole tank in 30 hours.

Ex. **Three taps *A*, *B* and *C* can fill a tank individually in 10 hours, 20 hours and 25 hours respectively. At first all the taps are opened simultaneously. After 2 hours tap *C* is closed and after further 2 hours tap *B* is also closed. Tap *A* is kept open till the tank gets completely filled. What fraction of the tank is filled by tap *A*?**

1. $\frac{2}{11}$ **2.** $\frac{9}{11}$ **3.** $\frac{18}{25}$ **4.** $\frac{7}{25}$

Sol: Let the capacity of the tank is 100 litres.

Pipe A can fill 10 litre/hour, Pipe B can fill 5 litre/hour and Pipe C can fill 4 litre/hour.

Water filled by pipe C in two hours = 2 × 4 = 8 litre

Water filled by pipe B in 4 hours = 4 × 5 = 20 litres

Water filled by pipe A = 100 – (8 + 20) = 100 – 28 = 72 litre

Therefore, pipe A filled 72/100 or 18/25th of the whole tank.

Ex. 4 men and 3 women can finish a piece of work in 6 days whereas if there were 5 men and 7 women, they would have taken 4 days to finish the work. How many days will 1 man and 1 woman take to finish the work?

1. $\frac{156}{5}$ **2.** $\frac{156}{7}$ **3.** $\frac{156}{9}$ **4.** $\frac{156}{11}$

Sol: If 1 man, working alone, takes m days to finish the entire work and 1 woman, working alone, takes w days to finish the entire work, then,

$\frac{4}{m}+\frac{3}{w}=\frac{1}{6}$ ..... (i)

$\frac{5}{m}+\frac{7}{w}=\frac{1}{4}$ ..... (ii)

4 × (ii) – 5 × (i) $\Rightarrow \frac{13}{w}=1-\frac{5}{6}=\frac{1}{6} \Rightarrow 1/w = 1/78$

Substituting 1/w in (i), $\frac{4}{m}=\frac{1}{6}-\frac{1}{26}=\frac{5}{39} \Rightarrow 1/m = 5/156$

Work done by 1 man and 1 woman in 1 day will be $\frac{1}{m}+\frac{1}{w}=\frac{5}{156}+\frac{1}{78}=\frac{7}{156}th$ of the work. Thus, the number of days taken is 156/7 days.

Ex. *A* and *B* can complete a work working individually in 60 days and 40 days respectively. Both start working simultaneously on the work, but 4 days before the work is scheduled to get over, *B* leaves. Find the total number of days taken for the work to be completed.

1. 24 **2. 30** **3. 26.4** **4. 28.2**

Sol: Let the total work is 120 units.

A can do 2 units/day and B can do 3 units/day. Together they can do 5 units/day.

The scheduled time = 120/5 = 24 days.

Now B left 4 days before the scheduled time, so they together worked for 20 days.

Work done by A and B in 20 days = 20 × 5 = 100 units.

The remaining 20 units will be completed by A alone and he will take 20/2 = 10 days to complete it. Therefore, the total days = 20 + 10 = 30.

Ex. *A* and *B* can complete a work working individually in 60 days and 40 days respectively. Both start working simultaneously on the work, but *B* did not work on the last 4 days. Find the total number of days taken for the work to be completed.

1. 24 **2. 25** **3. 26.4** **4. 28.2**

Sol: This problem is different from the previous one as here B did not work for the last 4 days, whereas in the last problem he left the work 4 days before the scheduled time. In this question he did not leave 4 days before the scheduled time. So, the scheduled time has no role here in this problem.

Let the total work is 120 units.

A can do 2 units/day and B can do 3 units/day

In the last 4 days, only A worked and he did 4 × 2 = 8 units.

The remaining 112 units are completed by A and B together and the took 112/5 = 22.4 days to complete it. So, the total time taken = 22.4 + 4 = 26.4 days.

Ex. ***A* is thrice as efficient as *B* and hence takes 12 days less to complete a work as compared to *B*. Find the number of days in which they can complete the work while working together?**

1. 3 **2. 4.5** **3. 6** **4. 6.5**

Sol: As A is thrice efficient than B, so he will take 1/3rd the time taken by B.

Let B takes 3x days to complete the work.

A will take x days to complete the work.

Given that 3x – x = 12 ⇒ x = 6 days.

So, A will take 6 days and B will take 18 days to complete the work.

Together they can do 1/6 + 1/18 = 4/18 or 2/9th of the work in one day.

So, together they will complete the work in 9/2 or 4.5 days.

QUANTIFIERS CAT ACADEMY

Exercise 13

1. If 15 men take 60 days to complete a job, find the time taken by 45 men to complete it.
15 2. 20 3. 18 4. 25

2. 18 men take 20 days to complete a job working 12 hours a day. Find the number of days that 15 men will take to complete it if they work 9 hours a day.
1. 32 2. 20 3. 25 4. 30

3. 20 men take 10 days to complete a job working 12 hours a day. Find the number of men required to complete a job, twice as large, in 30 days working 8 hours a day.
1. 15 2. 18 3. 20 4. 25

4. 10 boys or 20 girls can complete a job in 10 days. Find the time taken by 10 boys and 20 girls to complete it.
1. 4 2. 5 3. 6 4. 8

5. 3 men and 4 women can complete a job in 10 days. 24 men and 2 women can complete it in 2 days. Find the time taken by 11 men and 3 women to complete it.
1. 4 2. 6 3. 5 4. 8

6. A contractor decided to complete a job in 30 days for which he employed 20 men in the beginning. After 10 days he released that only $1/5^{th}$ of the job was completed. How many more men should be employed to complete the job on time.
1. 12 2. 10 3. 20 4. 15

7. X men can complete a work in 120 days. If there were 10 men more, the work would be completed in 20 days less. Find the value of X.
1. 40 2. 42 3. 45 4. 50

8. Ten men can do a piece of work in 15 days. How many men are needed to complete a work which is five times as large as the first one, in 10 days?
1. 50 2. 75 3. 80 4. 70

9. Nine men can complete a job in 15 days. If a man works thrice as fast as a woman, find the number of days taken by 15 women to complete the job.
1. 20 2. 24 3. 27 4. 36

10. A man can complete a job in four days working ten hours a day. In how many days can he complete it if he works eight hours a day?
1. 4 2. 5 3. 6 4. 8

11. Three men can complete a job in 3 days, working 3 hours a day. In how many days can 9 men complete it working 1 hour a day?
1. 6 2. 1.5 3. 3 4. 9

12. A group of 50 salesmen plan to achieve their target for the next 30 days by working 12 hours a day. Due to various reasons, they put in only 10 hours a day for the first 15 days. Now, if 10 men leave and the rest continue working for only 10 hours a day, how many days more than the initially estimated time will they require to meet their target?
1. 11¼ 2. 12¾ 3. 13¼ 4. 13¾

13. A contractor was assigned a task to be completed in 120 days. He initially hired 48 men to complete the task. But after 60 days, only two–fifths of the task was completed. How many more men should he hire after 60 days in order to complete the task on time?

1. 20 2. 25 3. 24 4. 30

14. If a piece of work can be done by 6 men and 8 women in 10 days or by 8 men and 22 women in 5 days, in how many days will 34 women do a piece of work thrice as large?
1. 10 2. 12 3. 20 4. 15

15. A gas station needs 80 people to fill air in 20,000 tyres in 84 days. How many people does the gas station need to fill air in 30,000 tyres in 63 days?
1. 140 2. 150 3. 160 4. 155

16. Two men or 5 women can complete a piece of work in 15 days. In how many days can 4 men and 5 women complete the same work?
1. 8 2. 7 3. 5 4. 4

17. The work done by a man in 3 days is equal to the work done by four boys in 2 days. How many men are needed to do a piece of work in 48 days which 72 boys can do in 16 days?
1. 10 2. 8 3. 9 4. 12

18. 40 men who can complete a job in 96 days started a job. After 24 days, 20 men joined. After 32 more days, N men left. The remaining men completed the remaining work in 20 more days. Find the value of N.
1. 10 2. 12 3. 15 4. 18

19. In a farm, each cow eats twice as much grass as each sheep. The cost of grass for 10 cows and 40 sheep for 20 days is Rs 900. Find the cost of grass for 20 cows and 10 sheep for 18 days (in Rs).
1. 600 2. 675 3. 750 4. 800

20. The cost of grass for 20 cows and 30 sheep for 30 days is Rs 720. If the 30 sheep eat double the grass eaten by the 20 cows, then what is the cost (in Rs) of grass eaten by 20 sheep in 15 days?
1. 150 2. 160 3. 140 4. 175

21. If 30 men working 7 hours a day can make 12 tables in 18 days, how many days will 45 women working 9 hours a day take to make 32 chairs? 4 men can make 3 tables in the same time as 3 women can make 4 chairs.
1. 12 days 2. 14 days 3. 28 days 4. 21 days

22. If 30 men complete $\frac{1}{7}^{th}$ the work in 2 days. How many more men should join in now, if the work has to be completed in 10 more days?
1. 2 2. 4 3. 5 4. 6

23. 10 students planned to complete a job in 8 days. But not everyone was present on all the days. On the 3rd and 4th day, 2 people were absent and on the last two days of work, 3 people were absent. By what fraction did the actual number of days taken to complete the work increase?
1. $\frac{1}{8}$ 2. $\frac{1}{9}$ 3. $\frac{1}{10}$ 4. $\frac{1}{11}$

24. If 18 pumps of 25 watts can raise 2170 tonnes of water to a height of 15 mts in 10 days working 7 hours a day, how many pumps of 40 watts will be required to raise 1736 tonnes of water to a height of 12 mts in 14 days, working 9 hours a day?
1. 4 2. 8 3. 9 4. 12

25. If 98 lamps can be lighted, 5 hours per day for 10 days for Rs. 12.25, then find the number of lamps which can be lighted 4 hours daily for 30 days in Rs. 76.50.
1. 165 2. 1484 3. 255 4. 164

26. Three pumps, a large, a medium and a small pump, are filling a tank. The large pump takes two-thirds the time taken by the medium pump to fill the entire tank. The medium pump takes two-third the time taken by the small pump to fill the entire tank. All the three pumps working simultaneously will take what fraction of the time taken by the medium pump alone to fill the tank?

1. 4/9 2. 1/9 3. 6/19 4. 1/2

27. 95 men went on a camp taking with them enough food that would last them 200 days. After 30 days, few men left the camp. Twice as many men as those who left joined the camp some days later. With all these changes if the food still lasted for 200 days, on which day, counted from the starting day, did the men join the camp?

1. 86th day 2. 114th day 3. 115th day 4. 116th day

28. Eight children and twelve men complete a certain piece of work in 9 days. If each child is half as efficient as a man, then in how many days will 12 men finish the same work?

1. 10 2. 12 3. 15 4. 11

Exercise 14

1. A and B together can complete a job in 12 days. A alone can complete it in 24 days. Find the time taken by B to complete it.
1. 12 2. 15 3. 18 4. 24

2. Ajay and Bala working together can complete a job in 16 days. Ajay alone can complete it in 18 days. Both work together for 4 days and then Bala leaves. Find the time taken by Ajay to complete the remaining work.
1. 13 2. 13.5 3. 14 4. 14.5

3. P and Q together can complete a job in 12 days. Q and R together can complete it in 15 days. P and R together can complete it in 18 days. All the three work together for 3 days and then R leaves. In how many days can P and Q complete the remaining work?
1. 7.2 2. 8.3 3. 9.4 4. 10

4. A can complete a job in 16 days. He started the work and after 4 days, B joined him. They completed the job in 4 more days. Find the number of days in which B alone can complete it.
1. 8 2. 6 3. 7 4. 9

5. Pipes X and Y take 10 minutes and 20 minutes respectively to fill an empty tank. Pipe Z takes 40 minutes to empty a full tank. Find the time taken (in minutes) to fill the empty tank if all the three pipes are opened simultaneously.
1. 6 2. 5 3. 8 4. 7.5

6. Pipes P and Q take 24 minutes and 36 minutes respectively to fill an empty tank. If both take 18 minutes to fill the tank along with an outlet pipe R, find the time (in minutes), R would take to empty the full tank.
1. 60 2. 72 3. 50 4. 64

7. Anand, Bhanu and Chandra can complete a work in 4 days. Anand and Chandra can do the same work in 8 and 16 days respectively. How many days will Bhanu alone take to complete the same work?
1. 16 2. 20 3. 8 4. 12

8. P and Q can complete a job in 12 days. Q and R can complete it in 20 days. R and P can complete it in 15 days. Find the time taken by P, Q and R working together, to complete it.
1. 10 days 2. 20 days 3. 5 days 4. 8 days

9. Somu and Ramu can do a job in 60 and 40 days respectively. They earned Rs 450 for completing work together. What is the share of Somu (in Rs)?
1. 100 2. 120 3. 150 4. 180

10. Gautham and Karan can complete a job in 4 hours and 12 hours respectively. If they work alternatively for one hour each, with Gautham starting the job, when will the job be completed (in hours)?
1. 5 2. 6 3. 4 4. 8

11. X and Y can do a piece of work in 12 days and 18 days respectively. If they work on alternate days starting with X how many days will they take to finish the job?
1. 14 1/2 2. 14 1/3 3. 15 1/3 4. 15 1/2

12. Tap X can fill a tank in 10 hours. Tap Y can fill it in 15 hours. If the two taps fill the tank together, what fraction of the tank is filled by X?
1. 1/10 2. 1/6 3. 2/3 4. 3/5

13. It takes 3 hours to fill a tank but due to a leakage it takes 4 hours. How long would the leak take to empty a full tank?
1. 14 hours 2. 16 hours 3. 10 hours 4. 12 hours

14. A tank can be filled by two pipes individually in 30 and 20 minutes respectively. There is an emptying pipe which can empty the full tank in 60 minutes. If all three pipes are opened simultaneously then, how much time does it take to fill the empty tank (in minutes)?
1. 10 2. 12 3. 15 4. 8

15. A team of two people A and B, who can complete a piece of work together in 40 days, begin the work. C joins them after 8 days and the work gets completed in 24 days after C joins them. In how many days can C alone complete the work?
1. 110 2. 100 3. 120 4. 80

16. Akbar can do a piece of work in 40 days and Ajay can do the same in 60 days. Akbar works on the piece of work alone for some days. Then Ajay joins him and together they complete the work 10 days earlier than the time Akbar alone would have taken. After how many days did Ajay join Akbar?
1. 19 2. 12 3. 15 4. 20

17. P can complete a job in 27 days. He starts it and after three days Q joins him. They work together for six days. P then leaves and R takes his place. Q and R complete the job in 12 more days. If Q takes at most 54 days to complete the job, then which of the following cannot be a possible value of the number of days taken by R to complete it?
1. 36 2. 34 3. 40 4. 38

18. P, Q and R are three machines. They produce electronic gadgets. The ratio of the rates of P, Q and R is 3: 4: 5. P worked for 6 days, Q worked for 8 days and R worked for 10 days. They manage to produce 400 gadgets. How many gadgets would they have produced in the same time if P's rate doubled and Q's rate tripled?
1. 728 2. 648 6. 664 4. 784

19. A man builds 1/8th part of a wall every day. Out of the length of the wall built per day, 20% falls off at the end of the day till the wall is completely built. In how many days can he complete the construction of the wall?
1. 8 2. 10 3. 9 1/5 4. 9 4/5

20. Raman can do a piece of work in half the time taken by Kapil. Sunil can do the same work in one-third of the time taken by Raman. All three of them work on it for 30 days after which Kapil leaves. Sunil and Raman complete the remaining work in 18 more days. How many days would it take for Raman alone to complete the total work?
1. 52 2. 414 3. 138 4. 207

21. The efficiency of a man is reduced by half every two hours. At maximum efficiency, he could have completed the job in 150 hours. How many hours does it take him to complete the job, if his efficiency becomes maximum after every 8 hours, and then reduces as mentioned above?
1. 300 2. 280 3. 320 4. 250

22. A man starts a piece of work. Starting from the second day onwards, every day a new man joins. With every new man joining, the work that each man can do per day doubles. The work is completed in 5 days. On which day would they have completed the work, if the work that each of them could do per day had remained constant?
1. 14^{th} 2. 15^{th} 3. 16^{th} 4. 18^{th}

23. Beginning with the second day, the amount of work Ram can do per day keeps doubling every day he works. Saleem can always do the work done by Ram in 40% less time. Owing to this increasing ability, Saleem and Ram complete the job together in 2 days. How many days would it have taken if their efficiencies had remained constant?
1. 4 2. 6 3. 5 4. 3

24. Vivek, Rameshwar and Bhuvan divide a work amongst themselves in the ratio of 2: 3: 5. Their rates of work are in the ratio 1: 2: 3. It takes Vivek 12 days to complete his part. What is the amount of work completed by them in 8 days from the start?
1. 29/40 2. 11/45 3. 4/5 4. 31/45

25. Pipe A can fill a cistern in 18 minutes. Pipe B can fill the cistern in 24 minutes. Pipe C can empty the cistern in 36 minutes. At 10 a.m., pipe A is opened. At 10: 10 a.m., pipe C is opened. At 10: 15 a.m., pipe B is opened. The time at which the cistern would be full is approximately.
1. 10: 40 2. 10: 36 a.m. 3. 10: 27 a.m. 4. 10: 20 a.m.

26. Niranjan, Rajesh and Vinayak can complete a piece of work in 10, 15 and 12 days respectively. All the three of them started working and after an integral number of days (y) Niranjan stopped working and Rajesh stopped working exactly (y) days before the work got completed. If the number of days, taken to complete the work is an integer, in how many days did the work get completed?
1. 4 2. 6 3. 8 4. 10

27. Krishna and Rama can complete a piece of work in 20 days and 30 days respectively. With the help of Gopi, they complete the work in 6 days. If the total amount paid for the work is Rs 720, what are the daily earnings of Gopi?
1. Rs 40 2. Rs 480 3. Rs 360 4. Rs 60

28. P, Q and R together can complete a job in 5 days. The wages paid to P, Q and R for completing the job were Rs 4050, Rs 5400 and Rs 6750 respectively. In how many days can P working alone, complete the job?
1. 20 2. 24 3. 30 4. 36

29. Alok and Sachin agree to complete a piece of work in 20 days. They also agree to forfeit double the amount of wages corresponding to the uncompleted part of work, if they fail. If Alok alone can complete the work in 40 days and they lost $1/3^{rd}$ of the pay due for the total work, in how many days can Sachin alone complete the work?
1. 30 2. 60 3. 40 4. 80

30. A tank is full of water. A drain pipe, which can empty the full tank in 60 minutes, is opened. 18 minutes later another pipe which can fill the empty tank in 30 minutes is opened. After how much time, in total, is the tank full again (in minutes)?
1. 18 2. 20 3. 36 4. 40

31. P and Q are filling pipes which can fill a tank in 15 and 20 minutes respectively. R is an emptying pipe which can empty the full tank in 30 minutes. The three pipes are operated continuously one after the other in the order of P, Q and R, each being kept opened for 2 minutes until the tank is filled. After how much time will the tank be full?
1. 30 minutes 2. 32 2/3 minutes 3. 36 minutes 4. 34 minutes

32. Two taps can normally fill a cylindrical tank in 16 hours and 48 hours. But a leak which can empty the tank in 24 hours is present at $3/4^{th}$ of the tank's height from the base. Find the time taken to fill the tank if the taps are opened simultaneously (in hours).
1. 10 2. 12 3. 15 4. 20

33. X and Y are filling pipes. If the bottom 2/3rd of a tank is filled by X and the rest is filled by Y, the tank will be filled in 14 minutes. If the bottom 2/3rd of the tank is filled by Y and the rest is filled by X, the tank will be filled in 16 minutes. Find the time taken by the two pipes to fill the tank together (in minutes).

1. 6.5 2. 8.4 3. 5.5 4. 7.2

34. Raman and Rajan together complete half the work in 18 days. Rajan and Rajiv complete the remaining half in 12 days. If Rajiv alone can complete the work in 36 days, then how many days will it take for Raman alone to complete the work?

1. 50 2. 60 3. 72 4. 64

35. Kaushik is one and a half times more efficient than Ravi. Kaushik can do a piece of work in 20 days. What portion of the total work can both of them together complete in 10 days?

1. 3/10 2. 4/5 3. 9/10 4. 7/10

Exercise 15

1. In 8 days, Peter can do as much work as Pan can do in 12 days. To do a certain job both together take 36 days. In how many days can Pan, working alone, complete the job?
1. 60 days 2. 80 days 3. 108 days 4. 90 days

2. Pradeep can work 2/3 times as fast as Abishek and Antony together. Pradeep and Antony together can work twice as fast as Abishek. If Antony alone takes 45 days to complete a job, how long (in days) would Pradeep and Abishek individually take to complete the job?
1. 30, 36 2. 36, 30 3. 45, 60 4. 60, 45

3. Jadeja, Bhangar, Balaji and Dravid can do a piece of work in 8, 16, 32 and 64 days respectively. Dravid starts the work and Balaji joins him after $1/4^{th}$ of the work is done, Bhangar joins them after half the work is done and Jadeja joins them after $3/4^{th}$ work is done. How many days does it take to complete the work?
1. 30 2. 32 3. 25 2/35 4. 24 24/35

4. Machines P, Q and R can do a piece of work in 20, 30 and 60 days respectively. Machines P, Q and R start the work together. Machine P goes out of order after 5 days. After 3 more days machines Q and R also go out of order. Machine P got repaired by then and it completes the remaining work. What portion of the total work did machine P do?
1. 3/4 2. 11/20 3. 7/20 4. 3/5

5. Some workers have been divided into two groups – A and B – depending on their rate of doing work. Three workers from A and six from B take 20 days to complete a job. Eight from A and 4 from B take 10 days to complete it. Find the time taken to complete the work if one worker from each group is doing it. (in days).
1. 90 2. 108 3. 72 4. 54

6. Chakri and Bharat can complete a piece of work individually in 24 days and 36 days respectively. They work on alternate days starting with Chakri. Further, they get a holiday after every four days and after the holiday the person who worked on the last day before the holiday now starts the work. After how many days from the start will the work get completed?
1. 14 2/5 days 2. 36 days 3. 35 2/3days 4. 35 days

7. Sonia can complete a piece of work in 20 days, while Priyanka and Anjali can do it in 30 days each. Sonia works on the first day. Sonia and Priyanka work on the second day and Sonia, Priyanka and Anjali work on the third day. Also, Sonia works on the fourth day, Sonia and Priyanka work on the fifth day and Sonia, Priyanka and Anjali work on the sixth day and this cycle is repeated till the work is completed. In how many days is the work completed?
1. 12 days 2. 13 1/3 days 3. 14 days 4. 16 1/2 days

8. A man starts a job, working at the rate of 1 unit a day. After each day, another man joins the working group. The man who joins on the x^{th} day of the job works until the job is completed, at x units per day. The job is completed in 4 days. Find the time it would have taken have the man who started the work to complete the job alone (in days).
1. 20 2. 10 3. 16 4. 24

9. Prakash, Pranay and Pramod working alone can do a piece of work in 20, 30 and 60 days respectively. All three of them start the work together, but after x days Prakash leaves and then after y more days Pranay leaves and Pramod completes the remaining

work. If Pranay had not left, Pramod and Pranay would have completed the remaining work in (y + 6) days after Prakash had left. If both Prakash and Pranay had stayed, the work would have been completed in (x + 6) days. What is the total number of days taken to complete the work?
1. 22 2. 24 3. 26 4. 28

10. A group of men are building a wall. After half the wall has been built, double the number of men join the original group. The wall gets completed 6 days earlier than scheduled. What is the total number of days the initial group of men would have taken to complete the wall?
1. 18 2. 16 3. 14 4. 12

11. Two pipes P and Q can fill a cistern in 12 and 18 hours respectively. Both the pipes were opened at 10:00 a.m. and the cistern was full at 6:00 p.m. What could be the minimum possible duration for which one of the pipes must have been closed during that interval?
1. 4 hours 2. 2.5 hours 3. 2 hours 4. 4/3 hours

12. N taps numbered from 1 to N are fitted to a tank. The rate at which the nth tap, for n = 1, 2, ……. N fills the tank, equals the sum of the rates of all the taps numbered below it. If the sixth tap can fill it in 80 minutes, find the time in which the ninth tap can fill it (in minutes).
1. 10 2. 12 3. 15 4. 18

13. A pipe can fill a tank in 4 hours, while a leak which is at one-fourth the height of the tank can empty up to that part in 2 hours. If both are operated simultaneously and initially the tank is full, then when will it be one-fourth full?
1. 2 hours 2. 2 1/3 hours 3. 1 1/2 hours 4. 6 hours

14. A pipe is filling a tank of 200 litres capacity at 5 litres/hr. Because of a leak it takes 80 hours more to fill the tank. What is the rate in litres/hr at which the water is leaking?
1. 6 2. 3 3. 3.33 4. 8

15. A and B together can complete a piece of work in 12 days. A and C together can complete the work in 20 days. If three of them together can complete the work in 8 days, then in how many days can A alone complete the work?
1. 24 2. 13 1/3 3. 120 4. 60

16. Each of Atul, Yogesh and Riyaz can complete a piece of work in 12 days, 36 days and 54 days respectively. They started the work together, but Atul left after working for 6 days and Yogesh left 8 days before the completion of the work. In how many days was the work completed?
1. 16 2. 17 3. 20 4. 15 3/5

17. A, B and C can complete a piece of work in 8, 12 and 24 days respectively. If A and B work on the first day, B and C work on the second day and A and C work on the third day and the same pattern continue 4^{th} day onward, then in how many days will the work be completed?
1. 4 2. 6 3. 8 4. 9.33

18. A can complete a piece of work in 30 days and B can complete the same work in 20 days. If A and B started working together and A quit 5 days before the completion of the work, then for how long they worked together?
1. 8 days 2. 12 days 3. 9 days 4. 5 days

19. If two pipes are opened simultaneously, the reservoir will be filled in 6 hr. One pipe fills the reservoir 5 hr faster than the other. How much time does the slower pipe take to fill in the reservoir?

1. 15 hr 2. 20 hr 3. 10 hr 4. 25

20. Two cigarette smokers A and B take puffs on the same cigarette alternately starting with A. A takes a puff for 2 s and then B takes a puff for 3 s. Rate of burning while taking a puff is 3 mm/s and while not puffing is 1 mm/s. After how much time will the cigarette be completely burnt, if its length is 84 mm and the gap between any two consecutive puffs is 3s?
1. 40 sec 2. 35 sec 3. 44 sec 4. 36 sec

21. Two inlet pipes can separately fill a tank in 20 minutes and 15 minutes respectively and a waste pipe can empty the completely filled tank in 12 minutes. On a certain day, the two inlet pipes are turned on simultaneously to fill the empty tank. But after 9 minutes it was found that the waste pipe was also left opened, it was closed immediately. How much more time is required to fill the tank completely?
1. 5.5 min 2. 5 min 3. 6 min 4. 6.2 min

22. An iron water tank with a capacity of 10,000 gallons has a water supply inlet, which feeds water at the rate of 300 gallons per hour. This water inlet starts automatically when the water level goes below the 5000-gallon mark. If there is a hole formed at the bottom of the tank when the water tank is full, how long (in hours) will it take for the whole tank to be emptied if the water goes out of the hole at the rate of 500 gallons per hour?
1. 20 2. 10 3. 50 4. 35

23. It takes six technicians a total of 10 hr to build a new server from Direct Computer, with each working at the same rate. If six technicians start to build the server at 11 am, and one technician per hour is added beginning at 5 pm, at what time will the server be completed?
1. 6.40 pm 2. 7 pm 3. 7.20 pm 4. 8 pm

24. Sharat, Chandra and Mayank completed a job in 10 days in the following manner: They worked together for 3 days in which they completed 37% of the total work. But due to unavoidable circumstances, Sharat quits the job at this point and Chandra and Mayank continued to work together to complete the job. If the work done by Chandra in 4 days is equal to the work done by Mayank in 5 days, then what is the time required by the fastest worker amongst the three mentioned persons to complete the whole work alone?
1. 20 days 2. 25 days 3. 30 days 4. 15 days

25. In a tank there are four outlets A, B, C and D. These outlets can empty the tank in 10 minutes, 12 minutes, 15 minutes and 18 minutes respectively. If the tank is full and all the outlets are opened how much time (in minutes) will it take to empty the whole tank, if the outlets A and B are closed after 1 minute and 2 minutes respectively, and outlet C is closed 5 minutes before the tank gets empty.
1. 8 8/11 2. 4 8/13 3. 7 4. 6 8/11

26. In a factory's total daily wages of 20 men, 30 women and 36 children is Rs. 78. The ratio of work done by a man, a woman and a child in a day is 3: 2: 1 respectively. What will be the total wages of 15 men, 21 women and 30 children for 18 weeks?
1. Rs. 7,371 2. Rs. 9,585 3. Rs. 6,956 4. Rs. 5,000

27. A tank can be filled in by an inlet pipe of circular cross section in 10 min. Another tank twice as big is filled in by 3 inlet pipes whose diameters are in the ratio 1: 2: 3. If the smallest pipe of these is four times as much in cross section as the pipe of the first tank, find the time taken to fill the second tank. [Speed of inflow of water in the second tank is half the speed of inflow of the water in the first tank.]
1. 7/5 min 2. 5/7 min 3. 5/6 min 4. 6/5 min

Time, Speed & Distance

The topic of Speed, time and distance is one of the most important topics while preparing for CAT or other management entrance examinations. Let us start it with some few basic discussion.

Speed: Speed is defined as distance covered per unit time. Thus,

$$\text{Speed } (S) = \frac{\text{Distance } (D)}{\text{Time } (T)}$$

One should be very conversant with all the possible arrangements of the three terms:

$D = S \times T \Rightarrow T = \frac{D}{S}$

Both of these formats will be used equally frequently as the first relation. So be very swift with these.

Looking at the expression of speed, the unit of speed can be deduced as $\frac{\text{km}}{\text{hr}}$ (represented in this text as kmph) or $\frac{m}{s}$ (represented in this text as m/s). In questions one would require to convert the units and hence it's good idea to memorise the conversions.

1 kmph = $\frac{1000}{3600}$ m/s i.e., $\frac{5}{18}$m/s.

Thus, kmph $\xrightarrow{\times\frac{5}{18}}$ m/sand m/s $\xrightarrow{\times\frac{18}{5}}$ kmph

Conversion from kmph to m/s

More often than not, in questions that require conversion of speed from kmph to m/s, the speed used will be some comfortable multiple of 18 kmph. One should not spend time in using the above conversion process, instead should be ready with the following:

18 kmph = 5 m/s	36 kmph = 10 m/s	45 kmph = 12.5 m/s
54 kmph = 15 m/s	72 kmph = 20 m/s	90 kmph = 25 m/s

The following examples and exercise is just a sort of primer to make you acquainted with using the relations learnt above. As we will learn ahead, they can be solved, using proportionality, as of now stick to learning how to form equations using the above relations.

Ex. **I travel three distances in the ratio 1: 2: 3 at speeds of 20 m/s, 30 m/s and 40 m/s. If the total time taken is 23 minutes, find the total distance covered.**

Sol: In this case there are three distances travelled at three different speeds and hence there are three different time taken for the three stretches. And we are given the sum of the time take i.e.

$\text{Time}_1 + \text{Time}_2 + \text{Time}_3 = 23 \times 60$ secs.

And we know that time is found as $\frac{\text{Distance}}{\text{Speed}}$.

Thus, $\frac{x}{20} + \frac{2x}{30} + \frac{3x}{40} = 23 \times 60$

Multiplying by 120, $6x + 8x + 9x = 23 \times 60$ i.e. $x = 60$.

Thus, the total distance covered = $x + 2x + 3x = 6x$ = 360 meters.

Ex. **I travel for three stretches of time, in the ratio 1: 2: 3, at speeds of 20 kmph, 30 kmph and 40 kmph. If cover a total distance of 100 km, find the total time for which I am travelling.**

Sol: In this case there are three time intervals travelled at three different speeds and hence there are three different distance covered in the three stretches. And we are given the sum of these distances i.e.

$Distance_1 + Distance_2 + Distance_3$ = 100 kms.

And we know that distance is given by D = S × T

Thus, $20 \times x + 30 \times 2x + 40 \times 3x = 100$ i.e., $200x = 100$ i.e. $x = ½$

We need total time i.e., $x + 2x + 3x = 6x$ and will be $6 \times ½ = 3$ hours.

Forming equations

Very often the journey would be broken into two or three parts. You have to be very agile in forming equations. You need quickly ascertain whether the equation can be formed in terms of time ($Time_1 + Time_2 + Time_3 + \ldots\ldots$ = Total time) or whether the equation can be formed in terms of distance ($Distance_1 + Distance_2 + Distance_3 + \ldots\ldots$ = Total distance).

Next step is to be very conversant that Time is given as $\frac{\text{Distance}}{\text{Speed}}$ whereas Distance is given by Speed × Time.

Hence look out for relations of the type

$\frac{d_1}{s_1} + \frac{d_2}{s_2} + \frac{d_3}{s_3} + \ldots\ldots = T$ or $(s_1 \times t_1) + (s_2 \times t_2) + (s_3 \times t_3) + \ldots\ldots$ = D

Sometimes you can have the liberty to frame the equation in whichever term you wish, as the following example shows.

Ex. **I travel between Pune and Mumbai via Lonavala. The total journey of 160 km takes me 2 hours. If I travel between Pune and Lonavala at speed of 60 kmph and between Lonavala and Mumbai at speed of 90 kmph, find the time for distance between Lonavala and Mumbai and also the time taken for this stretch.**

Sol: Approach 1: Forming equation in terms of distance

If the time taken between Pune and Lonavala is assumed as t, then the time taken between Lonavala and Mumbai will be $(2 - t)$. Thus, adding the distances covered in the two stretches,

$60 \times t + 90(2 - t) = 160$ i.e., $30t = 20$ i.e., $t = 2/3$ i.e., 40 minutes. Thus, the time taken between Lonavala and Mumbai will be 120 min – 40 min = 80 min and required distance will be $90 \times \frac{80}{60}$ = 120 km.

Approach 2: Forming equation in terms of time

If the distance between Pune and Lonavala is assumed as d, then the distance between Lonavala and Mumbai is $(160 - d)$. Next, adding the time taken in the two stretches,

$\frac{d}{60} + \frac{160-d}{90} = 2$ i.e., $3d + 2(160 - d) = 2 \times 180$ i.e., $d = 40$

Thus, the distance between Lonavala and Mumbai = 160 – 40 = 120 and the time taken = $\frac{120}{90} = \frac{4}{3}$ hr i.e., 80 mins.

Proportionality b/w Time, Speed and Distance

This is probably the most important topic in this chapter. And also, the most thought-intensive. It provides the foundation to solve many tough problems orally!

Usually in questions there are two scenarios. And in the two scenarios, one among the three variables, S, D and T, is a constant. If one of these variables is a constant, we can use the proportionality relation among the other two to solve the question orally.

Scenarios

When we say that in questions there are two scenarios, it requires some efforts on your part to identify the two scenarios. The following are some common instances of how to tune your thought process towards identifying the two scenarios ……

(Complete questions are not given, just the crucial info is given, so don't be too argumentative about the conclusions)

1. Today I travelled 20 % faster from home to office …

The two scenarios are 'everyday' and 'today'. And while it is directly given that speeds are not constant, you need to realize the stated assumption that distance covered, home to office, will not change between 'everyday' and 'today' and thus, distance is constant

2. Two friends started simultaneously from their homes towards each other to meet ……

The two scenarios are that of the two individuals. Since they start simultaneously and they meet, both of them are travelling for the same time. Thus, the time is constant when we consider the case of the two individuals separately.

3. A train takes 10 secs to cross a pole and 15 seconds to cross a platform ……

While we will see this scenario in details later on, the two scenarios are obvious – one is 'train crossing pole' and other is 'train crossing platform'. And in the two scenarios, the speed of the train is going to be constant.

It will be on immense help to think on the lines of two scenarios. And then using proportionality. Most questions in the following text will be done by the traditional approach and then solved by the approach of using proportionality.

Distance being constant

Time is inversely proportional to speed, when distance is constant.

This is to say that, over a same distance, if the ratio of speeds is a : b, the ratio of the time taken will be b : a.

And this should be obvious, because over a same distance, if I double my speed (ratio of speeds 1: 2), the time taken will be half (ratio of time 2: 1).

If I travel at $\frac{1}{3}^{rd}$ the usual speed (ratio of speed 3: 1), I would take thrice the time taken earlier (ratio of time 1: 3)

If I reduce my speed to $\frac{3}{5}^{th}$ of the usual speed (ratio of speed 5: 3), the time taken will be $\frac{5}{3}$ times the usual time (ratio of time 3: 5).

Ex. **From home to office, if I travel at $\frac{3}{5}^{th}$ of my usual speed, I am late by 12 minutes. Find the time that I take usually and the time taken at the reduced speed.**

Sol: In such problems, 'late by 12 minutes' implies that 12 more minutes will be taken to travel the same distance, or in other words, the difference in the time taken at the usual speed and the reduced speed will be 12 minutes.

Had my usual speed been s, the reduced speed would be $\frac{3}{5}s$. Thus, the ratio of usual speed to reduced speed would be 5: 3. (From next problem onwards, this step will be done directly).

Since time is inversely proportional to speed, the ratio of the time is 3: 5. We also know that the difference in the time taken will be 12 minutes.

Thus, we are looking for two numbers that are in the ratio 3: 5 and the difference between them is 12 minutes. In the chapter on ratios, we learnt an oral way to do these problems.

Thus, the usual time taken is 3 × 6 = 18 minutes and the time taken at reduced speed is 5 × 6 = 30 minutes.

Ex. **If a man walks at the rate of 5 kmph, he misses a train by 7 minutes. However, if he walks at the rate of 6 kmph, he reaches the station 5 minutes before the departure of the train. Find the distance to the station.**

Sol: The ratio of speeds is 5: 6 and since distance is constant the ratio of the time taken will be in the ratio 6: 5.

Missing the train by 7 minutes and reaching early by 5 minutes implies that the time taken at speed of 6 kmph is 12 minutes less than the time taken at speed of 5 kmph.

Thus, we need to find two numbers in ratio 6: 5 with a difference of 12. Thus, a difference of 1 on the ratio scale is a difference of 12 in actual values. Hence the multiplying factor is 12. Thus, time taken at 5 kmph is 6 × 12 = 72 minutes and at 6 kmph is 5 × 12 = 60 minutes.

Either of these speed and time combination can be used to find the distance. Since, 60 minutes is 1 hour, it is easier to find the distance as 6 kmph × 1 hr = 6 km. Check that the same distance is found using the other combination of 5 kmph and 72 minutes. Thus, the required distance is 6 km.

Ex. **A train meets with an accident and travels at $\frac{4}{7}^{th}$ of its regular speed hereafter and hence it reaches its destination 36 minutes late. How much time does the train take to reach its destination from the site of the accident had it traveled at its regular speed?**

Sol: The two scenarios are that at 'regular speed' and that at 'decreased speed' between the distance from site of accident to destination.

Ratio of speeds in the two scenarios is 7: 4. Ratio of time taken from site of accident to destination = 4: 7. And the difference is given to be 36. Thus, a difference of 3 on the ratio scale is a difference of 36 in actual values. Hence the multiplying factor is 12. Thus, time taken at regular speed = 4 ×12 = 48 minutes.

Ex. **A train meets with an accident and travels at $\frac{4}{7}^{th}$ of its regular speed hereafter and hence it reaches its destination 36 minutes late. Had the accident occurred**

30 kms further, the train would have been late by only 21 minutes. Find the regular speed of the train.

Sol: Let's say the point where the accident occurred was A when the train was late by 36 minutes and was point B when the train was late by 21 minutes. Let the destination be D.

Comparison over distance AD: The two scenarios are 'at regular speed' and 'at reduced speed'. Ratio of speeds 7: 4. Ratio of time 4: 7. Difference in time is 36 minutes. Thus, time taken at regular speed for the distance AD is 4 × 12 = 48 minutes.

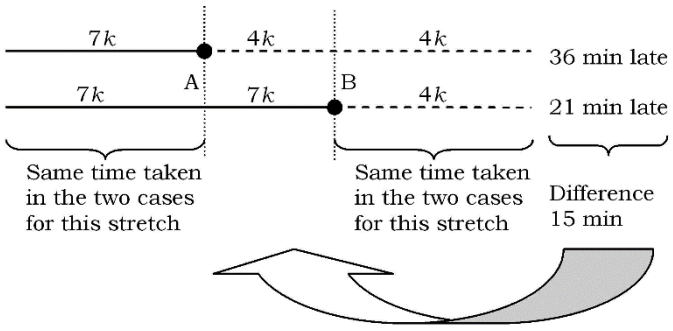
Comparison over distance BD: The two scenarios are 'at regular speed' and 'at reduced speed'. Ratio of speeds 7: 4. Ratio of time 4: 7. Difference in time, over this distance is 21 minutes. Thus, time taken at regular speed for the distance BD is 4 × 7 = 28 minutes.

At its regular speed, the train takes 48 minutes to travel A to D and takes 28 minutes to travel B to D. Hence it must be taking 20 minutes to travel from A to B, a distance of 30 kms. Thus, it's regular speed = $\frac{30 \text{ km}}{\frac{20}{60} \text{hr}}$ = 90 kmph.

Shortcut:

While in the above solution we had the comparisons (at regular speed & at reduced speed) made twice, once over distance AD and once over distance BD. We could make do with just one comparison as well.

Consider the stretch AB, when accident occurs at A, this stretch is travelled at reduced speed. And when accident occurs at B, this stretch is travelled at regular speed. And the difference in the time taken **just over this stretch** is 36 – 21 = 15 minutes. (At destination, why is the train late for 21 mins, when accident occurs at B as compared to 36 mins, when accident occurs at A? The train would have been lesser late because, over AB, it would have not have got late) See figure if this is not yet clear.



Comparing the two scenarios, ratio of speeds over AB is 4: 7. Hence ratio of time will be 7: 4 and difference in time taken over AB is 15 minutes. The multiplying factor to go from ratio scale to actual values will be 5. Thus, at regular speed it would take 4 × 5 = 20 minutes to cover AB, a distance of 30 km.

Time being constant

Distance is directly proportional to speed, when the time is same

This is to say, if time is constant and the ratio of speeds is *a*: *b*, the ratio of the distance covered will also be *a*: *b*.

The most common case of time remaining same would be when two persons, trains or objects start from two points simultaneously and meet each other. In this case the time that the two objects are traveling is the same for both the objects. So, they will cover distances in proportion to their speeds.

Ex. **Two trains start simultaneously, one from Bombay to Kolkata and other from Kolkata to Bombay. They meet each other at Nagpur which is at a distance of 700 kms from Bombay. If the distance between Bombay and Kolkata is 1600 km, find the ratio of their speeds.**

Sol: Since the trains started simultaneously, the time they have been traveling till they meet is equal. And hence the distance they cover will be in ratio of their speed. Since the train from Bombay has covered 700 km and the train from Kolkata has covered 1600 – 700 = 900 kms, the ratio of their speeds will be 700: 900 i.e., 7: 9.

Ex. **Two friends start walking towards each other with speeds in the ratio 3: 4. When they meet it is found that the faster of them has covered 25 meters more than the slower. Find the distance that separated them initially if they are walking in opposite directions, but obviously towards each other.**

Sol: Again, in this case since they are walking for the same amount of time the ratio of the distance covered will also be 3: 4. The distance given in the question, 25 meters, refers to the difference in the distances covered by them. Thus, a difference of 1 in the ratio scale corresponds to an actual distance of 25 i.e., multiplying factor will be 25.

We are required to find the distance separating them i.e., the sum of distances travelled by both. In the ratio scale this will be 3 + 4 = 7 and the actual distance will be 7 × 25 = 175.

Ex. **A police-man starts chasing a thief. The ratio of the speeds of the thief and the policeman is in the ratio 9: 11 and when the policeman catches the thief it is found that the policeman has covered 60 meters more than the thief. How much distance did the police have to run to nab the thief?**

Sol: Since the chase starts with both of them running simultaneously, from this point onwards to the time the police has caught the thief, they are running for same duration. Thus, the distance covered will be proportional to their speeds. So we are searching for two distances in the ratio 9: 11 and the difference being 60 meters i.e. 2 of the ratio scale corresponds to 60 mts, implying that the multiplying factor is 30. Hence distance run by police will be 11 × 30 = 330.

Ex. **In the movie Ghulam, Aamir is able to spot the approaching train when it is 2 km away. He has to run towards the train and reach the red kerchief hung on a pole 400 meters away from him before the train reaches the pole. How fast must Aamir run if the speed of the train is 36 kmph so that he just manages to reach the kerchief at the same time as the train reaches it?**

Sol: From the point when the distance between them is 2000 m, Aamir has to cover a distance of 400 m and the train will cover the rest of the 1600 m i.e. the distance will be in the ratio 1: 4. Since they are running for the same duration, the speed will be proportional to the distance covered. Since the speed of the train is 36 kmph, the speed of Aamir should be 9 kmph.

Ex. **In a race of 100 meters, A beats B by 10 meters and C by 20 meters. By how many meters does B beat C in the same race?**

Sol: Most of the problem on race are based on the proportionality of speed and distance because in the case of race, the runners start running simultaneously and thus at

any point of time (when the runners are yet running), they have been running for the same time.

When A has run 100 meters, B would have run 90 meters and C would have run 80 meters. Since this has happened in the same time, the ratio of the speeds of B and C is 9: 8. Now we have to find the difference between B and C when B has run 100 meters. So again, we are searching for difference between two numbers in the ratio 9: 8 when the first of them is 100 meters. Since 9 of ratio scale corresponds to 100 m, 8 of the ratio scale will be equal to $\frac{8}{9} \times 100$= 88.88 m. Thus, B beats C by i.e., 11.111... mts.

Speed being constant

Distance is directly proportional to time when speed is constant

At same speed, if the ratio of speed is *a*: *b*, the ratio of time will also be *a*: *b*.

By now, you would have got the hang of solving questions based on proportionality and thus we will solve just one example for this proportionality.

Ex. **A car overtakes an auto at point A at 9 am. The car reaches point B at 11 am and immediately turns back. It again meets the auto at point C at 11:30 am. At what time will the auto reach B?**

Sol: Since the car takes 2 hours to travel AB and 0.5 hours to travel BC, The ratio of the distances AB: BC will be 2: 0.5 i.e. 4: 1. (Distance is proportional to time)

Since C lies in between A and B, the ratio of the distance AC to CB will be 3: 1.

The auto has traveled AC in 2.5 hours (from 9 am to 11:30 am). To travel CB (one-third distance of AC) he will take $\frac{2.5 \text{ hrs}}{3} = \frac{150 \text{ min}}{3}$ = 50 more minutes. Thus, the auto will reach B at 12:40 pm.

Relative Speed

When two objects are moving simultaneously with speeds S_1 and S_2, the speed of any object when observed from the other object's perspective is called the relative speed. And it is distinct from S_1 or S_2.

E.g., Consider yourself to be sitting in a moving train and another train passes your train. If the other train is in opposite direction to your train, the speed of the other train appears to be far far more than it actually is. And if the other train is in the same direction as yours, it just appears to be inching ahead of your train very very slowly. This observed/perceived speed is called as relative speed and is calculated as:

If two objects are moving with speeds S_1 and S_2, their relative speed is

$S_1 + S_2$, if they are moving in opposite direction and

$S_1 - S_2$, if they are moving in same direction.

Relative speed is usually considered when one has to find the time taken to meet or catch and it can be found as follows:

$$\text{Time taken to meet/catch} = \frac{\text{Initial distance separating them}}{\text{Relative Speed i.e., } S_1 \pm S_2}$$

Ex. **A thief escapes from a prison at 2 pm and travels away at a speed of 30 kmph. The police realize the escape at 3:30 pm and start the chase then at a speed of 40 kmph. At what time will the police catch the thief? At what distance from the prison is the thief caught.**

Sol: Approach 1: Using Relative Speed

When the chase starts i.e., at 3: 30 pm, the thief has already run 30 × 1.5 = 45 kms. Thus, this is the distance that separated the police and the thief and so the time taken to catch is $\frac{45}{40-30} = \frac{45}{10}$ = 4.5 hours. This 4.5 hour is measured from the time chase starts i.e., from 3:30 and thus the thief is caught at 8 pm.

The distance run by the police is 40 kmph × 4.5 hours = 180 kms and so the thief is caught at a distance of 180 kms from the prison.

Approach 2: Using Proportionality

Method 1: Considering distance constant

Thief escapes from police station and say is caught at point X. The police also run the same distance, from police station to X. Thus, the distance run by police and thief is the same. Hence the time for which they are running will be inversely proportional to their speeds. Since ratio of speeds of police and thief is 4: 3, time for which they are running will be 3: 4. Also the difference in the time for which they are running is 1½ hour (thief starts running 1½ hours earlier). Thus, police and thief have been running for 3 × 1.5 = 4.5 hours and 4 × 1.5 = 6 hours respectively. And so, the thief is caught at 3:30 pm + 4.5 hours i.e., 8 pm (or 2 pm + 6 hours i.e., 8 pm)

Method 2: Considering time as constant

If just consider the time interval for which the chase is on i.e., from the time police start chasing to the time thief is caught, then the time run will be the same. However, in this case the distance run is different (because we are considering the event from 3:30 pm when thief has a head-start of 30 × 1.5 = 45 km).

With time being constant, the distances run will be proportional to speed. Thus, ratio of speed and distance will be 4: 3. And we know that since 3:30, the police have to run a distance of 45 km more than the thief. Thus, 1 of ratio scale corresponds to 45 km and police will cover a total of 4 × 45 = 180 km in catching the thief. Time taken for this will be $\frac{180}{40}$= 4.5 hrs.

Ex. **A starts from *X* at 9:00 am and reaches *Y* at 1:00 pm. B starts from *Y* at 9:00 am and reaches *X* at 3 pm. At what time do the two meet?**

Sol: **Approach 1:** Using Relative Speed

Assuming the distance between *X* and *Y* to be '*d*', since *A* and *B* take 4 hours and 6 hours respectively to cover the distance, their speeds are $\frac{d}{4}$ and $\frac{d}{6}$ respectively. Thus, the time taken to meet will be $\frac{d}{\frac{d}{4}+\frac{d}{6}} = \frac{1}{\frac{1}{4}+\frac{1}{6}} = \frac{4\times6}{6+4}$ = 2.4 hours, after 9 am i.e.,

at 11:24 am.

Short-cut:

Since in the above solution, we see that the answer is independent of the distance between *X* and *Y*, we can assume any value for the distance such that our calculations are easy. Since the time taken by *A* and *B* are 4 hours and 6 hours, we take the distance *XY* to be the LCM of 4 and 6 i.e., 12 km. So, the speeds of *A* and *B* are 3 kmph and 2 kmph respectively.

Now time taken to meet = $\frac{12}{3+2} = \frac{12}{5}$= 2.4 hours, just as found above.

Approach II: Use of proportionality

Considering the case when individually both A and B cover the entire distance XY. A takes 4 hours to cover XY and B takes 6 hours to cover XY. Since the distance covered is same, the ratio of their speed will be in inverse proportion to the time taken by each. Thus, ratio of speed of A and B is 3: 2.

Now considering that they start simultaneously from X and Y respectively and meet. In this case, the time for which they are travelling is the same. So, the distance that they cover will be in ratio of their speeds. Thus, A will cover $\frac{3}{5}^{th}$ of XY (and B will cover $\frac{2}{5}^{th}$ of XY).

A takes 4 hours to cover complete XY and hence will take $\frac{3}{5}^{th} \times 4 \text{ hrs} = \frac{12}{5}$= 2.4 hours to cover 3/5th of XY.

Ex. **A starts from X at 9:00 am and reaches Y at 1:00 pm. B starts from Y at 10:00 am and reaches X at 4 pm. At what time do the two meet?**

Sol: This question is very similar to the above question except that both A and B are not starting simultaneously. A takes 4 hours to travel XY and B takes 6 hours to travel XY. Assuming the distance to be 12 km (for reasons given in the above solution), the speeds of A and B are found to be 3 kmph and 2 kmph.

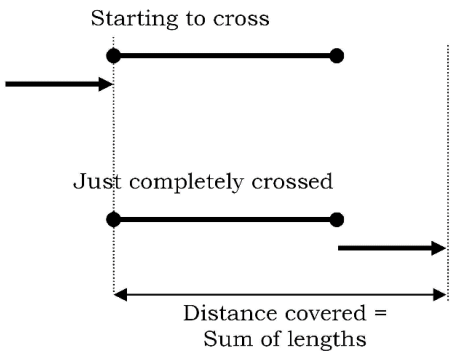
Since A starts 1 hour earlier than B starts, by 10 am he would have covered 3 kmph. So, when both of them are moving (i.e. when the relative speed is 3 + 2 = 5 kmph), the distance separating them is 12 – 3 = 9 km. Thus, time taken to meet = $\frac{9}{5}$ = 1.8 hours, after 10 am i.e., at 11:48 am.

If one is wondering how can we assume any distance, just see the following expression and realize that the answer is independent of the distance. Assuming the distance XY as *d*, speeds of A and B are $\frac{d}{4}$ & $\frac{d}{6}$

Time taken to meet = $\frac{\text{distance separating them}}{\text{relative speed}} = \frac{d-\frac{d}{4}\times 1}{\frac{d}{4}+\frac{d}{6}}$

Train crossing a pole/man/platform/bridge/train

Whenever one object of finite length crosses another object of finite length, in crossing the object completely it covers a distance equal to the sum of the lengths. The distance to be covered is independent of the direction in which it is crossing i.e., whether the two objects are crossing each other in opposite directions or if one is overtaking the other i.e. in same direction, the distance to be covered for one to completely cross the other is the sum of the lengths.



In the problems asked, usually a train is crossing either a pole or a man (stationary or walking) or a platform or bridge or another train (obviously moving). For all such cases you could use the following with the modifications as noted below:

Time taken to cross = $\frac{L_1+L_2}{S_1 \pm S_2}$

If one of the objects is a pole or a man, its length will be 0 (zero)

If one of the objects is stationary (e.g. pole, platform, bridge), its speed will be 0 (zero) and the relative speed will just be the speed of the moving object.

Ex. **A train running at 72 kmph crosses a telephone pole in 7 sec. What is the length of the train?**

Sol: Since a telephone pole is a stationary object of negligible length, the distance the train covers is just its own length.

Converting 72 kmph into m/s, since 72 is 4th multiple of 18, speed in m/s is 4th multiple of 5 i.e., 20 m/s

Distance = 20 × 7 = 140 m

So, length of the train = 140 m

Ex. **A train crosses 2 platforms of length 400 m and 600 m in 6 seconds and 8 seconds respectively. What is the length of the train?**

Sol: Approach 1: Using formula

Let the length of the train be x.

Then, $\frac{400+x}{S_t} = 6$ and $\frac{600+x}{S_t} = 8$

Since we want to find x, eliminating S_t, we have $\frac{400+x}{6} = \frac{600+x}{8} \Rightarrow 2x = 400$ i.e., $x = 200$.

Hence the length of the train is 200 m.

Approach 2: Using proportionality

In the two scenarios 'crossing platform of 400 m in 6 sec' and 'crossing platform of 600 m in 8 sec', the speed is the same. Hence distance will be proportional to time taken. The distances covered are 400 + x and 600 + x i.e., a difference of 200. And the time taken are 6 sec & 8 sec. Thus, ratio of time is 3: 4. This will also be the ratio of distances and we know that the difference in distances is 200. Thus, the distances covered in the two scenarios is 600 m and 800 m. Thus, length of train is 200 m.

Ex. **A train crosses two persons who are walking at 2 kmph and 4 kmph, in the same direction in which the train is going, in 9 and 10 seconds respectively. Find the length of the train.**

Sol: Approach 1: Using formula

If the length (in kms) and speed (in kmph) of the train is L and S_t respectively, we have

$\frac{L}{S_t-2} = \frac{9}{3600}$ and $\frac{L}{S_t-4} = \frac{10}{3600}$

i.e., $400L = S_t - 2$ and $360L = S_t - 4$

Subtracting $40L = 2$ i.e., $L = \frac{1}{20}$ km $= \frac{1000}{20}$ m = 50 m.

Approach 2: using Proportionality

In the two scenarios, 'train crossing man walking at 2 kmph' and 'train crossing man walking at 4 kmph', the distance covered will be the length of the train itself i.e., will be same in both the cases. Thus, ratio of speed will be inverse of ratio of time taken. Since ratio of time taken is 9: 10, ratio of speed will be 10: 9. Further we also know that the difference in the speeds in the two scenarios will be 2 kmph (speeds will be S – 2 and S – 4). Thus, the speeds in the two scenarios are 20 kmph and 18 kmph.

At 18 kmph i.e., 5 m/s, distance covered in 10 sec will be 50 m. This has to be the length of the train.

Boats and Streams

When a boatman is rowing in still water, say a lake, he would be moving at a speed at which he can row. This speed is called the speed of boat in still water or simply speed of boat. But consider the same boatman in a stream. Because of the current he is either aided (if rowing in the direction of the stream, this is called Downstream) or will be opposed (if rowing against the stream, called Upstream).

If the speed of boatman in still water is *B* and the speed of the stream is *S*, we have

Downstream Speed = B + S

Upstream Speed = B – S

Do not confuse with relative speed: In relative speed we saw that when two objects travel in same direction with speeds S_1 and S_2, the relative speed is $S_1 – S_2$. In the case of boats and streams, when boat is travelling in the direction of stream, the downstream speed of the boat is taken as B + S. Since they are in same direction, should we not consider relative speed to be B – S? No.

In case of relative speeds, the two objects are moving independently i.e., say police running to catch thief, the speed of police does not get 'transferred' or effects the speed of the thief. This is a case of relative speeds.

Whereas in the case of Boats and Streams, the speed of the Stream gets 'transferred' to the speed of the boat, the boat is travelling 'on' the stream. This is not a case of relative speed. Here the stream is 'aiding' the boat, unlike the case of police and thief where neither 'aids/hinders' the other.

If the speed of boatman is lesser than the speed of the current or stream, the upstream speed will be negative i.e., he is trying to row upstream, but rather than move in that direction, he is taken in opposite direction by the stream. But such situations do not occur in math problems on this topic.

The above relations can be modified for cases when the effective downstream speed, D and upstream speed, U is given and we are supposed to find B and S.

$$B = \frac{D+U}{2};\ S = \frac{D-U}{2}$$

Ex. **A boat covers a distance of 16 km in 2 hours when rowing downstream and in 4 hours if rowing upstream. What is the speed of the boat in still water?**

Sol: Speed Upstream = $\frac{16}{2}$= 8 km/hr and Speed Downstream =$\frac{16}{4}$ = 4 km/hr

From the above formula, speed of boat in still water =$\frac{D+U}{2} = \frac{8+4}{2}$= 6 km/hr

Ex. **The rowing speed of a man in still water is 7.5 kmph. In a river flowing at 1.5 kmph, it takes the same boatman 50 minutes to row a certain distance and come back. Find the distance.**

Sol: Here Speed Downstream = 7.5 + 1.5 = 9 km/hr and Speed Upstream = 7.5 – 1.5 = 6 km/hr

Approach 1: Formulaic approach

Let the required distance be *x*. Equating the time taken, $\frac{x}{9}+\frac{x}{6}=\frac{50}{60}$ hrs

Solving we get, x = 3. Hence the place is 3 km away.

Approach 2: Use of proportionality.

In going upstream and downstream, the distance covered is same. Hence time taken are in inverse proportion to their speeds. Ratio of upstream and downstream speeds is 6: 9 i.e., 2: 3. Thus, ratio of time travelling upstream and downstream is 3 : 2. And we know the total time is 50 minutes. Thus, time travelled upstream is 30 minutes and time travelled downstream is 20 minutes. Now the distance can be found using speed × time i.e., 6 kmph × 30 min = 6 kmph × ½ hour = 3 km (it could also be found using downstream speed and time i.e. 9 kmph × 20 min = 9 kmph × 1/3 hour = 3 km)

Ex. **A boatman rows to a place 48 km distant and back in 14 hours. He finds that he can row 4 km with the stream in the same time as 3 km against the stream. Find the rate of the stream.**

Sol: Since the time taken to row 4 km downstream is same as the time taken to row 3 km upstream, $\frac{4}{B+S}=\frac{3}{B-S}\Rightarrow$ B = 7S

Since the boatman takes 14 hours to row to a place 48 km distant and come back, $\frac{48}{B+S}+\frac{48}{B-S}=14$

Substituting B = 7S, we have $\frac{48}{8S}+\frac{48}{6S}=14\Rightarrow\frac{6}{S}+\frac{8}{S}=14\Rightarrow\frac{14}{S}=14\Rightarrow$S = 1.

Approach 2: Using Proportionality

Consider the statement, 'he can row 4 km with the stream in the same time as 3 km against the stream'. Obviously, the time is constant and thus, downstream speed and upstream speed are in the ratio of distances covered i.e., 4: 3.

Next, consider travelling 48 km downstream and 48 km upstream. Again, since distance is constant, time taken will be inversely proportional to speed i.e., time taken to travel downstream and that taken to travel upstream are in ratio 3: 4. And we know that the total time taken is 14 hours. Thus, 6 hours is taken to travel 48 km downstream (speed = 8 kmph) and 8 hours is taken to travel 48 km upstream (speed = 6 kmph). Thus, speed of stream = $\frac{D-U}{2}=\frac{8-6}{2}$=1

Anatomy of standard questions

Usually in the case of questions on boats and streams, the data involves a boat travelling some distance downstream and some distance upstream. Either the total time is given or the total distance is given. Thus, we should have the flexibility to form equations regarding the total distance or the total time.

Equation for total time: T_{down} + T_{up} = Total Time

i.e., $\frac{D_{down}}{B+S}+\frac{D_{up}}{B-S}=$ Total Time

Equation for total distance: D_{down} + D_{up} = Total Distance

i.e. (B + S) × T_{down} + (B – S) × T_{up} = Total Distance

Familiarity with the above and ability to quickly realise which of the two equations can be formed from the given data will go a long way in building your speed of solving.

While trying the following questions of the exercise, see if you can identify that invariably most questions are of the above form.

Average Speed

During a journey, different parts of the journey could be covered at different speeds. Average speed is that uniform speed at which the total distance could be covered in the same time as it took in the actual case.

Thus, $\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}}$

It is not necessarily the average of different speeds, as seen in the following example:

Ex. **Atul covered 20 kms at a speed of 30 kmph and next 30 kms at a speed of 90 kmph. Find his average speed.**

Sol: The total distance that Atul covered was 20 + 30 = 50 kms. To find the average speed, we would also need to find the total time take. This can be found by adding the individual time taken over the two stretches.

Total time taken = $\frac{20}{30} + \frac{30}{90} = \frac{2}{3} + \frac{1}{3}$ = 1 hr.

Thus, average speed = $\frac{50 \text{ km}}{1 \text{ hr}}$ = 1 kmph

Please note that 50 kmph is not the arithmetic mean of 30 kmph and 90 kmph. Whereas in the following example the average speed turns out to be the arithmetic mean…

Ex. **Atul covered one fourth of a distance at a speed of 30 kmph and rest of the distance at a speed of 90 kmph. Find his average speed.**

Sol: Since the distance is broken into one-fourth and three-fourth, let's assume the total distance to be $4d$

To find the average speed, we would also need to find the total time take. This can be found by adding the individual time taken over the two stretches.

Total time taken = $\frac{d}{30} + \frac{3d}{90} = \frac{6d}{90} = \frac{d}{15}$ hrs

Thus, average speed = $\frac{4d \text{ km}}{\frac{d}{15} \text{ hr}}$ = 60 kmph

In this case the average speed 60 kmph is the arithmetic mean of 30 kmph and 90 kmph.

Thus, the average speed need not necessarily be equal to the arithmetic mean of the different speeds. But it can also be equal to the arithmetic mean. Later we will decipher when is it going to be the arithmetic mean.

Ex. **Atul travelled for one fourth of the total time of a journey at a speed of 30 kmph and rest of the time at a speed of 90 kmph. Find his average speed.**

Sol: In this case the journey is broken into stretches of time, one-fourth of time and three-fourth of time. So let us assume the total time as $4t$.

To find the average speed, we would also need to find the total distance covered. This can be found by adding the individual distances covered over the two stretches.

Total distance covered = $30 \times t + 90 \times 3t = 300t$

Thus, average speed $\frac{300t}{4t}$= 75

Please compare e.g., 25 and 26, while the numbers used were exactly similar, the average speed are different. Also, in the earlier example distance was assumed as $4d$ and in the latter one time was assumed as $4t$. These two examples basically cover most of the types of the questions in this topic.

Stretches in terms of distance or time

As seen in the above solved examples, when "stretches" of the journey are traveled at different speeds, the "stretches" could be expressed in terms of distance or in terms of time:

"Atul traveled $\frac{1}{3}^{rd}$ of a distance at a speed of 30 kmph and the rest of the distance at a speed of 60 kmph" is different from the data "Atul traveled for $\frac{1}{3}^{rd}$ of the time at a speed of 30 kmph and the rest of the time at a speed of 60 kmph"

And the expression to find the average speed visually appears different but is essentially $\frac{\text{Total Distance}}{\text{Total Time}}$.

Stretches in terms of distance:

If d_1, d_2, d_3, … are the distances run at speeds s_1, s_2, s_3, …. respectively,

Average speed = $\frac{d_1+d_2+d_3+\ldots\ldots}{\frac{d_1}{s_1}+\frac{d_2}{s_2}+\frac{d_3}{s_3}+\ldots\ldots}$

The numerator is the total distance covered and denominator is the total time taken and is found by adding the time taken in each stretch.

Stretches in terms of time:

If one travels for t_1, t_2, t_3, … hours at speeds s_1, s_2, s_3, …. respectively,

Average speed = $\frac{(s_1\times t_1)+(s_2\times t_2)+(s_3\times t_3)+\ldots\ldots}{t_1+t_2+t_3+\ldots\ldots}$

The numerator is the total distance traveled and is found by adding the distances traveled in each time interval. The denominator is the total time taken.

In certain cases, you can have the liberty to assume variable for distances or for times (see next solved example). Looking at the two expressions, assuming variables for time is always going to be lesser on calculations.

Ex. **I travel two different stretches at speeds of 25 kmph and 40 kmph such that my average speed for the entire journey is 30 kmph. What fraction of the entire distance was travelled at 25kmph?**

Sol: Approach 1: Assuming distances

Since the question requires us to find a fraction of distance, a logical way would be to assume the distance travelled at 25kmph and 40 kmph as d_1 and d_2.

Writing the term for average speed, $\frac{d_1+d_2}{\frac{d_1}{25}+\frac{d_2}{40}} = 30$. This expression would involve more calculation than necessary. Cross-multiplying, $d_1 + d_2 = \frac{6}{5}d_1 + \frac{3}{4}d_2$. Multiplying both sides by 20, $20d_1 + 20d_2 = 24d_1 + 15d_2$ i.e., $4d_1 = 5d_2$ i.e., $\frac{d_1}{d_2} = \frac{5}{4}$.

Thus, the fraction of distance covered at speed of 25 kmph is $\frac{5}{9}$.

Approach 2: Assuming time

Even though the question requires us to find a fraction of total distance, a lesser-calculation intensive approach would be to assume time travelled at 25 kmph and 40 kmph as t_1 and t_2.

Writing expression for average speed, $\frac{25t_1+40t_2}{t_1+t_2} = 30 \Rightarrow \frac{t_1}{t_2} = \frac{10}{5} = \frac{2}{1}$

Now the ratio of distances can be found using D = S × T as $\frac{d_1}{d_2} = \frac{25\times2}{40\times1} = \frac{5}{4}$

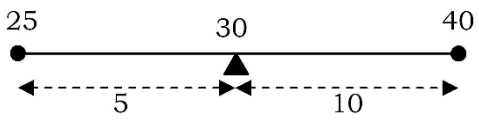
Thus, the fraction of distance covered at speed of 25 kmph is $\frac{5}{9}$.

Average Speed as Weighted Average of speed.

Average speed as the term suggests has to involve an average. And indeed, average speed is the average of the individual speeds, just that it is a case of weighted average and the weight for each speed is the **time** traveled at that speed.

Looking back at the expression, Average speed = $\frac{(s_1\times t_1)+(s_2\times t_2)+(s_3\times t_3)+\ldots\ldots}{t_1+t_2+t_3+\ldots\ldots}$, one can relate the expression to that of finding the weighted average.

Thus, looking back at the just solved example, we are given that the average of 25 kmph and 40 kmph is 30 kmph. Thus, we could have just allegation/scale approach and found the weights visually as:



Thus, the weights would be 2: 1. However what is important is to understand that this is the ratio of the time travelled at the individual speeds and not the distances covered.

Now that we know that average speed is the weighted average of the individual speed and the weights are the time, we can answer the question – 'When would average speed be equal to the arithmetic mean of the speeds?'. Obviously, when the weights are all equal. Thus, if equal stretches of time are travelled at different speed, the average speed will be equal to the arithmetic mean of the speed. Recall e.g., one fourth of a distance at a speed of 30 kmph and rest of the distance at a speed of 90 kmph implies that the ratio of time will be $\frac{d}{30}$: $\frac{3d}{90}$ i.e. 1 : 1. And hence the average speed was found to be the arithmetic mean of 30 and 90 i.e. 60 kmph.

This interpretation of the average speed being equal to the weighted average of the speeds with the weights being time, can be used to quickly solve few questions of average speed as follows:

Ex. **A man travels $\frac{1}{3}^{rd}$ of a distance at a speed of 45 kmph, $\frac{1}{2}$ the distance at 60 kmph and rest of the distance as speed of 36 kmph. Find his average speed for the entire journey.**

Sol: If total distance is taken to be 6, rest of the distance will be 6 – 2 – 3 = 1.

Rather than calculate, $\frac{6}{\frac{2}{45}+\frac{3}{60}+\frac{1}{36}}$, which is going to be tedious (try it), a better approach would be to find the ratio of time taken and then use weighted averages …

Ratio of distance is 2: 3: 1 and ratio of speeds is 15: 20: 12. Thus, ratio of time taken will be $\frac{2}{15}:\frac{3}{20}:\frac{1}{12}$ i.e., 8: 9: 5

Average speed = $\frac{45\times8+60\times9+12\times5}{8+9+5} = \frac{360+540+180}{22} = \frac{1080}{22}$= 49.09

Ex. **A man covers equal distances at a speed of 30 kmph and 60 kmph. Find his average speed.**

Sol: Since the distances are equal, assuming them to be '*d*',

the average speed = $\frac{2d}{\frac{d}{30}+\frac{d}{60}} = \frac{2d\times60}{3d}$= 40.

Special Case: Two stretches of equal distances, Harmonic Mean

A special case is when two equal distances are traveled at two different speeds, say u and v. In this case, the average speed is $\frac{d+d}{\frac{d}{u}+\frac{d}{v}} = \frac{2}{\frac{1}{u}+\frac{1}{v}} = \frac{2uv}{u+v}$

Remember to use the formula only when two equal distances are run at speeds of u and v. Also remember how the formula is arrived at to understand why you might get weird results using the formula in some cases (next solved e.g.)

Harmonic Mean

The expression $\frac{2uv}{u+v}$ is the Harmonic mean of u and v. Thus, when equal distances are run at two speeds, the average speed is the Harmonic Mean (HM) of the two speeds. And when equal time durations are run at two speeds, the average speed is the Arithmetic Mean (AM) of the two speeds. And for any two positive numbers, HM ≤ AM. Thus, when equal distances are run at two speeds, the average speed is always less than the Arithmetic Mean of the two speeds.

Ex. **A man covers equal distances at a speed of 30 kmph and *x* kmph such that his average speed for the entire journey is 80 kmph. Find *x*.**

Sol: Again since the distances are equal, using the formula, we have $\frac{2\times30\times x}{30+x} = 80 \Rightarrow 60x = 2400 + 80x$ i.e. $x = -120$???

Obviously, something is wrong some place, but then the formula will not help us identify what's wrong.

Since the working is independent of the equal distances, let's assume the distances to be 240 kms (LCM of 30 and 80) to avoid calculations and also to realize what is going wrong.

To travel the first 240 km at 30 kmph, one would take $\frac{240}{30}$= 8 hours.

Since average speed of the entire journey is given, the time take for the entire journey is $\frac{480}{80}$ = 6 hours. How can this be possible? This means the second stretch of 240 kms should be covered in –2 hours! Hence, we are getting the weird answer. The working would be similar whatever value of the distance you take.

What this means is that if the first stretch of two equal distances is run at s kmph, the average speed for the entire journey in no case can be equal to $2s$ kmph or more than that. Why?

Some time is taken for first stretch of two equal stretches. Over entire journey, distance has doubled, and for average speed also to double, the second stretch has to be covered in 0 time. Even if one travels the second stretch in 1 sec or lesser than

that, yet one would have covered the entire journey (twice the distance) in little more than the time taken on first stretch, so the average speed cannot be twice or more than that.

So, in the above example, the data given is inconsistent and an average speed of 80 kmph is not possible with first stretch of equal distance being run at 30 kmph.

Circular Motion

These types of problems deal with athletes running on a circular track and questions being asked about when, where and how often would two or more athletes cross each other.

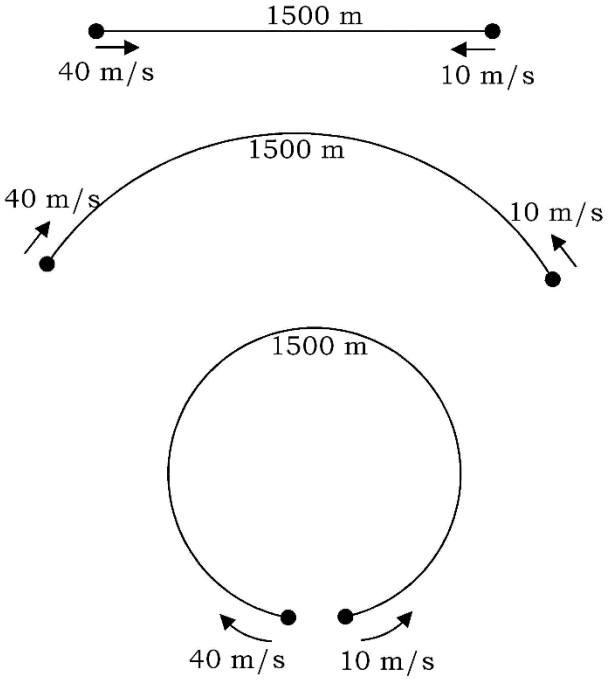
Meeting for the First Time

When running in opposite directions:

Consider two friends, A and B, who are separated by 1500 meters and they move towards each other at speeds of 40 m/s and 10 m/s. After how much time would they meet?

This is a straight question on relative speed and the time taken to meet = $\frac{1500}{40+10}$= 30 sec.

Now instead of traveling in a straight line, consider that the road they are traveling on is curved as shown. Would it make any difference to the time taken for them to meet? See the figure …



If you answered – No, you are right. Even now A would cover 40 meters each second and B would cover 10 meters each second, so they would come closer by 50 meters each second.

If that is the case, if the road was even more curved i.e., almost circular as shown, how much time would they take to meet? (They cannot turn around and meet each other, they have to travel along the entire circular track to meet).

In this case also they would take the same time i.e., $\frac{1500}{40+10}$ = 30 sec

Thus, time taken to meet = $\frac{\text{Track Length}}{\text{Relative Speed}}$

When running in the same direction:

Consider a policeman traveling at 40 m/s is chasing a thief traveling at 10 m/s. If the distance between the police and thief when the chase starts is 1500 meters, find the time after which the police catches up with the thief.

Again, the solution should be known by now as $\frac{1500}{40-10}$ = 50 sec

What one should note in this problem and solution is that when the police catches-up with the thief, the police man would have run 1500 meters *more than* the thief i.e., the police man has to cover / make-up a distance of 1500 meters over the thief. The same situation exists when two athletes are running on a circular track in the same direction. An instant after the start, looking from the faster one's perspective, he sees a distance equal to the track length that is separating him and the slower one. To catch up with the slower one, he has to cover an entire track length *more than* the slower one. Thus, even when the two athletes are running in the same direction, the time taken to meet = $\frac{\text{Track Length}}{\text{Relative Speed}}$. In this case the relative speed will be the difference in speeds.

Difference between Linear & Circular

Running on a circular track was explained above by drawing an analogy with meeting/catching up when two persons are running on a linear track. And we saw that the time taken to meet is found using the same formula, $\frac{\text{Distance seperating them}}{\text{Relative Speed}}$, except that the distance now is more appropriately the Track Length.

So, then what is the difference between the two case? As long as one is interested only in the firsts meeting, there is essentially no difference. It is just that in the case of circular tracks if the two keep running after the first meeting, they will meet again and yet again and yet again. This does not happen in the case of a linear track. So tougher questions on circular tracks would not be about first meeting but about further meetings.

Frequency of meeting

Say two athletes start running from a same point on a circular track and meet after t units of time. When they meet they are again together at a point (not necessarily the same point where they started but nevertheless they are together). And if they continue running in the direction they were running and at their respective speeds, then this instance can be thought of as a new beginning, two athletes starting to run from a same point. And thus, from this instant onwards, they would again meet after t units of time. And the reasoning could be again argued similarly at their 2^{nd} meeting. So, considering the 2^{nd} meeting as a fresh start, they will again meet after t duration of time. Thus, the time after which they meet for the first time is also the frequency with which they keep meet if they continue running at their respective speeds and respective directions.

If the first meeting takes place after t duration after start, the n^{th} meeting will take place after $n \times t$ duration after start.

Where would the first meeting take place?

Once the time is known after which two athletes, running on a circular track, meet, we can find the distance run individually by them (rather by any one of them) and

find the exact position on the track where the meeting takes place. Rather than measuring this distance in absolute terms, it is more beneficial to measure this distance in terms of the track length.

In the above example, when running in opposite direction, the two athletes meet after 30 sec. In 30 sec, A would have run 40 × 30 = 1200 m. In terms of the track length this will be $\frac{1200}{1500} = \frac{4}{5}^{th}$ of the track length. It would be quite obvious that B in the same time would have run $\frac{1}{5}^{th}$ of the track length. And since they are in opposite directions both of them will be at the same point because measuring $4/5^{th}$ in one direction and $1/5^{th}$ in other direction will lead to the same point.

And when running in same direction, they meet after 50 secs and in this time, A would have run 40 × 50 = 2000 m. This in terms of track length will be $\frac{2000}{1500} = \frac{4}{3}^{th}$ or $1\frac{1}{3}^{rd}$ of the track length. B in the same time, would have run 10 × 50 = 500 i.e., $\frac{1}{3}^{rd}$ the track length. And since they are running in the same direction, they both are at the same point i.e., $\frac{1}{3}^{rd}$ of the track length, from the starting point in the direction they are running.

The following are simple questions based on the above, but will make you familiar and conversant with the approach.

Ex. **Three athletes A, B and C are running on a circular track of length 1200 meters with speeds 30 m/s, 50 m/s and 80 m/s. A is running clockwise and B and C are running anticlockwise. Find the time after which A and B will meet for the first time and the frequency (in seconds) after which they will keep meeting. Also find the sum of the distance (in fraction of the track length) run by them till their first meeting.**

Sol: Since A and B are running in opposite direction, they would meet after $\frac{1200}{30+50} = \frac{1200}{80}$= 15 sec.

From the first meeting onwards, one could say that A and B are again starting simultaneously afresh and running in opposite direction. Thus, they would meet again after further 15 seconds. Continuing the same argument, we can say that A and B meet after every 15 seconds.

In 15 seconds, A would have run 15 × 30 = 450 meters i.e., $\frac{450}{1200} = \frac{3}{8}^{th}$ of the track length.

In 15 seconds, B would have run 15 × 50 = 750 meters i.e., $\frac{750}{1200} = \frac{5}{8}^{th}$ of the track length.

Notice that, when running in opposite directions, on meeting for the first time, they have together run one full track length.

Meeting for the First Time <u>at Start</u>

The question of identifying after how much time would two or more athletes meet *at the starting point* is an application of LCM rather than a problem of Time Speed Distance. The key word here is meeting *at the starting point*. Please realize that this may not be the first time that they meet. They could have crossed each other (met) at some other point on the track but then that would not be counted as a meeting point for this question as it has not occurred at the starting point.

Consider two athletes, A and B running on a circular track and taking x and y units of time to complete one full circle.

A would reach the starting point for the first time after x units and thereafter would be at the starting point after every x units. Similarly, B would reach the starting point for the first time after y units and thereafter would be at the starting point after every y units.

Thus, A and B would both be at the starting time after common multiple of x and y and the first time that this would occur would be the LCM of x and y.

For these types of problems, it does not matter in which direction the two athletes are running. Even if both are running clockwise or if one is running clockwise and other anticlockwise, the time when they would be at the starting point would remain the same.

Ex. **Three athletes A, B and C are running on a circular track of length 1200 meters with speeds 30 m/s, 50 m/s and 80 m/s. A is running clockwise and B and C are running anticlockwise. Find the first time that A and B will meet at the starting point. Also find which numbered meeting will this be. Find the sum of the distance (in terms of the track length) that they have run so far.**

Sol: A would be at the start after every multiple of $\frac{1200}{30}$= 40 sec and B will be at the start after every $\frac{1200}{50}$= 24 sec. Thus, they would be simultaneously at the starting point for the first time at the LCM of 40 and 24 i.e., 120 seconds.

To find which numbered meeting will this be: In the above example we have found that A and B meet for the first time after 15 seconds and would keep meeting after every 15 seconds thereafter. Thus, the meeting after 120 seconds will be their $\frac{120}{15}$ i.e., 8th meeting.

The distance run by A so far = 120 sec × 30 m/s = 3600 meters i.e., 3 full rounds.

The distance run by B so far = 120 sec × 50 m/s = 6000 meters i.e., 5 full rounds.

Thus, *together* they have run eight full rounds and this is their eighth meeting. Remember they are running in opposite direction.

From these two solved examples, one should be clear that when two athletes are running in opposite direction, they meet whenever *together* they have run one full round.

Ex. **Three athletes A, B and C are running on a circular track of length 1200 meters with speeds 30 m/s, 50 m/s and 80 m/s. A is running clockwise and B and C are running anticlockwise. Find the time after which B and C will meet for the first time and the frequency (in seconds) after which they will keep meeting. Also find the difference of the distance (in fraction of the track length) run by them till their first meeting.**

Sol: Since B and C are running in same direction, they would meet after $\frac{1200}{80-50} = \frac{1200}{30}$= 40 sec.

From the first meeting onwards, one could say that B and C are again starting simultaneously afresh and running in opposite direction. Thus, they would meet again after further 40 seconds. Continuing the same argument, we can say that B and C meet after every 40 seconds.

In 40 seconds, B would have run 40 × 50 = 2000 meters i.e., $\frac{2000}{1200} = \frac{5^{th}}{3}$ or $1\frac{2^{rd}}{3}$ of the track length.

In 40 seconds, C would have run 40 × 80 = 3200 meters i.e., $\frac{3200}{1200} = \frac{8^{th}}{3}$ i.e., $2\frac{2^{rd}}{3}$ of the track length.

Notice that, when running in same direction, on meeting for the first time, the faster athlete has run one full track length *more* than the slower.

Ex. **Three athletes A, B and C are running on a circular track of length 1200 meters with speeds 30 m/s, 50 m/s and 80 m/s. A is running clockwise and B and C are running anticlockwise. Find the first time that B and C will meet at the starting point. Also find which numbered meeting will this be. Find the difference of the distance (in terms of the track length) that they have run so far.**

Sol: B would be at the start after every multiple of $\frac{1200}{50}$ = 24 sec and C will be at the start after every $\frac{1200}{80}$= 15 sec. Thus, they would be simultaneously at the starting point for the first time at the LCM of 24 and 15 i.e., 120 seconds.

To find which numbered meeting will this be: In the above example we have found that B and C meet for the first time after 40 seconds and would keep meeting after every 40 seconds thereafter. Thus, the meeting after 120 seconds will be their 3rd meeting.

The distance run by B so far = 120 sec × 50 m/s = 6000 meters i.e., 5 rounds

The distance run by C so far = 120 sec × 80 m/s = 9600 meters i.e., 8 rounds

The difference of the distance run by them is 3 rounds i.e., the faster athlete has run three full rounds *more* than the slower one and this is their third meeting.

From these two solved examples, one should be clear that when two athletes are running in same direction, whenever the faster one has run one full round *more* than the slower one they would meet.

Another Important Approach: 'Relative' Distances or Rounds run

When running in opposite directions ……

…… they meet whenever *together* they have covered one full round ……

…… and hence at the nth meeting, *together* they would have covered n rounds.

This is easy to understand. Since both start from common point and run in opposite directions, at 1st meeting, if one runs f fraction of the track, the other has to run $(1 - f)$ fraction of the track so that he is at same point. Thus, sum of distances run = $f + (1 - f)$ i.e. 1 full round.

This is new beginning, and from now onwards, till 2nd meeting they would together again cover 1 full round i.e., they would together cover 2 rounds since start. And so on.

When running in same directions ……

…… they meet whenever the faster one has covered 1 round more than the slower one ……

…… and hence at the n^{th} meeting, the faster one would have covered n rounds more than the slower one.

Consider both start from same point and run in same direction. Focus on the gap between them. The faster one will race ahead of the slower one and a gap will start emerging between them. As time passes the gap will start increasing. Slowing the gap will increase from being $\frac{1}{10}^{th}$ of the track to $\frac{1}{5}^{th}$ of the track and after some more time, the gap will increase to $\frac{1}{2}$ of the track (the gap increases linearly, the values chosen are random ones to explain). When the gap is $\frac{1}{2}$, they will appear diametrically opposite. Note that so far, they would not have met (and we also have no idea how many circles each would have done individually so far, could be any number – fraction of the track or many rounds of the tracks). As they keep running, the gap increases from $\frac{1}{2}$ to $\frac{3}{4}^{th}$ of the track length. (For an observer, the slower one will appear to be ahead of the faster one). And after some more time, the gap will be exactly one full round. When the gap is exactly one full round, the faster one will be at a point that is 1 round ahead of the point where the slower one will be. Both these points will be the same i.e., they will be together. From now again the gap again starts increasing, linearly, increasing to $1\frac{1}{10}$, then to $1\frac{1}{5}$, then to $1\frac{1}{2}$, then to $1\frac{3}{4}$ and finally to 2. It is only at this moment that the faster one would again be together with the slower one i.e., this will be the 2nd meeting.

The above logical way of looking at circular tracks can be used very effectively in solving questions that go beyond the first meeting.

Ex. **In a race of 4800 meters run on a circular track of 400 meters length, the ratio of the speed of the two athletes is 3: 5. If they run in the same direction, how many times do they meet in the entire race?**

Sol: Approach 1: Short-cut

Since it is a 4800 meter race on a track of 400 meters, the winner would have to complete $\frac{4800}{400}$ = 12 laps or rounds. Since the ratio of the speeds is 3: 5, when the faster guy completes 12 rounds, the slower guy would have done $\frac{3}{5} \times 12 = \frac{36}{5} = 7\frac{1}{5}$ or 7.2 rounds. The faster one has run 12 – 7.2 = 4.8 rounds more than the slower one and hence would have met him four times.

(The difference in the number of rounds is not an integer means that when the race ends the two athletes are not together, the faster athlete is 4.8 rounds ahead of the slower. The last meeting was when the faster one was 4 rounds ahead and since then he has gone ahead of the slower guy and the race ends when the faster one is 0.8 i.e. of the track ahead of the slower one. But since the race ends here, he does not overtake (meet) the slower guy for the fifth time.)

Approach 2: Multiple of 1st meeting

If the speed of the athletes is assumed as $3k$ and $5k$, then the two athletes would meet every $\frac{400}{5k-3k} = \frac{200}{k}$ secs.

And the race would last for $\frac{4800}{5k} = \frac{960}{k}$ secs.

If meeting occurs every $\frac{200}{k}$ secs, the number of times they would meet in $\frac{960}{k}$ secs is $\frac{\frac{960}{k}}{\frac{200}{k}}$ = 4.8 i.e., 4 times.

Ex. **In the above race, had they been running in opposite directions, how many times would they have met?**

Sol: When the faster one completes 12 rounds, the slower one completes 7.2 rounds. Thus, they have together done 12 + 7.2 = 19.2 rounds. So, they would have met 19 times.

Ex. **A and B can complete one full round of the circular jogging track in 15 and 40 minutes respectively. If they start at the same time form the same point and run in the same direction, after how much time will they meet for the first time?**

Sol: Approach 1: Short-cut

Since the ratio of time taken for 1 round, same distance, is 3: 8, the ratio of their speeds will be 8: 3. Thus, in the time A runs 8 rounds, B runs 3 rounds i.e. A runs 5 rounds more than B. Dividing this by 5, in the time A runs $\frac{8}{5}$ rounds, B will run $\frac{3}{5}$ rounds i.e. A runs 1 round more than B i.e., this is their 1st meeting. If A takes 15 minutes to run 1 1round, to cover $\frac{8}{5}$ rounds he will take $\frac{8}{5} \times 15$ = 24 minutes.

Approach 2:

One could also have solved like the traditional way by assuming the track length to be '*d*'. Thus, the speeds of A and B are $\frac{d}{15}$ and $\frac{d}{40}$. And the time for 1st meeting will be $\frac{d}{\frac{d}{15}-\frac{d}{40}} = \frac{15\times40}{40-15} = \frac{15\times40}{25}$= 3 × 8 = 24.

Ex. **A runs in the clockwise direction on a circular track whereas B runs on the same track but in the anti-clockwise direction, but they start from the same point. If their speeds are 25 m/s and 40 m/s respectively, at what fraction of the track-length, measured in clockwise direction from the starting point, does the 10th meeting take place?**

Sol: Ratio of speeds is 5: 8.

Thus, in the time A runs 5 rounds, B will run 8 rounds.

Dividing by 5 + 8 = 13, in the time A runs $\frac{5}{13}$ rounds, B will run $\frac{8}{13}$ rounds and this will be their 1st meeting.

Multiplying this by 10, at the time of the 10th meeting A will have run $\frac{50}{13} = 3\frac{11}{13}$ rounds i.e., the meeting place, measured in the clockwise direction, is $\frac{11}{13}^{th}$ of the track length.

Three or more people meeting

To find time when three or more people meet, find the time after which pairs of athletes meet, such that at least one athlete is common to all the pairs (say, A & B, A & C, A & D, ……). The respective pairs will keep meeting after any multiple of the time found. At the LCM of these durations of time, we are sure that A & B have met, that A and C have met, that A & D have met and so on. But the only way that A could have met all these people at the same instant is when all of them are together.

Ex. **If A, B and C take 12 mins, 15 mins and 20 mins to complete one full round of a circular track, after how much time will all three A, B and C meet for the first time, if they start from the same point at the same time, with A and C running in clockwise direction and B running in anti-clockwise direction?**

Sol: **Considering A & B:** Taking the LCM of their time, in 60 minutes, A runs 5 rounds and B will run 4 rounds. Since they are running in opposite directions, they would have met 5 + 4 = 9 times. Thus, in $\frac{60}{9} = \frac{20}{3}$ mins, they would meet for the first time and will meet every $\frac{20}{3}$ mins.

Considering A & C: Taking the LCM of their time, in 60 minutes, A runs 5 rounds and C will run 3 rounds. Since they are running in same directions, they would have met 5 – 3 = 2 times. Thus, in $\frac{60}{2}$= 30 mins, they would meet for the first time and will meet every 30 mins.

All three would meet after time equal to the LCM of $\frac{20}{3}$ & 30.

Using LCM of fraction = $\frac{\text{LCM of numerators}}{\text{HCF of denominators}}$, the required answer is $\frac{\text{LCM (20,30)}}{\text{HCF (3,1)}} = \frac{60}{1}$= 60 sec.

Position of the meeting points

These types of questions do not deal with when or how often do the athletes meet when they are running on a circular track. These questions pertain to the number of points and their placement on the circular track where the athletes can possibly meet.

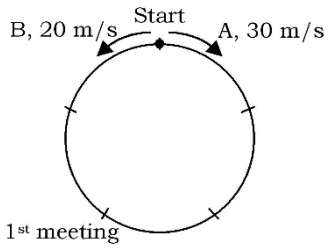
When running in the opposite directions:

If the ratio of speeds of two athletes (in the most reducible form) is a: b, the number of distinct meeting points on the track would be would be $a + b$. This funda is best understood with an example…

Consider athletes A and B running at speeds of 30 m/s and 20 m/s on a circular track of 1000 meters, A running clockwise and B anti-clockwise. If they keep running indefinitely, at how many distinct points on the circle would they meet?

A and B would meet for the first time after $\frac{1000}{30+20} = \frac{1000}{50}$= 20 sec.

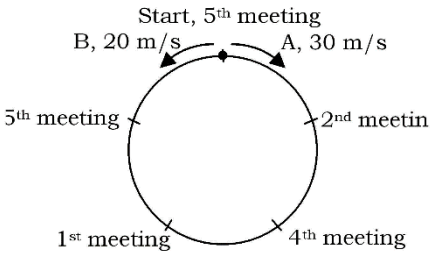
In 20 seconds, A would travel 30 × 20 = 600 i.e., $\frac{600}{1000} = \frac{3}{5}^{th}$ of the circle. To find this point precisely, it is best to divide the circle in 5 equal parts. Thus, they would meet at the point as shown in the figure:



Compared to the starting point, the meeting point is 3 segments clockwise.

Now this point could be considered as a fresh start and thus, they would again meet after A has run $\frac{3}{5}^{th}$ of the circle again i.e., 3 segments clockwise from the point of first meeting.

This logic can be extended and the following figure shows the 2nd, 3rd, 4th and 5th meeting points.



We see that the 5th meeting happens at the starting point. This is not just 'can be considered as a fresh start' but is exactly same as the start since not only are they together but they are also at the same point from where they started. Thus, after this the same pattern will start repeating and even if the runners keep running indefinitely, they would just meet at these 5 points.

In short, if the ratio of two runners running in opposite directions, in the most reducible form is a: b, they can meet at $a + b$ distinct points on the track.

When running in the same direction:

If the ratio of speeds of two athletes (in the most reducible form) is a: b, the number of distinct meeting points on the track would be would be $|a - b|$.

Consider a similar example as above but with different values: A and B are running with speeds 40 m/s and 10 m/s on a circular track of length 1500 meters, but this time they are running in the same direction, clockwise.

They would meet for the first time after $\frac{1500}{40-10}$= 50 sec.

In 50 seconds, A would have run 40 × 50 = 2000 meters i.e., $\frac{2000}{1500} = \frac{4^{th}}{3} = 1\frac{1^{rd}}{3}$ of the circle and B would have run 10 × 50 = 500 i.e., $\frac{500}{1500} = \frac{1^{rd}}{3}$ of the circle. As expected, B has run 1 round more than A and they have met. And this meeting point is $\frac{1^{rd}}{3}$ of a circle in the clockwise direction from start. Considering this as a new start, they should meet at $\frac{1^{rd}}{3}$ of a circle ahead (clockwise) from this point i.e., at a point $\frac{2^{rd}}{3}$ clockwise from the starting point. Considering this as a yet new start, they should meet at $\frac{1^{rd}}{3}$ of a circle ahead (clockwise) from this point i.e., at the start. And since this is exactly similar to the original start, even if they keep running indefinitely, they would just keep meeting at these 3 points.

If the ratio of two runners running in the same direction, in the most reducible form is a: b, they can meet at $|a - b|$ distinct points on the track.

Ex. **Which of the following cannot be the ratio of speeds of two joggers running on a circular jogging track if while running they meet at a diametrically opposite point to the point from where both of them started?**

1. 3: 5 **2. 1: 3** **3. 1: 5** **4. 2: 5**

Sol: Evaluating each option …

Option (1): With ratio of speeds being 3: 5, if they are running in same directions, then they would meet at 5 – 3 = 2 points, the starting point and the diametrically opposite point.

Option (2): With ratio of speeds being 1: 3, if they are running in same directions, then they would meet at 3 – 1 = 2 points, the starting point and the diametrically opposite point.

Option (3): With ratio of speeds being 1: 5, if they are running in same directions, then they would meet at 5 – 1 = 4 points, of which one will be the diametrically opposite point to the point from where they started.

Option (4): With ratio of speeds being 2: 5, if they are running in same directions, then they would meet at 5 – 2 = 3 points. The 3 points will be the starting point, point that is $1/3^{rd}$ of the track length from the starting point and point that is $1/3^{rd}$ of the track length from the starting point. Thus, they would not meet at the diametrically point if this is their ratio of speeds.

Even if they were running in opposite direction, they would meet at 7 points, the starting point and points at distance of $\frac{1}{7}^{th}$, $\frac{2}{7}^{th}$, $\frac{3}{7}^{th}$, ……, $\frac{6}{7}^{th}$ of track length. Even in this case, the diametrically opposite point will not be a meeting point.

QUANTIFIERS CAT ACADEMY

Exercise 16

1. The speeds of a scooter, a car and a train are in the ratio 1: 4: 16. If all of them cover equal distance, the ratio of time taken by each of the vehicle is:
1. 256: 16: 1 2. 1: 4: 16 3. 16: 4: 1 4. 16: 1: 4

2. By walking at $4/5^{th}$ of his usual speed, a man reaches office 10 minutes later than usual. What is the usual time taken by the man to reach office?
1. 30 min 2. 40 min 3. 35 min 4. 50 min

3. Walking at 3/4 of his normal speed, Bharat takes 45 minutes more than the normal time to reach the office. Find his normal time (in hour).
1. 2.25 2. 2 3. 4 4. 1.5

4. If a man walks from his house at 5 km/ hr, he reaches office 5 minutes late. If he walks at 6 km/hr, he reaches 5 minutes early. Then the distance from his house to the office is:
1. 3 km 2. 4 km 3. 5 km 4. 6km

5. A train travels at a uniform speed of 15 km/hr. If the speed had been 10 km/hr more, it would have taken 8 hours less to cover the distance between stations A and B. What is the distance (in km) between A and B?
1. 100 2. 300 3. 250 4. 200

6. If a man cycles at 10 km/hr, then he arrives at a certain place at 1 p.m. If he cycles at 15 km/hr from the same starting point, he will arrive at the same destination at 11 a.m. At what speed must he cycle to get there at noon?
1. 11 km/hr 2. 12 km/hr 3. 13 km/hr 4. 14 km/hr

7. A car has to cover 928 km in 10 hrs. If it covers first half of the journey in $3/5^{th}$ of the total time, what should be its speed (km/hr) for the remaining journey so as it covers the entire journey in a specified time?
1. 124 2. 164 3. 116 4. 120

8. A man leaves a point P at 6 a.m. and reaches the point Q at 10 a.m. Another man leaves the point Q at 8 a.m. and reaches the point P at 12 noon. At what time do they meet if they travel at a constant speed?
1. 9 a.m. 2. 10 a.m. 3. 8.30 a.m. 4. 9 p.m.

9. A man travels three-fifths of the total distance from A to B at a speed 3a, and the remaining at a speed 2b. If he goes from B to A and returns at a constant speed 5c in the same time, then which of the following is true?
1. $1/a + 1/b = 1/c$ 2. $a + b = c$ 3. $1/a + 1/b = 2/c$ 4. None of these

10. A train approaches a tunnel AB. Inside the tunnel is a cat located at a point that is $3/8^{th}$ of the distance AB measured from the entrance A. When the train whistles the cat runs. If the cat moves to the entrance of the tunnel A, the train catches the cat exactly at the entrance. If the cat moves to the exit B, the train catches the cat at exactly the exit. What is the ratio of the speed of the train to that of the cat?
1. 3: 1 2. 4: 1 3. 5: 1 4. None of these

11. Train X departs from station A at 11 a.m. for station B, which is 180 km away from A. Train Y departs from station B at 11 a.m. for station A on the same day. Train X travels at an average speed of 70 km/hr and does not stop anywhere until it arrives at station B. Train Y travels at an average speed of 50 km/hr, but has to stop for 15 minutes at station C, which is 60 km away from station B enroute to station A.

Ignoring the lengths of the trains, what is the distance, to the nearest integer kilometre, from station A to the point where the trains cross each other?
1. 112 km 2. 118 km 3. 120 km 4. None of these

12. After travelling for 30 minutes, a train met with an accident and was stopped there for 45 minutes. Due to the accident, its speed reduced to 2/3rd of its initial speed and the train reached its destination 90 minutes late. Had the accident occurred 60 km after the point it occurred earlier, the train would have reached 30 minutes earlier than it was reaching after the previous accident. Find the length of the journey.
1. 90 km 2. 120 km 3. 60 km 4. 180 km

13. The distance between Ankit's house and office is 60 km. One day, he covers half of the distance at twice of his usual speed and the rest of the distance at half of his usual speed. If he took 75 minutes more than the usual time, find the usual speed (in km/hr).
1. 12 2. 15 3. 20 4. 10

14. The distance between A and B is 19 km. A cyclist starts from A at a constant speed towards B. A car leaves from A 15 min later in the same direction. In 10 min it catches up with the cyclist and continues towards B; after reaching B, it turns around and in 50 min after leaving, car encounters the cyclist the second time. The speed of the car is:
1. 30 km/hr 2. 40 km/hr 3. 35 km/hr 4. 80 km/hr

15. Ram, Shyam and Hari went out for a 100 km journey. Ram and Hari started the journey in Ram's car at the rate of 25 kmph, while Shyam walked at 5 kmph. After sometime, Hari got off and started walking at the rate of 5 kmph and Ram went back to pick up Shyam. All three reached the destination simultaneously. The number of hours required for the trip was:
1. 6 2. 7 3. 8 4. 10

16. A car travels from A to B in 46 min. If the distance between A and B is 52.8 km, what is the (approximate) average speed of the car?
1. 1.14 km/hr 2. 40.5 km/hr 3. 68.86 km/hr 4. 84.2 km/hr

17. A person covers 180 km, 240 km and 300 km by car, train and bus respectively. Find the average speed (in km/hr) of the whole journey if the speed of car, train and bus is 60 km/hr, 80 km/hr and 50 km/hr respectively.
1. 63 1/3 2. 60 3. 62 1/3 4. 62

18. Anu covers some distance at 4 km/hr and the rest at 6 km/hr. If the average speed for the entire journey is 5 km/hr and total distance covered is 50 km, then how much distance (in km) is covered at 4 km/hr?
1. 25 2. 20 3. 30 4. None of these

19. Excluding stoppages, the average speed of a bus is 54 km/hr, and including stoppages it is 45 km/hr. For how many minutes does the bus stop per hour?
1. 12 2. 16 3. 8 4. 10

20. What is the time taken by a train 900 m long to cross an electric pole, if the speed of the train is 54 km/hr?
1. 60 sec 2. 80 sec 3. 75 sec 4. 72 sec

21. A train of length 500 m takes 25 seconds to completely cross a pole. Find its speed (in kmph).
1. 54 2. 72 3. 60 4. 90

22. What is the time taken by a train 650 m long travelling at 72 km/hr to cross a 750 m long platform?
1. 60 sec 2. 65 sec 3. 70 sec 4. 75 sec

23. How long will a train 250 m long travelling at 90 kmph take to cross a train 200 m long travelling at 36 kmph in the same direction?
1. 20 seconds 2. 22 seconds 3. 30 seconds 4. 18 seconds

24. What is the time taken (in sec) by a 750 m long train travelling at 99 km/hr to cross a boy running at 9 km/hr towards the train?
1. 25 2. 20 3. 24 4. 30

25. Walking from home at $3/4^{th}$ of his usual speed, a man reaches his office 20 minutes late. Had the person walked at $4/3^{rd}$ of his usual speed, find the time taken (in minutes) by the man to reach his office.
1. 30 2. 45 3. 40 4. 50

26. A boy walks from his home to school at 5 kmph, reaches the school 4 minutes early. If he walks from his home to school at 4 kmph, he reaches the school 2 minutes late. Find the time taken by him to go to school if he walks at 6 kmph.
1. 15 minutes 2. 20 minutes 3. 25 minutes 4. 30 minutes

27. M and N start at the same time from two ends S and T, at speeds of 3 m/sec and 4 m/sec towards each other, ST being 7 m. After reaching the opposite ends, they retrace their path and start moving towards each other. Find the total distance travelled by M when he meets N for the second time.
1. 20 m 2. 15 m 3. 9 m 4. 26 m

28. A train crosses two bridges 430 m and 550 m long in 30 seconds and 36 seconds respectively. Find the speed of the train.
1. 36 kmph 2. 72 kmph 3. 27 kmph 4. 45 kmph

29. A train travelling at 60 kmph, crosses a cyclist who is cycling at 12 kmph in the opposite direction in 20 sec. Find the time taken (in sec) by the train to cross another cyclist travelling at 20 kmph in the same direction as the train does.
1. 30 2. 32 3. 36 4. 40

30. Two men, M and N started walking towards each other simultaneously from two places F and G respectively, which are 50 km apart. They meet after 5 hours. After their meeting, M reduced his speed by 1 kmph and N increased his speed by 1 kmph. They arrived at G and F respectively at the same time. Find the initial speed (in kmph) of M.
1. 5.5 2. 5 3. 6.5 4. 6

31. A thief was running along to a median on a road at 6 kmph. He crossed a police jeep moving at 9 kmph in the opposite direction. The jeep had to continue for 10 more minutes before it could find a gap in the median and start chasing the thief. Find the total time taken by the jeep to catch the thief from the time it crossed him (in minutes).
1. 45 2. 50 3. 55 4. 60

32. Anand and Bala started from towns P and Q simultaneously towards Q and P respectively. After 6 hours, they crossed each other at town R. After that, Anand took 5 hours more to reach Q than Bala took to reach P. Find the sum of the times taken by them to reach their destinations from R (in hours).
1. 15 2. 11 3. 13 4. 17

33. Anwar leaves home every day at 4 p.m. to pick his wife from office and returns home at 6 p.m. One day, the office was over at 4 p.m. and his wife started walking home from office. Anwar, unaware of this, starts from home as usual and meets his wife on the way and returns home with wife 15 minutes early. If the speed of Anwar's car is 30 kmph, find the walking speed of his wife.
1. 30/7 kmph 2. 3.6 kmph 3. 34/5 kmph 4. 5.4 kmph

34. An artillery gun is fired twice at an interval of 24 seconds. A motorist moving towards the gun heard the sounds of the gunfire at an interval of 22 seconds. If the speed of sound is 330 m/s, what is the speed of the car (in m/sec)?
1. 20 2. 25 3. 30 4. 28

35. A car travels a total distance of 300 km. After travelling a part of the distance without any trouble, the car develops an engine problem and proceeds at 3/4th of its former speed and arrives at the destination 80 minutes late. Had the problem developed 50 km further on, the car would have arrived 20 minutes sooner. Find the original distance it travelled without any problem, and the speed over that part of the journey.
1. 100 km, 50 kmph 2. 50 km, 75 kmph
3. 150 km, 50 kmph 4. 50 km, 100 kmph

36. Mr. Ashok takes 16 hours to go by train to a certain town and return by car. He would save 4 hours if he travelled both ways by car. Find the time he would lose, if he travelled both ways by train (in hours).
1. 2 2. 3 3. 4 4. 3.5

37. A man reaches his destination which is 16 km away, 9 min late, if he travels at 8 kmph. What should his speed be if he wishes to reach 15 minutes ahead of the right time?
1. 10 kmph 2. 13 kmph 3. 12 kmph 4. 15 kmph

38. A person covered 50% of a certain distance at 15 kmph, 60% of the remaining distance at 18 kmph and the rest at 45 kmph. The total travel time of the person is 9.8 hours. Find the total distance covered. (in km)
1. 120 2. 140 3. 150 4. 180

39. The distance between two points P and Q is 84 km. Two persons start at the same time but one travelling from P towards Q and the other travelling from Q towards P. If their respective speeds are 36 kmph and 27 kmph, where do they meet each other?
1. 48 km from Q 2. 24 km from P 3. 36 km from P 4. 48 km from P

40. Towns P and Q are 80 km apart. Cars A and B are stationed at towns P and Q respectively. If they start simultaneously towards each other, they would meet in an hour. If both start simultaneously in the same direction, the faster car would overtake the slower car in 4 hours. Find the speed of the faster car (in kmph).
1. 50 2. 40 3. 60 4. 45

41. A cat on seeing a dog 100 m away turns around and starts running away at 24 kmph. The dog spots him one minute later and starts chasing the cat at a speed of 33 kmph. After how much time (in sec), from the start of the cat's run, will the chase end?

1. 200 2. 220 3. 260 4. 280

42. Train A starts at 6 a.m. from city P towards city Q at a speed of 54 kmph. Another train 'B' starts at 9 a.m. from P towards Q at 72 kmph. If the distance between P and Q is 1440 km, find at what distance from Q would the two trains meet each other?
1. 648 km 2. 792 km 3. 486 km 4. 954 km

43. Towns P and Q are 265 km apart. Car A started from P at 60 km/hr at 8:00 a.m. towards Q. Car B started from Q at 40 km/hr at 10:00 a.m. towards P. At 11:00 a.m., B stopped for half an hour. A stopped for 28 minutes at town S, which is at a distance of 210 km from P. Find the time at which B crossed A.
1. 11: 35 a.m. 2. 11: 54: 30 a.m. 3. 11: 52: 30 a.m. 4. 11: 48 a.m.

44. M and N are two points that are 8 m apart. Anand and Ajay started simultaneously from M and N respectively. Anand moved towards N at 3 m/sec. Ajay moved towards M at 5 m/sec. After reaching their destinations, both turned back at their original speeds towards their starting points. Find the total distance (in m) that Ajay would have travelled before crossing Anand for the second time.
1. 12 2. 13 3. 15 4. 14

45. P and Q are two points, 10 km apart. Anand started from P towards Q and at the same time, Ashok started from Q towards P. They crossed each other after 1 hour. After that, Anand reduced his speed by 2 kmph and Ashok increased his speed by 2 kmph. They reached their destinations simultaneously. Find Anand's initial speed (in kmph).
1. 5 2. 6 3. 8 4. 7

46. Mahesh travelled from Hyderabad to Tirupati at a certain speed and returned at a certain speed. His average speed for the entire trip was the average of his onward and return speeds. He travelled a total distance of 1200 km in 12 hours. Find his onward speed (in kmph).
1. 100 2. 80 3. 60 4. 40

47. Two cars C and D start from a junction along two perpendicular roads at 8:00 a.m. and 9:00 a.m. respectively. If at 12 noon, the cars, which travel at the same speed, are 150 km apart, then, find the speed (in kmph) of each car.
1. 30 2. 25 3. 40 4. 35

48. A car had to travel a total distance of 600 km. After travelling a part of the distance, it developed an engine problem. It travelled the remaining distance at $4/5^{th}$ of its usual speed. It arrived an hour late at its destination. Had the engine problem occurred after it had travelled 150 km more, the car would have arrived half an hour late at its destination. Find the distance (in km) it travelled without any problem.
1. 150 2. 250 3. 200 4. 300

49. Every day, Ashwin starts at 3.00 pm from his home to pick up his son from school. They reach their house at 5.00 p.m. One day, school was over at 3.00 p.m. Ashwin, not aware of this, started from home as usual. He met his son on the way and they reached home 20 minutes earlier than usual. If the speed of his car is 55 kmph, find his son's speed (in kmph).
1. 11 2. 10 3. 14 4. 13

Exercise 17

1. A man saw a train passing over a 10 km long bridge. The length of the train is one-fourth that of the bridge. If the train clears the bridge in 2.5 min, then speed of the train is:
1. 150 km/hr 2. 300 km/hr 3. 250 km/hr 4. 120 km/hr

2. Two boats, traveling at 5 and 10 kms per hour, head directly towards each other. They begin at a distance of 20 kms from each other. How far apart are they (in kms) one minute before they collide.
1. 1/12 2. 1/6 3. 1/4 4. 1/3

3. The distance between the two stations X and Y is 220 km. A train leaves station X for Y at a speed 80 km/hr. After half an hour, another train leaves station Y for X at a speed 100 km/hr. Find the distance of the point where the two trains meet from X.
1. 100 km 2. 120 km 3. 80 km 4. 140 km

4. Two trains start from A and B and head towards B and A respectively. Train from A starts at 11 p.m. and reaches B at 5 p.m. next day. Train from B starts at 10 p.m. the same day on which the train starts from station A and reaches A at 6 a.m. the next day. At what time (approximately) will they cross each other?
1. 4 a.m. 2. 3.55 a.m. 3. 2.51 a.m. 4. 3.51 a.m.

5. August Kranti Rajdhani Express leaves Delhi towards Mumbai at 12.20 p.m. and travels uniformly at 80 km/hr. Rajdhani Express leaves Chennai towards Delhi at 3.10 p.m. and travels uniformly at 120 km/hr. Both the trains cross at Baroda at 4.30 p.m. On a particular day, Rajdhani leaves at 3.20 p.m. and the other train leaves at its usual time. When will the two trains cross each other?
1. 4.32 p.m. 2. 4.38 p.m. 3. 4.40 p.m. 4. 4.36 p.m.

6. Singur, Nandigram and Kolkata are 3 villages on a straight road such that Singur is somewhere in between the other two villages. Distance of Kolkata from Singur is 10 km and the distance of Kolkata from Nandigram is 17 km. Buddha and Biman start running from Singur at 9:00 a.m. in opposite directions at 4 m/s and 3 m/s respectively. Mamta starts from Nandigram in a car moving at 36 km/hr towards Kolkata, but she started 10 minutes later than Buddha and Biman. If the interval between the time when Mamta met Buddha and the time when Mamta met Biman is approximately 15 10/21 minutes, then find the distance (in km) between Buddha and Biman when Mamta meets Biman.
1. 10 2. 12 3. 13 4. 14

7. Nikhil reaches school every day at 5 p.m. to pick up his children. On a Saturday, the school got over at 4 p.m. and the children started walking towards home. Nikhil met them on their way and returned home 30 min earlier. How long did the children walk?
1. 50 min 2. 55 min 3. 45 min 4. 30 min

8. A car overtakes a bus travelling from Bangalore to Chennai at 6:30 a.m. The car reaches Chennai at 8 a.m. After stopping there for 1 hour, it starts back towards Bangalore and meets the same bus at 9: 30 a.m., which was moving towards Chennai at that time. If both the car and the bus travel at uniform speeds on the same route, then what will be the time when the bus would reach Chennai?
1. 10:00 a.m. 2. 10:30 a.m. 3. 11:00 a.m. 4. 9:45 a.m.

9. Consider a road with two cars at a distance of 500 km driving in the opposite direction. The left-side car drives at a speed of 40 km/hr and the right-side car at a speed of 60 km/hr. A bird starts from the same location as the right-side car and flies at a speed of 80 km/hr. When it reaches the left-side car, it turns its direction;

and when it reaches the right-side car, it turns its direction again to the opposite, and so on. What is the total distance that the bird has covered till the moment the two cars reach each other?
1. 200 km 2. 400 km 3. 450 km 4. 300 km

10. A and B start simultaneously from P and reach R via Q along the same road. A travels at 15 km/hr from P to Q and at 20 km/hr from Q to R. B travels at 20 km/hr from P to Q and at 15 km/hr from Q to R. A reaches at R, 10 min before B reaches there. If the time taken to cover PQ to the time taken to cover QR by B is in the ratio 1:2, find the distance between P and R.
1. 35 km 2. 50 km 3. 55 km 4. 45 km

11. Navjivan Express from Ahmedabad to Chennai leaves Ahmedabad at 6.30 a.m. and travels at 50 kmph towards Baroda situated 100 km away. At 7.00 a.m. Howrah-Ahmedabad Express leaves Baroda towards Ahmedabad and travels at 40 kmph. At 7.30 a.m. Mr Shah, the traffic controller at Baroda realizes that both the trains are running on the same track. How much time does he have to avert a head-on collision between the two trains?
1. 15 min 2. 20 min 3. 25 min 4. 30 min

12. A boat sails m km upstream at the rate of 'r' km/hr. If the stream flows at the rate of 's' km/hr, then how long will the boat take for the return trip?
1. $\frac{m}{r+2s}\ hr$ 2. $\frac{r+2s}{m}\ hr$ 3. $\frac{m}{r+s}\ hr$ 4. None of these

13. A boat goes from point A to B in upstream and returns to A through the same path. If time of journey from A to B is twice that of from B to A, then the ratio of speed of the boat in still water to the speed of the current is:
1. 3: 1 2. 1: 3 3. 2: 1 4. Cannot be determined

14. A and B are stationed at two points Q and R respectively on a flowing river. The direction of flow of river is from Q to R. When A and B swim towards each other, they meet at P, which is at a distance of 20 m from the point R. When A swims towards B and B swims away from A, they meet at S, which is at a distance of 40 m from the point R. B's speed in still water is five times the speed at which the river flows. Find the ratio of speeds of A and B in still water.
1. 6: 1 2. 7: 1 3. 8: 1 4. 9: 1

15. Two boats start towards each other at different speeds from the parallel and opposite banks of a lake. When they pass each other for the first time, they are 300 m from one of the banks. After the first meeting, they continue to move towards the opposite banks. Once they reach the opposite banks, they turn around and move towards each other once again. When they pass each other the second time, they are 200 m from the other bank. Find the ratio of the speeds of the two boats. (Assume both the boats travel at uniform speed and neglect values of turnaround time.)
1. 2: 1 2. 4: 3 3. 7: 3 4. 5: 4

16. A swimmer jumps from a bridge over a canal and swims 1 km upstream. After that first kilometre, he passes a floating cork. He continues swimming for another half an hour and then turns around and swims back to the bridge in 30 mins. The swimmer and the cork arrive at the bridge at the same time. The swimmer has been swimming with constant speed. How fast does the water in the canal flow?
1. 0.5 km/hr 2. 2 km/hr 3. 3.5 km/hr 4. 1 km/hr

17. At his usual rowing rate, Rahul can travel 12 miles downstream in a certain river in 6 hr less than it takes him to travel the same distance upstream. But if he could double his usual rowing rate for this 24-mile round trip, the downstream 12 miles would then take only 1 hr less than the upstream 12 miles. What is the speed of the current in miles per hour?

1. 7/3 2. 4/3 3. 5/3 4. 8/3

18. Two friends Aku and Chika are stationed at two different points P and Q respectively along the same bank of a straight stretch of a flowing river. The ratio of the speeds of Aku and Chika in still water is 1: 3. When Aku and Chika swim in the opposite directions from P and Q respectively, they meet at a point 30 meters from Q. If the time taken by Aku to reach point Q and Chika to reach point P from their original positions P and Q respectively is the same, then the distance (in meters) between P and Q is:
1. 45 2. 90 3. 120 4. 60

19. A and B are 2 points on a river. Akbar took 6 hours to row from A to B and 8 hours to row from B to A. Find the ratio of the speeds of his boat in still water and the river.
1. 7: 1 2. 4: 3 3. 5: 3 4. 3: 1

20. Akbar takes a certain time to go downstream on a river between two villages. He takes 4 times as much time to cover the same distance upstream. What is the speed (in kmph) of the stream, if the distance between the villages is 10 km, which can be covered in 2 hours if the boat travels in still water?
1. 3 2. 3.5 3. 4.5 4. 4

21. A boat travels from point P to point Q upstream and returns from point Q to point P downstream PQ = 96 km. If the round trip takes 9 hours and the speed of the boat in still water is 8 kmph more than the speed of the stream, find the time taken for the downstream journey.
1. 3 hours 2. 4 hours 3. 8 hours 4. 6 hours

22. A boat takes 9 hours to make a round trip in a river between two points 24 km apart. It would have taken 4 hours to cover the upstream distance in still water. Find the speed of the stream (in kmph).
1. 1 2. 1.5 3. 1.75 4. 2

23. An escalator is moving downwards. Anil takes 60 steps to reach the bottom of the escalator starting from the top. Bala takes 140 steps to reach the top of the escalator starting from the bottom. The speeds of Anil and Bala are equal. Find the number of steps visible when the escalator is stationary.
1. 80 2. 84 3. 60 4. 75

24. A takes 120 steps to reach the bottom of an escalator which is moving upwards. B takes 60 steps to reach the top of the escalator. If A takes two steps for every step of B, find the number of steps visible when the escalator is stationary.
1. 80 2. 90 3. 100 4. 75

25. There are two identical escalators, one is ascending and the other is descending. A person took 90 seconds to go up using the descending escalator and 18 seconds to go up using the ascending escalator. Both times walking at the same speed. If the speed of the two escalators is the same, then the time taken (in sec) to go up/down using either of the escalators when they are switched off is:
1. 20 2. 25 3. 30 4. 40

26. An escalator is moving at 3 steps per second. Harish walks in the same direction as the moving escalator at 2 steps per second and takes 3 second less to get out of the escalator than when he was moving on the stationary escalator. How many steps are there in the stationary escalator?
1. 15 2. 10 3. 20 4. 18

27. Rajat takes 11 steps to reach the bottom of an escalator in 35 seconds. Sharad walks on the same escalator and takes 39 steps in 21 sec to reach the bottom. How many

steps are visible on the escalator when it is not moving? (**Note:** The escalator is coming down in both the cases)

1. 75 2. 81 3. 85 4. 71

28. An escalator is coming down at a constant speed. Vivek takes 40 steps to reach the bottom. The time in which he takes 15 steps, Samar manages to reach the bottom by taking 45 steps. If the escalator were turned off, how many steps would they have to take to walk down?

1. 48 2. 50 3. 50 4. 54

29. Shyama and Vyom walk up an escalator (moving stairway). The escalator moves at a constant speed. Shyama takes three steps for every two of Vyom's steps. Shyama gets to the top of the escalator after having taken 25 steps, while Vyom (because his slower pace lets the escalator do a little more of the work) takes only 20 steps to reach the top. If the escalator were turned off, how many steps would they have to take to walk up?

1. 50 2. 40 3. 60 4. 75

Exercise 18

1. In a 500 m race, the ratio of the speeds of two contestants A and B is 2: 3. If A has a start of 170 m, then A wins by:
1. 5 m 2. 10 m 3. 15 m 4. 20 m

2. In a 1 km race, A beats B by 40 m and B beats by C by 25 m. By how many meters A beats C in the same race?
1. 65 2. 60 3. 64 4. 66

3. P beats Q by 20 m in a 120 m race, and Q beats R by 24 m in the same race. By what distance will P beat R in a race of 150 m?
1. 54 m 2. 50 m 3. 48 m 4. 45 m

4. A and B stay in the same apartment and study in the same school. A and B start walking towards their school at speeds of 2 km/hr and 3 km/hr respectively. The faster of the two reaches the school first, turns around and starts walking back. If A and B meet 200 m away from the home then what is the distance between their apartment and their school?
1. 260 m 2. 240 m 3. 200 m 4. 250 m

5. A and B start from two locations P and Q respectively at the same time and meet every day at X on the way. Uniform speeds of A and B are 20 km/hr and 15 km/hr respectively. One day B starts t min late and meets A, 5 km ahead of X at Y. Find t.
1. 15 min 2. 20 min 3. 35 min 4. 25 min

6. Karan and Arjun run a 100-meter race, in which Karan beats Arjun by 10 metres. To do a favour to Arjun, Karan starts 10 metres behind the starting line in a second 100 metre race. They both run at their earlier speeds. Which of the following is true in connection with the second race?
1. Karan and Arjun reach the finishing line simultaneously.
2. Arjun beats Karan by 10 metres
3. Arjun beats Karan by 11 metres.
4. Karan beats Arjun by 1 metre.

7. Three runners A, B and C run a race, with runner A finishing 12 m ahead of runner B and 18 m ahead of runner C, while runner B finishes 8 m ahead of runner C. Each runner travels the entire distance at a constant speed. What was the length of the race?
1. 36 m 2. 48 m 3. 60 m 4. 72 m

8. A and B run a 5 km race on a round course of 400 m. If their speeds are in the ratio 5: 4, then how many times will the winner of the race cross the other participant?
1. 1 2. 2 3. 3 4. 4

9. Amol and Avash start running simultaneously around a circular track from the same point in the same direction. Amol can complete 1 round in 16 seconds and Avash can do the same in 28 seconds. Find the time (in sec) after which they would be together for the first time.
1. 40 2. 112/3 3. 27 2/3 4. 37

10. Two runners A and B are running on a circular track of length 14 km. If A and B started simultaneously from the same point but in opposite directions with speeds 15m/s and 20m/s respectively. What is the distance (in km) covered by A by the time they cross each other for the first time?
1. 4 2. 8 3. 5 4. 6

11. A and B run along a circular track, of unknown radius, in the same direction at speeds of 3m/sec and 5m/sec respectively. Find at how many points will they meet.

1. 2 2. 3 3. 1 4. Cannot be determined

12. A and B start running simultaneously from the same point on a circular track with speeds 5 m/s and 15 m/s respectively. If they run in opposite directions, find the ratio of distances covered by them till their first meeting.
1. 1: 3 2. 2: 5 3. 2: 3 4. 1: 2

13. Two persons start cycling simultaneously from a point P on a circular track. If they travel at the speeds of 3 km/hr and 10 km/hr respectively, then what is the ratio of number of distinct points that they would meet on that track then when they travel in the same direction first to that when they travel in the opposite direction?
1. 7: 13 2. 3: 10 3. 3: 7 4. 30: 13

Directions for questions 14 and 15: Answer the questions on the basis of the information given below.
Chiku and Charu are running along a circular path with a diameter of 1050 m at the speeds of 15 m/s and 25 m/s respectively. Chaman is running along the path AB (diameter of the circular path) at the speed of 35 m/s. Chaman turns back immediately after reaching the point B and does the same as he reaches the point A. All of them start from point A simultaneously.

14. If Charu and Chiku are running in the opposite direction, what is the shortest distance (in metres) along the circular path between point A and the point at which Charu and Chiku meet for the third time?
1. 247.5 2. 412.5 3. 660 4. 1320

15. If Charu and Chiku are running in the opposite direction, what is the shortest distance (in metres) between Charu and Chiku along the circular path when Chaman reaches the point B for the fifth time?
1. 150 2. 270 3. 412.5 4. 900

16. In a 4000-meter race around a circular stadium having a circumference of 1000 meters, the fastest runner and the slowest runner reach the same point at the end of the 5^{th} minute, for the first time after the start of the race. All runners have the same starting point and each runner maintains a uniform speed throughout the race. If fastest runner runs at twice the speed of slowest runner, what is the time taken (in min) by fastest runner to finish the race?
1. 10 2. 12 3. 15 4. 8

17. Two boys start simultaneously from the same point on a circular track and run in opposite directions at speeds of 2.25 km/hr and 3.5 km/hr respectively. If circumference of the track is 736 m, after how much time they meet for the first time?
1. 7.5 min 2. 7.68 min 3. 8 min 4. 6.86 min

18. A sprinter starts running on a circular path of radius r metres. Her average speed (in metres/minute) is πr during the first 30 seconds, πr/2 during next one minute, πr/4 during next 2 minutes, πr/8 during next 4 minutes and so on. What is the ratio of the time taken for the nth round to that for the $(n-1)^{th}$ round (n > 1)?
1. 4: 1 2. 8: 1 3. 16: 1 4. 32: 1

19. In a 100 m race, Akbar gives Birbal a start of 2 seconds. Birbal covers 10 m by the time Akbar starts. If both of them finish together, find Akbar's speed (in m/sec).
1. 5 2. 5 5/9 3. 4 1/2 4. 4

20. Peter and Paul are running in opposite directions along a circular track of 600 m with initial speeds of 3 m/s and 6 m/s. They start at the same point and at the same time. Whenever they meet, they exchange their speeds and carry on in their

respective directions. Find the distance (in m) between Peter and Paul (when measured along the track) when Peter completes 4 1/4 rounds.
1. 120 2. 150 3. 140 4. 160

21. Two runners run in the same direction along a circular track 3 km in length. The faster runner overtakes the slower every 1 hour. Find the speed of the slower runner, if the faster one completes one length of the track 2 minutes sooner than the other.
1. 20 kmph 2. 18 kmph 3. 15 kmph 4. 12 kmph

22. Prakash and Pramod are running along a circular track having started at the same time from the same point, in the same direction. How much more distance (in m) would Prakash have travelled compared to Pramod by the time they meet for the 11th time given that the radius of the track is 7 m, Prakash and Pramod run at 22 m/s and 11 m/s respectively?
1. 456 2. 490 3. 484 4. 478

23. P and Q started moving simultaneously from points X and Y respectively on a circular track in the same direction. XY = 2 km and the length of the track is 6 km. The initial position of P was behind that of Q. P caught up with Q for the second time in 40 minutes. Had P started in the opposite direction, they would have met for the first time in 12 minutes. Find the speed of P (in km/hr).
1. 16 2. 14 3. 6 4. 4

24. In a race of 500 m, L beats M by 40 seconds and beats N by 125 m. If M and N run a 500 m race, M beats N by 40 seconds. Find the time taken (in seconds) by M to run the race.
1. 160 2. 240 3. 280 4. 320

25. Rahim and Saleem are competing in a 1000 m race. Rahim gives Saleem a head start of 200 m, but his speed is twice that of Saleem. When Rahim reaches the 600 m mark, Rahim reduces his speed by half but still beats Saleem by 20 s. What is Saleem's speed?
1. 14 m/s 2. 5 m/s 3. 3.5 m/s 4. 7 m/s

26. In a 100 m race, Ajay gives Bala a start of x meter ($x \geq 20$) and is beaten by him by y meter ($y \leq 20$). If the speeds of both are distinct, which of the following cannot be the ratio of the speeds of Ajay and Bala?
1. 6: 5 2. 4: 3 3. 7: 5 4. 9: 10

27. In a 200 m race, Raja gives Rakesh a start of 20 m and beats him by 20 m. If Raja beats Rakesh by 4 seconds, find Raja's speed (in m/sec).
1. 6 2. 6.25 3. 6.5 4. 6.75

28. In a kilometre race, Ram beats Shyam by one minute and Shyam beats Tarun by 30 seconds. If Ram beats Tarun by 250 m in the same race, find the time taken by Ram to run the race (in seconds).
1. 180 2. 270 3. 360 4. 330

29. Girish takes 1 minute to complete a round around a circular track. Harish is twice as fast as Girish, Suresh is thrice as fast as Harish. All three start at the same point. Find the time taken by them to meet at the starting point for the first time (in minutes).
1. 1 2. 1.5 3. 2 4. 3

30. Tony and Harry begin to run in opposite directions on a circular path of radius 35 m at 20 m/sec and 11 m/sec respectively from the same point. What is the time taken by them to meet for the third time at the starting point?
1. 11 min 2. 220 sec 3. 8 min 40sec 4. 325 sec

31. P and Q start cycling simultaneously around a circular track 48000 m long with speeds of 10 m/sec and 30 m/sec respectively in opposite directions. After every crossing, P's speed increases by 10 m/sec and Q's speed decreases by 10 m/sec. Find the time taken by them to cross for the third time (in seconds).

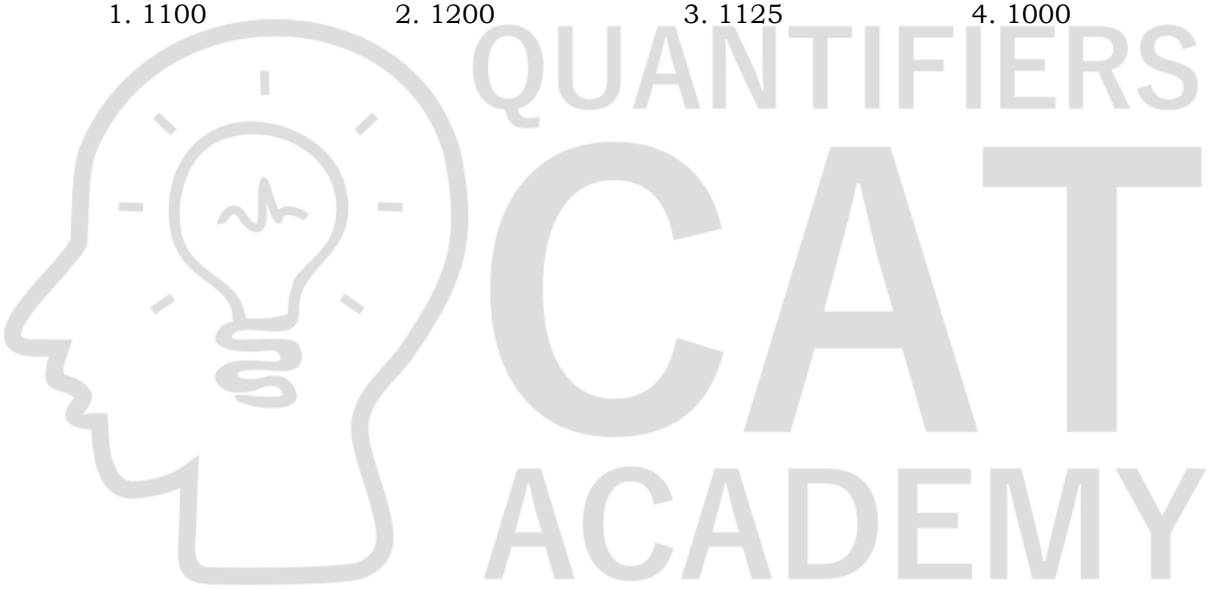
1. 3600 2. 7200 3. 4800 4. 9600

32. Ram and Shyam started simultaneously from the same point on a circular track 1800 m long in opposite directions with speeds of 6 m/sec and 12 m/sec respectively. After every crossing, they exchanged their speeds and continued to travel in the same direction. Find the distance (in m) between them (measured along the track) when Ram completed 3 1/4 rounds.

1. 400 2. 375 3. 425 4. 450

33. P and Q are two points on a 1 km long circular track. The distance PQ, along the track is 200 m. Rohan started running from P. Sohan started running simultaneously in the same direction from Q. Both reached P for the first time simultaneously. If both started simultaneously from P in opposite directions, they would take 111 1/9 seconds to meet for the first time. Find the time taken by them to meet for the first time (in seconds), if they started simultaneously from Q in the same direction.

1. 1100 2. 1200 3. 1125 4. 1000



Solutions

Exercise 1

1. Let a – b = x, b – c = 2x and c – d = 3x

∴ c = 3x + d

b = 2x + c = 5x + d, a = x + b = 6x + d

Hence, $\frac{a+d}{c} = \frac{6x+d+d}{3x+d} = \frac{2}{1}$

2. Here a/b = 2/3 ⇒ a/2 = b/3, b/c = 3/5 ⇒ b/3 = c/5, c/d = 5/6 ⇒ c/5 = d/6

So, a/2 = b/3 = c/5 = d/6 = k (say)

⇒ a = 2k, b = 3k, c = 5k and d = 6k

So, $\frac{3a+2b+4c+d}{3d+4c} = \frac{3\times 2k+2\times 3k+4\times 5k+6k}{3\times 6k+4\times 5k} = \frac{38}{38} = 1.$

3. Let $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a-3b+5c}{k} = p$

⇒ a = 2p, b = 3p, c = 4p. So, $\frac{2\times 2p-3\times 3p+5\times 4p}{k} = p$

⇒ 15p/k = p ⇒ k = 15.

4. Let income of X be Rs. p and income of Y be Rs. 3p Let expenses of X be Rs. 19q and expenses of Y be Rs. 40q. Hence, savings of X = Rs. (p – 19q) and savings of Y = Rs. (3p – 40q)

Total savings = (4p – 59q) = 36020

Difference of savings = (2p – 21q) = 18860

Solving the above equations, we get p = 10480, Thus, income of X and Y is Rs. 10,480 and Rs. 31,440 respectively.

5. Let the number of one-rupee coins, 50 paise coins and 25 paise coins be 3x, 4x and 12x respectively. Value of one-rupee coins = 3x, 50 paise coins = 4x/2 = 2x and 25 paise coins = 12x/4 = 3x.

Thus, 3x + 2x + 3x = 600 ⇒ x = 75

Therefore, value of 25 paise coins = (12 × 75)/4 = Rs 225.

6. Ratio of amounts collected from first and second classes = (3 × 1) : (1 × 27) = 1 : 9

So, amount collected from first class passenger is = = 1/10 × 2700 = Rs 270

7. Let present ages (in years) of mother and daughter be 9x and 4x respectively.

So, $\frac{9x-10}{4x-10} = \frac{7}{2}$, ⇒ x = 5. Hence, present ages of mother and daughter are 45 years and 20 years respectively.

8. On the basis of the given information, we can conclude that ratio of present ages of Chinta, Mani and Suknaya is 14: 19: 10. Let the ages (in years) of Chinta, Mani and Sukanya be '14x', '19x' and '10x' respectively.

⇒ $\frac{14x+19x+10x}{3} = 43$ ⇒ x = 3. ∴ Age of Mani = 19x = 19 × 3 = 57 years.

9. Let the numbr of chocolate with Sheela, Munni and Chameli be 's', 'm' and 'c' respectively. s = c + 5 and s = m + 3 ⇒ m – c = 2 ⇒ 3c/2 – c = 2 (As m: c = 3: 2)

⇒ c = 4 ∴ s = 9 and m = 6. ∴ m + c + s = 6 + 4 + 9 = 19.

10. The required ratio = (4 × 1.1): (3 × 1.2) = 11: 9.

11. Each person receives = 8/3 = 2.67 apples

The ratio of the amount received by A and B from C = $\frac{5-2.67}{3-2.67} = 7:1$

Hence, the respective share of A and B from the amount of C is Rs. 70 and 10 respectively.

12. Let his marks be 5x, 4x, 6x, 8x and 7x respectively. Aggregate marks obtained = 30x = 50% of aggregate maximum marks. Thus, aggregate maximum marks = 60x.

⇒ Maximum marks per subject = 12x. 55% of maximum marks per subject = 6.6x. Hence in two subjects, marks are more than 6.6x.

13. Ratio of four numbers = 1^{st}: 2^{nd}: 3^{rd}: 4^{th} = 40: 60: 72: 81

Therefore, sum = 40 + 60 + 72 + 81 = 253.

Hence, average of 2nd and 3rd number is 66.

14. Let a, b and c be the cost-prices of three brands of juices per litre.

Therefore, $\frac{3a+4b+5c}{12} = 20 \Rightarrow 3a + 4b + 5c = 240$.

And $\frac{4a+5b+6c}{15} = 25 \Rightarrow 4a + 5b + 6c = 375$

Hence, $\frac{6a+7b+8c}{21} = \frac{3(4a+5b+6c) - 2(3a+4b+5c)}{21} = \frac{3(375)-2(240)}{21} = Rs\ 30.7$

15. Let initial value be v_1. Here $v_1 = k \times w^2$, where k is a proportionality constant, and w is the weight.

According to the question, $w = 4 \Rightarrow v_1 = 16k$

Suppose the diamond breaks into two pieces of weights x kg and (4 – x) kg, and let their values be v_2 and v_3 respectively.

Hence, $v_2 = kx^2$ and $v_3 = k \times (4-x)^2 = k(x^2 + 16 - 8x)$

Also, $(v_2 + v_3)$ = (62.5) % of $v_1 \Rightarrow 2k(x^2 - 4x + 8) = 5/8 \times 16k \Rightarrow x = 1$ or 3

Therefore, weight of the parts are 1 kg and 3 kg.

16. Let the weights (in kg) of the father, the son and the mother be x, y and z respectively. According to the question, x: y = 3: 2 and y: z = 5: 6

Therefore, x: y: z = 15: 10: 12

Weight of the father = 2 × 35 = 70 kg. Weight of the mother = 12/15 × 70 = 56 kg

17. Let the incomes (in Rs.) of Aishwarya, Babita and Charu be 7x, 9x and 12x and their expenditures (in Rs.) be 8y, 9y and 15y respectively. Then, 7x – 8y = 7x/4

⇒ 21x/4 = 8y ⇒ x = 32y/21.

So, their respective incomes (in Rs.) are 32y/3, 96y/7 and 128y/7 and respective savings (in Rs.) will be 8y/3, 33y/7 and 23y/7.

Hence, the required ratio is 56: 99: 69.

18. Assume the last year Harsha's salary and Mandar's salary were 3x and 5x respectively. Then their salaries in this year are 4.5 x and 6.25 x. So, the ratio of their salaries in this year is 4.5 x: 6.25 x, i.e., 18: 25 respectively.

∴Present salary of Harsha = 86000 × 18/43 = Rs 36000.

19. Let the number of students in Batch A in January be 2x. Let number of students in Batch A in February be 5y. Total numbers of students in Batch A and Batch B combined in January and February are 5x and 13y respectively. Therefore 13y: 5x = 26: 5 or y: x = 2: 1 or y = 2x. From here, we can compile the following table:

	Batch A	**Batch B**
Number of students in January	2x	3x
Number of students in February	10x	16x
Rate of increase from Jan to Feb	400%	1300/3 %
Rate of increase from Feb to March	800%	1300%
Number of students in March	90x	224x

Hence, the required Ratio = 45: 112.

20. Suppose the person buys A apples and M mangoes and cost price of an apple is Rs. x. Therefore, cost price of a mango will be 2x. Total cost price = Ax + 2Mx.

Now selling price of an apple is 2x. ∴ SP of a mango will be 6x.

Total SP = 2Ax + 6Mx. Now, we have 2Ax + 6Mx = 5/2 (Ax + 2Mx) or M/A = 1/2.

21. Let x be the fixed cost and y the variable cost. Then,

17500 = x + 25y … (i)

30000 = x + 50y … (ii)

Solving the equation (i) and (ii), we get x = 5000, y = 500

Now if the average expense of 100 boarders be 'A', then 100 × A = 5000 + 500 × 100

⇒ A = Rs. 550.

22. Here, $A \propto B$; $B \propto 1/C$. $C \propto D^2$; $D \propto E^{1/3}$

⇒ $A = k_1B = k_2/C = k_3/D^2 = k_4/E^{2/3}$. So as A increases, E decreases.

23. $P \propto T$ and $P \propto 1/V$, combining, we get $P \propto T/V$ or $P = kT/V$

⇒ $PV/T = k$

So, we have $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = k$

or $\frac{150 \times 20}{100} = \frac{150 \times 8}{T_2} \Rightarrow T_2 = 40°$ kelvin

24. Profit = Rs. 8,0000 × 40/100 = Rs 32000

Ratio of shares of A, B and C in the profit = Ratio of amount invested

B's share in the profit = Rs. 32000 × 6/16 = Rs 12000

C's share in the profit = Rs. 32000 × 7/16 = Rs 14000

Sum of the shares of B and C in the profit = Rs. 12,000 + Rs. 14,000 = Rs. 26,000

25. Ratio of profits is A: B: C = 2000: 5000: 4000 = 2: 5: 4

Profit share of A = 2/11 × 1210 = Rs. 220

Profit share of B = 5/11 × 1210 = Rs. 550

Profit share of C = 4/11 × 1210 = Rs. 440

26. Profit received by Z = Rs. {1500 – (450 + 550)} = Rs. 500

Ratio in capital = X: Y: Z = 450: 550: 500 = 9: 11: 10

∴ Z's capital = 10/30 × 3300 = Rs. 1,100

27. Ratio of investments is A: B: C = 1/2: 1/3: 1/4 = 6: 4: 3

Ratio of time period is t_A: t_B: t_c = 1: 1/2: 1/4 = 4: 2: 1

∴ Ratio of profit shared is At_A: Bt_B: Ct_C = 24: 8: 3

As B's share is 1/3rd of A's share. hence, share of B = 1/3 × 30000 = Rs 10000

28. Ratio of profit sharing = 4500 × 12: 5400 × n, where n is the period for which B invested. So, ratio = 4500 × 12: 5400 × n = 10: n. Now, this ratio is equal to 2: 1.

∴ 10: n = 2: 1 ⇒ n = 5 months. Thus, B joined A after 12 – 5 = 7 months

29. The ratio in which profit will be shared among Ajay, Boman and Chirag

= 28000 × 12: 40000 × 7: 560000 × 5 = 6: 5: 5.

Hence, share of Boman = 5/16 × 32000 = Rs. 10,000.

30. In terms of Rupee-Month for the entire year,

(i) Abhishek's Investment = (100 × 3) + (75 × 3) + (50 × 6) Lakhs R-month

= 825 lakhs R-month

(ii) Rani's Investment = (200 × 3) + (150 × 3) + (100 × 6) = 1650 Lakhs R month

(iii) Susmita's Investment = 50 × 6 = 300 Lakhs R-month

(iv) Randeep's Investment = 100 × 3 = 300 Lakhs R-month

∴Their ratio of Investment = 11: 22: 4: 4

∴Abhishek and Rani's combined share of profit = (11 + 22)/41 × 41 = Rs 33 lakh.

QUANTIFIERS CAT ACADEMY

Exercise 2

1. Let the ratio of the number of boys and girls in the class be a: b. Number of boys and girls will be ak and bk.

Given ak + bk = 70 ⇒ k = 70/(a + b)

Since k is an integer a + b must be a factor of 70; from the options.

The sum of the ratio in options (A), (B), (C) are factors of 70. But the sum of ratio 9: 2 is not a factor of 70. So, 9: 2 is not a possible ratio.

2. Let the numbers of students in A, B and C be 3x, 7x and 8x respectively.

If 10 students leave C and join B, C and B would have 8x - 10 students and 7x + 10 students respectively.

⇒ 7x + 10 = 80 ⇒ x = 10. Total number of students = 18x = 180.

3. Let the incomes of A and B be Rs 4x and Rs 3x. Their savings be Rs 5y, Rs 6y

Their expenditures are Rs (4x – 5y) and Rs (3x – 6y)

⇒ 4x – 5y = 3/4 (4x) ⇒ x = 5y

Therefore, the expenditure of A = 4x – 5y = 4(5y) – 5y = 15y

And the expenditure of B = 3x – 6y = 3(5y) – 6y = 9y

Hence, the ratio of the expenditure of A and B is = 15y: 9y = 5: 3.

4. If $\frac{s_1}{s_2} < \frac{e_1}{e_2}$ *then* $\frac{s_1}{s_2} < \frac{s_1 + e_1}{s_2 + e_2} < \frac{e_1}{e_2}$

In the given problem, $\frac{s_1 + e_1}{s_2 + e_2} = \frac{3}{4}$ and $\frac{e_1}{e_2} = \frac{4}{5}$, therefore, $\frac{s_1}{s_2} < \frac{3}{4}$.

Only, $\frac{s_1}{s_2} = \frac{13}{20}$ satisfies this condition.

5. Given that $\frac{q+r}{p} = \frac{p+r}{q} = \frac{p+q}{r} = k$

⇒ q + r = pk, p + r = qk and p + q = rk

⇒ q + r + p + r + p + q = k(p + q + r)

⇒ (p + q + r) (2 - k) = 0

∵ p + q + r ≠ 0, ⇒ 2 - k = 0 ⇒ k = 2

6. We have, Area α (Diagonal)^2

⇒ A = kD^2, where k is the constant of proportionality.

Now, D = 8, A = 32, therefore, 32 = k × 8^2 ⇒ k = 1/2

So, we have A = D^2/2

Now, when A= 72, then, 72 = D^2/2 ⇒ D^2 = 144 ⇒ D = 12 sq. units.

7. From the given information, A varies directly with the product of B and C.

⇒ $\frac{A_1}{A_2} = \frac{B_1C_1}{B_2C_2}$

Given that, A_1 = 6000, B_1 = 20, C_2 = 30, B_2 = 40 and C_2 = 60

⇒ $A_2 = \frac{A_1B_2C_2}{B_1C_1} = 24000.$

8. Let a, b and c be the weights of the Bimal, Basu and Bali respectively.

a/b = 2/3 = 8/12; c/b = 3/4 = 9/12; a: b: c = 8: 12: 9

Given the sum of the weights (8 + 12 + 9 = 29 parts) is 203 kg i.e., one part is 7 kg. So, Basu's weight i.e., 12 parts is 12 x 7 = 84 kg.

9. Two numbers are in the ratio 4: 7. Let the numbers be 4k and 7k. Let the number added to each be L.

Given that, (4k + L) + (7k + L) = 75

⇒ 11k + 2L = 75 --------- (1)

It is also given that, $\frac{4K + L}{7K + L} = \frac{8}{17}$

⇒12k + 9L = 0 ---------- (2)

⇒ k = - 3L /4. Substituting the value of k in (1) we get L = –12.

10. Let the amounts initially with Mohan and Sohan be 8x and 5x respectively. Let us say Mohan spends y each day.

We have, $\frac{8x - 9y}{5x + 9y/6} = \frac{10}{11}$ ⇒ x = 3y.

Let us say Mohan and Sohan have amounts which are in the ratio 18: 35 after t more days. So, $\frac{8x - ty}{5x + ty/6} = \frac{18}{35}$ $i.e.\frac{8(3y) - ty}{5(3y) + ty/6} = \frac{18}{35}$ ⇒ t = 15. The ratio of the amounts with them would be 18: 35 after 6 more days.

11. Ratio of prices = 5: 8: 13

Ratio of number of balls = 5: 4: 3

Ratio of amounts spent = (5 × 5): (8 × 4): (13 × 3) = 25: 32: 39

Total number of parts of the ratio=25 + 32 + 39 = 96

Total amount spent, as per data = Rs 768

Value of each part of the amounts' ratio=768/96 = 8

Amount spent on costliest variety = Value of 39 parts = 39 × 8

Number of costliest variety balls Paul purchased is = (39 × 8)/13 = 24

12. Let the amount given to the first brother is Rs x. The second will get 2x + 2, the third will get 3x + 3, the fourth will get 4x + 4 and so on, the ninth brother will get 9x + 9.

Given that (9x + 9)/x = 10 ⇒ x = 9. So, the 5th brother will get 5x + 5 = 5 × 9 + 5 = Rs 50.

13. David received 9/17th of the number of toffees distributed by Alok. Amitha received 1/16th of the number of toffees distributed by David.

⇒ $\frac{1}{16}\left(\frac{9T}{17}\right) = 18$ ⇒ T = 544.

14. Quantity of diesel purchased = x litre

So, the cost per litre of diesel = 510/x

⇒ Cost per litre of kerosene = $\frac{510 \times 3}{x \times 5} = \frac{306}{x}$

Since equal volumes are purchased, amount spent on kerosene = $\frac{306}{x} \times x = Rs\ 306$.

15. Amount spent on kerosene = x

Amount spent on petrol = 5x

⇒ Cost involved = 6x ---------- (1)

Ratio of prices per litre of Petrol, diesel and kerosene is 15: 5: 3.

When equal volumes of all are purchased, the amounts are also in the ratio 15: 5: 3.

Amount spent on diesel, in this case = Rs 510

Total amount spent = (23 × 510)/5 = 23 × 102----- (2)

From (1) and (2), 6x = 2346 ⇒ x = 391

Therefore, the amount spent on petrol = 391 × 5 = Rs 1955.

16. In the two cases, the greatest parts are 15N/42 and 12N/42. Their difference is 3N/42. Given that 3N/42 = 6 ⇒ N = 84.

17. Let the two numbers be "a" and "b" ⇒ $\sqrt{ab} = 9$

Squaring on both sides, we get

ab = 81 ——— (1)

$b^2 = 243a$ --- (2)

multiplying both the sides with b, we get, $b^3 = 243 \times 81$

⇒ $b = (3^5 \times 3^4)^{1/3} = 3^3 = 27$ ⇒ a = 3. The larger of the two numbers = 27.

18. We have, V α r^2h ⇒ V = $k.r^2h$ ⇒ $V_1/(r^2_1\ h_1) = V_2/(r_2^2\ h_2) = k$

Hence, $66/(9 \times 7) = 308/(6r^2)$ ⇒ r = 7.

19. Let the fixed income and the royalty that he gets be denoted by F and R respectively.

Total income (T) = F + kB (As R α B).

So, 46000 = F + 2000k

66000 = F + 3000k

Solving these, k = 20 & F = 6000

Total income on setting 6000 books = F + 6000k = 6000 + 6000 × 20

Income per book = (6000 + 6000 × 20)/6000 = 21.

20. Here v = $40 - k\sqrt[3]{n}$, where v is the speed when n wagons are attached.

If n = 27, v = 34, we have $34 = 40 - k\sqrt[3]{27}$

⇒ 34 = 40 – 3k ⇒ k = 2

Therefore, v = $40 - 2\sqrt[3]{n}$

Minimum speed required = 30

⇒ $30 = 40 - 2\sqrt[3]{n}$ ⇒ $5 = \sqrt[3]{n}$ ⇒ n = 125.

So, a maximum of 125 wagons can be attached.

21. Let the initial weight of the stone (before breaking) be 6 w.

Weights of the broken stones are w, 2w and 3w respectively.

Initial value of the stone = Rs 10, 872

⇒ 10,872 α $36w^2$

⇒ $10,872 = 36w^2k$

Total value of the pieces of the stone = $(1^2 + 2^2 + 3^2)\ w^2k = 14\ w^2k$

Loss in the value = $22w^2k = 22 \times 10872/36$ = Rs 6644.

22. Fixed charge = x, unit rate = y, x and y are in Rupees.

5/4(x + 100y) = x + 200y

5x + 500y = 4x + 800y -------- (1)

x + 50y = 700 ----------(2) (given) ⇒ y = 2, x = 600

23. Let the numbers of sweets received by Ram, Shyam and Tarun be r, s and t.

Given that r: s = 5:4 and s: t = 2: 3 ⇒ r : s : t = 5 : 4 : 6 ⇒ t = 6 × 60/15 = 24.

24. Let the quantities of milk be 3xml, 4x ml, 5x ml and 6x ml and 7x ml.

Let the capacity of each vessel be 100 ml.

Total capacity = 500 ml

⇒ 3x + 4x + 5x + 6x + 7x = 3 × 500/5 ⇒ x = 12.

So, 3x = 36, 4x = 48, 5x = 60, 6x = 72, 7x = 84. The last 3 vessels contain at least 50% milk.

25. Manoj's expenditure is 2/3rds of his income. Let Manoj's monthly income be Rs 36. (An arbitrary number is chosen in such a way that no fractions are encountered in the calculation)

Hence, Manoj's expenditure is 36 x (2/3) = Rs 24 ---- (1)

Hence, Manoj's savings is 36 - 24 = Rs 12

Ratio of savings of Manoj and Shiva = 2: 3

Hence, Shiva's savings = 3 x (12/2) = Rs 18 ------- (2)

Income Ratio = 3: 4

As Manoj's income = Rs 36, Shiva's shall be 4 x (36/3) = Rs 48---------- (3)

Shiva's expenditure = Income – Savings = 48 - 18 = Rs 30 --------- (4)

From (1) and (4); Ratio of expenditures of Manoj and Shiva is 24: 30 = 4: 5.

26. We have.

	Larger Plot	Smaller Plot	Combined
Area M	8x	29y – 8x	29y
Area B	9x	33y – 9x	33y

We have, $\frac{29y - 8x}{33y - 9x} = \frac{13}{15}$ ⇒ x = 2y.

Ratio of the area under maize cultivation in the larger plot and that under barley cultivation in the smaller plot = $\frac{8x}{33y - 9x} = \frac{16y}{33y - 18y} = \frac{16}{15}$.

27. Let Ahmed's age = x, Ahmed's brother's age = a

Let Mohammed's age = y, Mohammed's brother's age = b

x/y = a/b ⇒ $\frac{x + y}{x - y} = \frac{a + b}{a - b}$ $(using\ componendo\ and\ dividendo)$

⇒ $\frac{x + y}{a + b} = \frac{x - y}{a - b} = \frac{1}{2} \Rightarrow \frac{a + b}{x + y} = 2.$

28. Let the total annual profit be Rs x

Ratio of the part of the total annual profit which is shared in the ratio of the (investments × time) = 20000 × 12: 40000 × 8 = 3: 4.

Hence the share of Sachin = $\frac{3}{7}\left(\frac{84x}{100}\right) + \frac{16x}{100} = 39000$ ⇒ x = 75000.

Therefore, Annual profit share of Sunil = Rs 75000 – Rs 39000 = Rs 36000.

29. Let us say C joined after x months.

Profit is shared in the ratio 20000 ×12: 40000 × 9: 100000 × 12 – x

= 24: 36: 10 (12 - x)

Given that $\frac{36}{180-10x} = \frac{18}{55} = \frac{36}{110} \Rightarrow 180 - 10x = 110 \Rightarrow x = 7$.

30. Ratio of investment of Serena and Venus is 50000 × 12: (50000 × 3 + 40000 × 9) or 20: 17.

Share of Venus in the total profit = $\frac{17}{37} \times 100 = 45\frac{35}{37}\%$.

31. Investment Ratio of Mehta, Mehra, and Mihir = 1: 2: 3

Ratio of time periods of Mehta, Mehra and Mihir = 12: 8: 4 = 3: 2: 1

Profit would be distributed in the ratio of (Amount Invested × Time)

Here the ratio = 3: 4: 3.

Mehta gets 3 × 600000/10 = 180000.

32. Let Goyal's salary be C and the total profit be p.

⇒ C = 0×2 (p - C) ⇒ C = p/6.

Hence 5p/6 is shared in the ratio of their capitals i.e., 20,000: 30,000 or 2: 3 and Goyal's share will be $\frac{2}{5} \times \frac{5p}{6} = \frac{p}{3}$.

Now, $\frac{p}{6} + \frac{p}{3} = 4800 \Rightarrow$ p = Rs 9600

33. Let the profit be p. Hence half the profit i.e., p/2 will be shared equally i.e., p/4, p/4 and remaining p/2 will be shared in the ratio 24: 36 or 2: 3

So, Gayathri will get $\frac{3}{5} \times \frac{p}{2} = \frac{3p}{10}$.

Total earnings of Gayathri = $\frac{3p}{10} + \frac{p}{4} = \frac{11p}{20}$ .....(i)

Had the entire profit been shared in the ratio of their capitals, Gayathri would have got 3p/5.....(ii)

Given that $\frac{3p}{5} - \frac{11p}{20} = 2500 \Rightarrow$ p = Rs 50000.

34. Ratio of the profit shares of Gopal, Hari and Karthik = 8000 ×12: 12000x: 16000x = 96: 12x: 16x

Now given that 16x > 96 > 12x ⇒ x > 6 and x < 8

As x is an integer, so, x = 7. Therefore, the required ratio = 24: 21: 28.

35. Let the investments of P and Q be `p and `q respectively.

Salary of Q = Rs 10000.

Now, $\frac{p}{p+q} \times 50000 = \frac{p}{p+q} \times 40000 + 8000$

⇒ p: q = 4: 1. So, Profit share of p = $\frac{4}{5} \times 40000$ = Rs 32000.

36. Let the investments of A, B, C and D be Rs 3x, Rs 4x, Rs 5x and Rs 6x respectively. Let the salary of B be Rs S.

Given, $\frac{4x+S}{5x+S} = \frac{9}{10} \Rightarrow$ S = 5x

Total amount of profit = 18x + 2S = 18x + 10x = 28x.

⇒ 28x = 84000 ⇒ x = 3000. Therefore, B's salary is 5x i.e., Rs 15000.

Exercise 3

1. If $\frac{a}{b} = \frac{c}{d} = k$ then $a = bk$, $c = dk$ and if $\frac{e}{f} = \frac{g}{h} = n$ then. $e = fn$, $g = hn$

$\frac{ae+bf}{ae-bf} = \frac{bk \times fn+bf}{bk \times fn-bf} = \frac{kn+1}{kn-1}$ and $\frac{cg+dh}{cg-dh} = \frac{dk \times hn+dh}{dk \times hn-dh} = \frac{kn+1}{kn-1}$

Thus, $\frac{ae+bf}{ae-bf} = \frac{cg+dh}{cg-dh}$

Alternately,

Since $\frac{a}{b} = \frac{c}{d}$ and $\frac{e}{f} = \frac{g}{h}$, multiplying them, $\frac{ae}{bf} = \frac{cg}{dh}$

By componendo and dividendo, $\frac{ae+bf}{ae-bf} = \frac{cg+dh}{cg-dh}$

2. We have, $x^2 \propto y^3 \Rightarrow \frac{x^2}{y^3}$ = constant

$\Rightarrow \frac{{x_1}^2}{{y_1}^3} = \frac{{x_2}^2}{{y_2}^3} \Rightarrow \frac{3^2}{4^3} = \frac{\left(\frac{1}{\sqrt{3}}\right)}{{y_2}^3}$ i.e., ${y_2}^3 = \frac{4^3}{27}$ i.e., $y_2 = \frac{4}{3}$

3. Adding all the numerators and all the denominator, i.e. using law of equal ratios,

$\frac{y}{x-z} = \frac{y+x}{z} = \frac{x}{y} = \frac{2x+2y}{x+y}$ i.e., $\frac{2}{1}$.

Using the last two ratios, we have $\frac{x}{y} = \frac{2}{1}$. Thus, taking $y = k$ and $x = 2k$ and substituting in $\frac{y+x}{z} = \frac{2}{1}$, we get $z = \frac{3}{2}k$. So the required ratio is 2: 1: $\frac{3}{2}$ i.e., 4: 2: 3.

4. $A - 25 = k$, $B - 25 = 3k$ and $C - 25 = 2k$. Adding the three, $A + B + C - 75 = 6k$ i.e., $k = 110$. Thus $C = 2k + 25 = 195$.

5. Here, $\frac{F}{2} = 2k, \frac{S}{3} = 3k, \frac{T}{6} = 6k$ i.e., $F = 4k$, $S = 9k$, $T = 36k$

i.e., $4k + 9k + 36k = 5390$ i.e., $k = 110$. Thus, $F = 440$, $S = 990$, $T = 3960$.

6. Let A, B, C, and D get Rs. a, b, c and d respectively.

A and D together get as much as B and C get. Since the total amount distributed is Rs. 500, $a + d = b + c$ = Rs. 250. Trying to write all variables in terms of d, we have $a = 250 - d$.

C gets as much as A & B together get. After distributing Rs. d to D, we are left with Rs. $500 - d$ and this has to be distributed in the ratio 1: 1 between C and (A & B). Thus, $c = \frac{500-d}{2}$.

D gets as much as A and C together get i.e., $d = (250 - d) + \frac{500-d}{2}$. Solving we get $4d - 500 = 500 - d$ i.e., $d = 200$. Thus, $c = \frac{500-200}{2} = 150$.

7. Since the ratio of C's share and the sum of A's and B's share is 1: 2, C gets $\frac{1}{3} \times 3000 = 1000$. Since, the ratio of A's share and the sum of B's share and C's share is 2: 3, A gets $\frac{2}{5} \times 3000 = 1200$. Thus, B's share = 3000 – 1200 – 1000 = 800.

8. Let number of boys and girls be $3a$ and $5a$ respectively. And let the amount received by each boy and girl be b and $2b$ respectively. Thus, $\frac{3a \times b+5a \times \mathbf{2b}}{3a+5a} = 27.5 \Rightarrow \frac{13b}{8} = 27.5$. Thus, b can be found out but there is no way of finding a. Hence only II can be accurately determined.

9. Let the income be $3x$ and $2x$ respectively and the expenditure be $5y$ and $3y$ respectively. Thus, we have two equations and two variables…

$3x - 5y = 1000$ and $2x - 3y = 1000$. Solving we get $x = 2000$ and $y = 1000$. Thus, income of A is $3x$ i.e., Rs. 6000.

10. Denoting number of men, women and children by m, w and c, we have $m: w = 4: 3$ and $c: m$ is 5: 3. Since m is common to the two relations, making m have equal values in the two ratios we have $m: w: c = 12: 9: 20$. Since women are 81,180, we have $9k = 81{,}180$ i.e., $k = 9{,}020$. So, the total number of children will be $20k$ i.e., 180,400.

11. Let the number of skilled, semi–skilled and unskilled workers be $5a$, $2a$ and $3a$ respectively and let the wages of each skilled, semi–skilled and unskilled worker be $8b$, $5b$, b respectively. Thus, we have $5a \times 8b + 2a \times 5b + 3a \times b = 3975$ i.e., $53ab = 3975$ i.e., $ab = 75$.

Now we know that both a and b has to be a natural value. Further $10a$ should be a value between 20 and. 50. This is possible only if a is a value between 2 and 5. Thus, a can only be 3 or 4 out of which only $a = 3$ results in b being a natural value, $b = 25$. Thus, wages paid to each skilled, semi–skilled and unskilled worker is 200, 125 and 25 respectively.

12. The profit has to be divided in the ratio $(16 \times 3 + 11 \times 9): (12 \times 3 + 17 \times 9): (21 \times 6)$ i.e., 147: 189: 126 i.e. 7: 9: 6. Thus $7k + 9k + 6k = 26{,}400$ and we have to find $9k - 6k$. Since $22k = 26400$, $3k = \frac{3}{22} \times 26400 = 3 \times 1200 = 3600$.

13. Comparing speed of cat and dog:

In terms of distance, 3 leaps of dog = 4 leaps of cat. Thus, distance covered per leap by dog and cat is 4: 3.

Next, in same time, cat takes 5 leaps and dog takes 4 leaps. Thus, in same time, distance covered by cat and dog is $5 \times 3 = 15$ and $4 \times 4 = 16$. Since time is same, the ratio of their speeds of cat and dog will also be 15: 16

Comparing speed of dog and fox:

In terms of distance, 2 leaps of fox = 3 leaps of dog. Thus, distance covered per leap by fox and dog is 3: 2.

Next, in same time, fox takes 3 leaps and dog takes 5 leaps. Thus, in same time, distance covered by fox and dog is $3 \times 3 = 9$ and $5 \times 2 = 10$. Since time is same, the ratio of their speeds of fox and dog will also be 9: 10

Combining the two ratios, since dog is common, making the speed of dog equal to LCM of 16 and 10 i.e., 80, the ratio of speed of cat and fox will be 75: 72 i.e., 25: 24.

14. The value of the 4 gm diamond is $16k$. Since its value reduces by 37.5%, i.e. $3/8^{th}$, the reduced value $16k - 6k = 10k$.

If the weights of the two pieces are x and $(4 - x)$ gm, their combined value is $x^2k + (4 - x)^2k = 10k$. Solving this we get $x = 1$ or 3 and thus the weights of the two pieces are 1 gm and 3 gm.

Once you arrive at the relation $x^2 + (4 - x)^2 = 10$, rather than solving, one should just try out few values for x. E.g. if $x = 2$, then $2^2 + 2^2 \neq 10$, if $x = 1.5$, then $1.5^2 + 2.5^2 = 2.25 + 6.25 \neq 10$, if $x = 1$, then $1^2 + 3^2 = 10$ and thus one can arrive at the answer a little faster than solving a quadratic.

15. $A - B = 2k$ and $B - C = 3k$. Adding the two, $A - C = 5k = 125$ i.e., $k = 25$

Thus, $A - B = 50$ i.e. $A = B + 50 = 225$

$B - C = 75$ i.e., $C = B - 75 = 100$

Thus, the sum distributed is $A + B + C = 225 + 175 + 100 = 500$.

16. **Method 1:** Since a, b, c and d are in continued proportion, we have $\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$.

Cubing, $\frac{a^3}{b^3} = \frac{b^3}{c^3} = \frac{c^3}{d^3}$. Using law of equal ratios, $\frac{a^3}{b^3} = \frac{b^3}{c^3} = \frac{c^3}{d^3} = \frac{a^3+b^3+c^3}{b^3+c^3+d^3}$.

Thus, in terms of b and c the answer is $\frac{b^3}{c^3}$.

Method 2: Since a, b, c and d are in continued proportion, they are in a GP and we can take them to be a, ak, ak^2 and ak^3. Plugging these values of b, c and d, we get

$$\frac{a^3+b^3+c^3}{b^3+c^3+d^3} = \frac{a^3+a^3k^3+a^3k^6}{a^3k^3+a^3k^6+a^3k^9} = \frac{1}{k^3} = \frac{(ak)^3}{(ak^2)^3} = \frac{b^3}{c^3}$$

17. We have, $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{a+b}{b+c} = \frac{b+c}{c+d}$. The last two ratios imply that $(b + c)$ is the mean proportional between $(a + b)$ and $(c + d)$.

18. Let, $X - a = 2k$, $X - b = 5k$ and $X - c = 7k$. ……(i)

Adding, $3X - (a + b + c) = 14k$.

Substituting $a + b + c = 2X$, $X = 14k$.

Putting $X = 14k$ in (i), we get $a = 12k$, $b = 9k$ and $c = 7k$. Thus, a: b: c is 12: 9: 7.

19. By the relation given, $y = k_1x + \frac{k_2}{x}$. Now we have two unknowns, k_1 and k_2, and we have two sets of $\{x, y\}$. Substituting these, we get two equations as

$6 = 4k_1 + \frac{k_1}{4}$ i.e. $16k_1 + k_2 = 24$ and $\frac{10}{3} = 3k_1 + \frac{k_2}{3}$ i.e., $9k_1 + k_2 = 10$.

Solving we get $k_1 = 2$ and $k_2 = -8$. Thus, $y = 2x - \frac{8}{x}$.

20. $y = k_1 + k_2x + k_3x^2$

Putting $x = 1$, $y = 0$; $0 = k_1 + k_2 + k_3$ ……(i)

Putting $x = 2$, $y = 1$; $1 = k_1 + 2k_2 + 4k_3$ ……(ii)

Putting $x = 3$, $y = 4$; $4 = k_1 + 3k_2 + 9k_3$ ……(iii)

(ii) – (i) $1 = k_2 + 3k_3$ ……(iv)

(iii) – (ii) $3 = k_2 + 5k_3$ ……(v)

(v) – (iv) $2 = 2k_3$ i.e., $k_3 = 1$

Substituting $k_3 = 1$ in (iv), $k_2 = -2$.

Substituting values of k_2 and k_3 in (i), $k_1 = 1$

Thus $y = 1 - 2x + x^2$. Putting $x = 7$, $y = 1 - 14 + 49 = 36$.

21. If distance fell in t seconds is d, then $d = kt^2$

Putting $t = 5$ and $d = 175$, we get $k = \frac{175}{25} = 7$.

Please note that in this question the relation of the total distance the body has fallen from start and the time it has been falling is given and the question is asking about the distance fallen in the 10th second.

Distance fell in the 10th second = total distance fell in 10 sec – total distance fell in 9 sec = $7 \times 10^2 - 7 \times 9^2 = 7 \times 19 = 133$ m.

22. This is a case of direct relation and thus the difference between the costs and the difference between the number of boys would be directly proportional.

From 150 boys to 120 boys: when the number of boys decreases by 30, the cost decreases by 1600.

Thus, from 150 boys to 330 boys: when the number of boys increases by 180, the cost would increase by 1600 × 6 = 9600. Thus, total expenses = 10,000 + 9,600 = 19,600.

23. Similar to the above question, this is also a case of direct relation and thus the difference between the costs and the difference between the number of units would be directly proportional.

From 540 to 620 units, an increase of 80 units increases bill by Rs. 240

From 540 to 500 units, a decrease of 40 units would decrease the bill by Rs. 120. Thus the bill for 500 units would be 1800 – 120 = Rs. 1680.

24. Here the reduction in speed of the engine has to be measured from the case when there are no bogies. Else the data would become inconsistent. Please understand that this is not an assumption and has to be deduced logically from the questions. Consider the train runs at speeds of S_1, S_2 and S_3 when there are 2, 4 and 6 bogies (the values are taken arbitrarily to explain the above). Now the reduction of speed when there are 6 bogies would be measured from where? From the case when there are 2 bogies or from the case when there are 4 bogies? If we measure from the case when there are 2 bogies, we get $S_3 - S_1 = 36k$ and if we measure form the case when there are 4 bogies, we get $S_3 - S_2 = 36k$. If we measure the reduction from the case when there are 3 bogies we will get yet another relation.

Also consider the statement "the speed reduced when there were 6 bogies attached". Since the reduction in speed is due to the 6 bogies attached, we should measure the reduction from the case when there were no bogies attached.

Say, the speed of the train when number of bogies is zero is S.

Then $S - 120 = 14^2 \times k$ and $S - 100 = 16^2 \times k$.

Subtracting the two, we get $60k = 20$ i.e., $k = 1/3$.

Substituting $k = 1/3$ in either equation we get $S = 185.33$

The reduction in speed when there are 20 bogies will be $\frac{20^2}{3} = 133.33$.

Thus, speed of the train = 185.33 – 133.33 = 52 kmph

25. At speed of 40, consumption is 2 tonnes per hour.

At speed of 50, consumption will be $2 \times \left(\frac{50}{40}\right)^2 = \frac{25}{8}$ tonnes per hour.

A 100 km journey, covered at speed of 50 kmph, will take 2 hours. So, coal consumed = 25/4 tonnes and cost will be Rs. 250/4 = 62.5.

26. The total cost of the boarders is directly related. And hence the change in expenses is directly proportional to the change in number of boarders.

When there are 400 boarders, the total cost is 400 × 120 = 48,000.

When there are 500 boarders, the total cost is 500 × 100 = 50,000.

An increase of 100 boarders, increases the cost by Rs. 2000.

A further increase of 300 boarders would increase the cost further by 6000 i.e., the total cost of 800 boarders will be 56,000 i.e., a per head cost of Rs. 70.

Exercise - 4

1. Let x be the initial average age of persons and y be the age of new persons. Then

$(23x - 20 + y)/23 = x + 4/12$

$\Rightarrow 23x - 20 + y = 23x + 23/3 \Rightarrow y = 23/3 + 20 = 83/3 = 27$ years 8 months.

2. The change in average age of the group = 58 – 56 = 2 years.

Actual change in years = 30 years. Therefore, total number of members = 30/2 = 15.

3. Average got reduced by 2 cm. So, overall decrease of 26 cm for 13 people. Height of new person = 184 – 13 × 2 = 184 – 26 = 158 cm.

4. Sum of marks for 24 students = 24 × 89 = 2136. Average marks of a student cannot increase beyond 100. So, total marks for 21 students cannot exceed 2100. So, maximum increase in average = 100 – 89 = 11.

5. 60 is wrong because then to arrive at a total of 121, the other box will have to weight 61 kg which will be obviously not the highest. 64 is wrong too, because then to add up to 121, the other weight will have to be 57 and to make up to a total of 120, the next box shall have a weight 63 which obviously makes the maximum possible total as 64 + 63 = 127. 62 is the correct answer because the other boxes shall be 59, 54, 58, 56. These will give all the totals given above.

6. Suppose the number of boys be S excluding a 100 kg boy.

Then, $\frac{S+100}{40} + \frac{1}{4} = \frac{S+New}{40} \Rightarrow$ New boy's weight = 110 kg.

7. Total rainfall for the week = 3.7 × 7 = 25.9 cm ...(i)

Rainfall for Monday, Tuesday and Wednesday = 4.3 × 3 = 12.9 cm ...(ii)

Rainfall for Friday, Saturday and Sunday = 3.9 × 3 = 11.7 cm ...(iii)

Rainfall for Thursday = Equation (i) – Equation (ii) – Equation (iii) = 25.9– 12.9 – 11.7 = 1.3 cm.

8. Let us assume that the average age of 5 members 3 years ago be M years. Total age of 5 members 3 years ago = 5M. Let the age of the old member and new member be O years and N years respectively. Present age of these 5 members = 5M + 15.

$\Rightarrow$ New Average age (M) = $(5M + 15 - O + N)/5 \Rightarrow N - O = 5M - 5M - 15 \Rightarrow O - N = 15$ years.

9. Let, the number of candidates who passed in the examination be 'p' and number of candidates who failed in the examination be 'f'.

Therefore, $43 < \frac{60p+25f}{p+f} \leq 49 \Rightarrow 43 - 25 < \frac{60p+25f}{p+f} - 25 \leq 49 - 25$

$\Rightarrow \frac{18}{35} < \frac{p}{p+f} \leq \frac{24}{35}$. Hence, 18/35 is not possible.

10. Let the number of boys and girls in the class be 'b' and 'g' respectively and the average age (in years) of boys and girls be 'x' and 'y' respectively.

Given, b + g = 60 and b/2 + 7g/4 = 60 $\Rightarrow$ b = 36 and g = 24.

Now, 36x + 24y = 720 and 18x + 42y = 810 $\Rightarrow$ x = 10 and y = 15

Hence, the average age of the boys is 10 years.

11. Let the initial strength of the students in the institute be x.

Then, total age of x students = 15x.

According to the question, 15x + (10 × 12) = 14.8(x + 10) $\Rightarrow$ 15x + 120 = 14.8x + 148 $\Rightarrow$ 0.2x = 28 $\Rightarrow$ x = 140

12. Marks obtained should be less than 50 in each paper.

There are three cases possible.

(a) Marks are 8, 9, 10, 13, 14, 15.

(b) Marks are 16, 18, 20, 26, 28, 30.

(c) Marks are 24, 27, 30, 39, 42, 45.

If we take case (a), then he required 81 marks more to get 50% of the total.

If we take case (c), then he got more than 50% marks.

In case (b) he required 12 marks more to get 50% marks, which is less than the case (a). So, this is the right answer.

13. Let the initial number of people in the group be n. The total age of the initial group of friends = 37n years.

The total age of the six friends who joined the group = 35 × 6 i.e., 210 years.

Given that, 37n + 210 = 36 (n+6) ⇒ 37n – 36n = 216 – 210 ⇒ n = 6

14. Let the number of boys in the class be 3x.

⇒ Number of girls in that class = 2x

Total age of the boys = 18 × 3x = 54x years.

Total age of the girls = 12× 2x = 24x years.

Average age of the class (54x + 24x)/5x = 15.6 years

15. Let the numbers of boys and girls in the class be b and g respectively. Let the average weights of the boys and the girls be B and G respectively.

We have $\frac{Bb + Gg}{b + g} = \frac{B + G}{2}$

⇒ 2Bb + 2Gg = Bb + Bg + Gb + Gg ⇒ B (b – g) + G (g – b) = 0 ⇒ (B – G) (b – g) = 0 ⇒ B = G or b = g or both.

16. Temperatures on Sunday, Monday and Tuesday were 28, 30, 32. (all values in degrees celsius). Temperature on Tuesday was 1 degree less than that on Wednesday.

Hence, temperature on Wednesday was 32 + 1 = 33.

Using the same logic, the temperatures on Thursday, Friday and Saturday were: 34, 35, 36 respectively.

Average of first four days = (28 + 30 + 32 + 33)/4 = 123/4 = 30.75 -------- (1)

Average of the last three days = (34 + 35 + 36)/3 = 35 ---------- (2)

Required difference = 35 – 30.75 = 4.25° C

17. Let the present average age of ten advisors as well as their average 3 years ago, be A. Also let the age of the younger man be Y and that of the director replaced by him be D.

So, (10A + 30 – D +Y)/10 = A, as the present average is same as the earlier average. Hence, D – Y = 30 years.

18. Here, an = y + k …….. (1)

And bn = x + k …….. (2), where k is the sum of the (x – 1) numbers which are other than y and x.

Subtracting equation (2) from equation (1), we get n(a – b) = y – x ⇒ $\frac{1}{n} = \frac{a - b}{y - x}$.

19. Let the average of classes I and II be A kg and B kg. Let the number of students in the classes I and II be a and b respectively.

We have, $\frac{Aa+Bb}{a+b} = \frac{A+B}{2}$

⇒ 2Aa + 2Bb = Aa + Bb + Ab + Ba

⇒ (a – b) (A – B) = 0

As a ≠ b, A = B ---------- (1)

and 2 A + B = 60 ---------- (2)

Solving (1) and (2), we get A = B = 20.

20. As each student missed a different number, the sum of the all the numbers added by the students = (N – 1) (sum of the first N natural numbers) = $\frac{(N-1)\,N\,(N+1)}{2}$.

The number of numbers added by the students in total = (N - 1) (N).

Therefore, the average = $\frac{(N-1)\,N\,(N+1)}{2\,(N-1)\,N} = 21$ ⇒ N = 41.

21. Let A's, B's, C's, and D's weights be a kg, b kg, c kg and d kg respectively.

a = (b + c + d)/3 ⇒ 3a = b + c + d ---------- (1)

b = (a + c + d)/3 ⇒ 3b = a + c + d ---------- (2)

Adding (1) and (2), 3 (a + b) = a + b + 2 (c + d) ⇒ a + b = c + d

Therefore, (c + d)/2 = 30 ⇒ (a + b)/2 = 30.

22. Here, y's age is 32 – 4 = 28

Let the average age of 11 people be A.

11A = 32 + 28 + k …… …(1), where k is the sum of the remaining ages.

11(A – 1) = a + b + k……. (2), where a and b are the ages of the two people replacing x and y.

Subtracting equation (2) from equation (1), we get 11 = 60 – (a + b) ⇒ a + b = 49 and the average age of the people replacing x and y = 49/2 = 24.5 years.

23. Let the number of goals in the first match be a. The number of goals in successive matches are a, a + 2, a + 4, a + 6, a + 8, a + 10, a + 12, a + 14, and a + 16.

The average number of goals = (9a + 72)/9 = a + 8 = 16.

The average number of goals in the second and eighth matches = (2a + 16)/2 = a + 8 = 16.

24. Let the average of the ages of the 11 men be A years. Let the ages of the oldest and the youngest be x years and y years respectively. If any one person leaves the group the maximum and minimum possible average age of the remaining occur if the person aged y years and the person aged x years, respectively, leave, so we have,

11 A – y = 10 × 11 = 110 ---------- (1)

11 A – x = 10 × 12 = 120 ---------- (2)

Adding (1) and (2), 22A – (x + y) = 230

⇒ 22A – 22 = 230 ⇒ 22A = 252 ⇒ A = 252/22 = 11 5/11 years.

25. Let the present ages of the father, his only son and his wife be f years, s years and m years respectively.

Now, (f + s)/ 2 = 5/4 {(s + m)/2}

⇒ 4 (f - m) = s + m ⇒ f – m = 10 and m - s = 30

⇒ s + m = 40 and m – s = 30 ⇒ s = 5.

26. As the average weight of A decreased after the student left, his weight must be more than the average weight of A. As the average weight of B decreased after the student joined, his weight must be less than the average weight of B. His weight must be between 40 kg and 60 kg.

27. Let the cost of each apple be Rs A and that of each mango be Rs M

5A + 4M = 9 × 48 = 432

7A + 8M = 15 × 60 = 900.

Adding both the equations and multiplying with 2, we get, 24A + 24M = Rs 2,664

28. Let the total marks of P and Q, P and R and Q and R be a, b and c respectively. Let the number of students in P, Q and R be p, q and r respectively.

a = 60 (p + q)

b = 64 (q + r)

c = 72 (p + r)

⇒ a + b + c = 132 p + 124 q + 136 r

$$x = \frac{a+b+c}{2(p+q+r)} = \frac{66p+62q+68r}{p+q+r}$$

$\Rightarrow \frac{62(p+q+r)}{p+q+r} < x < \frac{68(p+q+r)}{p+q+r}$ ⇒ 62 < x < 68. Therefore, 62.5 is the only possible value.

Exercise - 5

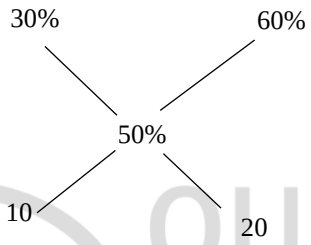
1. In 20 L solution, wine = 20 × 0.8 = 16 litre and water = 4 litre

Let x L water be added. Then, $\frac{4+x}{16} = \frac{60}{40} \Rightarrow$ x = 20 litre.

2. Here, 30 litre of mixture is drawn out. So, in 90 litre of mixture, milk = 90 × 2/5 = 36 L and water = 90 × 3/5 = 54 litre. Now, milk = 36 + 18 = 54 litre and water = 54 + 12 = 66 litre.

∴ New ratio of milk and water = 54: 66 = 9: 11.

3. By the rule of alligation, we have,



Therefore, Required ratio = 10: 20 i.e., 1: 2.

4. Here, CP = Rs 10 (at 10% profit)

Using alligation, we have $\frac{3}{2} = \frac{10-(x-2)}{x-10} \Rightarrow$ x = 10.8

5. Total volume = 105 litre

Milk: Water = 8: 7

14 litre of water was removed, ∴ Amount of mixture removed = 15/7 × 14 = 30 litre

Volume of mixture left = 105 – 30 = 75 litre.

Individual content in the mixture: Milk = 8/15 × 75 = 40 litre

Water = 75 – 40 = 35 litre. 5L of water was added afterwards

Now, the new ratio of milk and water is: 40: 35 + 5 = 40: 40 = 1: 1

Alternate Method:

Total volume = 105 litre

Total amount of Milk = 8/15 × 105 = 56 litre and total volume of water = 7/15 × 105 = 49 litre. Now, when 14 litre of water is removed, i.e., 16 litre of milk is removed, new ratio is 40: 49 – 14 + 5 = 40: 40 = 1: 1.

6. Removal of 25 litre at stage I will result in volume of milk being reduced by 80% of 25 litre i.e., 20 litre and volume of water being reduced by the remaining 5 litre. So, M = 60 litre and W = 15 litre. Addition of 25 litre water will finally give M = 60 litre and W = 40 litre. Hence the ratio of W and M = 40: 60 = 2: 3.

7. Let initial capacity of the can be x.

After the process, proportion of A = $\frac{\frac{7}{12}(x-9)}{x} = \frac{7}{16} \Rightarrow$ x = 36 litre

So, A = 7/12 × 36 = 21 litre.

8. Let I be the initial quantity of diluted milk in the can and the initial concentration of milk be m%.

So, Im/(I + 35) = 30 and Im/(I + 75) = 20

⇒ 2(I + 75) = 3(I + 35) ⇒ I = 45 and mI = 2400.

If w litre of water is required to be added to the can to make the concentration of milk in the can 8% then, Im/(I + 75 + w) = 8 ⇒ 120 + w = 2400/3 = 800 ⇒ w = 180.

9. Let the quantity of each alloy be 1.

Copper in first alloy = 7/9

Copper in second alloy = 7/18

Copper in third alloy = 7/9 + 7/18 = 21/18

Total quantity of third alloy = 2. Aluminium = 2 – 21/18 = 15/18

Copper and aluminium in the third alloy = 21: 15 = 7: 5.

10. Let the total quantity of alloy be x.

So, $\frac{82\%\ of\ x}{18\%\ of\ x + 18} = \frac{7}{3} \Rightarrow \frac{246x}{100} = \frac{126x}{100} + 126 \Rightarrow \frac{126x}{100} = 126 \Rightarrow$ x = 105

⇒ Quantity of new alloy = 105 + 18 = 123.

Here, copper = 70/100 × 123 = 86.1 and Zinc = 123 – 86.1 = 36.9.

11. Let a and b be the quantity and concentration of the sugar solution in the vessel having the least quantity.

We have, $\frac{ab}{100} + \frac{(a+5)b}{100} + \frac{(a+10)b}{100} = 5.4$

Also, b – a = 8. Hence, the required quantities are 7 kg, 12 kg, 17 kg.

12. For every litre of mixture sold Ramu received Rs. 2.5. Therefore, the amount of Ramu's share of milk in the final mixture is 2.5/20 = 1/8.

So, Out of 8 litre of milk 1 litre was contributed by Ramu and rest 7 litre was contributed by Kamu and Namu. Now, amount of milk contributed by Kamu = 2/5 × 10 = 4 litre and Namu = 1/5 × 15 = 3 litre. Between Kamu and Namu, they contributed 7 litre of Milk and 18 litres of Water. But total mixture is of 5 + 10 + 15 = 30 litre Hence, Ramu had 1 litre of Milk and 4 litres of water. Required ratio = 1: 4.

13. Amount of alcohol that goes out in the 10% liquid taken out is 20 ml. To this mixture, 100 ml of pure alcohol is added. So total volume of alcohol in the new mixture = 200 – 20 + 100 = 280 ml. Therefore, percentage of alcohol in the new mixture = 280 / 1000 × 100 = 28%.

14. Each day 2/3 of wine is remaining. So, at the end of third day $(2/3)^3 \times 180 = 160/3$ litre.

15. We have, $24 = 54\left(1 - \frac{x}{54}\right)^2$

$\Rightarrow \frac{24}{54} = \left(1 - \frac{x}{54}\right)^2 \Rightarrow$ Therefore, x = 18 litre.

16. Amount of milk left = 20 × 4/5 × 4/5 × 4/5 = 10.24 litre

Amount of water left = 9.76 litre

The required ratio = 10.24: 9.76 = 512: 488

17. Let x be the total volume. We have, $\left(\frac{x-a}{x-2a}\right)^2 = \frac{25}{16}$ or a = x/6

18. Let x be the initial amount of the mixture. ∴ Amount of milk in litre = 5x/8 and water in litre = 3x/8.

In 6 litres of mixture, Milk = 6 × 5/8 = 15/4 litre and water = 6 × 3/8 = 9/4 litre

Therefore, $\frac{5x}{8} - \frac{15}{4} = \frac{3x}{8} - \frac{9}{4} + 6 \Rightarrow 2x = 60 \Rightarrow x = 30$ litre

∴ Initial amount of milk = 30 × 5/8 = 18.75 litre.

19. Then, fraction of B in Product M = $\frac{5}{9} \times \frac{3}{4} + \frac{4}{9} \times \frac{2}{3} = \frac{77}{108}$

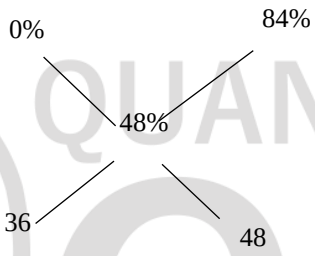
In 864 units of Product M, amount of B = 77/108 × 864 = 616.

In the final mixture 50% is B. Then amount of final mixture = 616 × 2 = 1232. So the water added = 1232 – 864 = 368.

20. Here 8 litre is 1/10th of 80 litre.

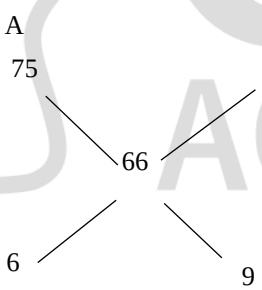
So, the amount of milk left = 80 × 9/10 × 9/10 = 64.8 litre.

21. Pure water has no milk, i.e., 0% milk. So, by the rule of alligation, we have,



So, the required ratio is 3: 4.

22. By the rule of alligation, we have,

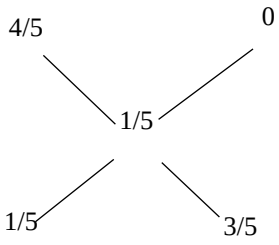


Therefore, the ratio of the contents of vessel A and B is 2: 3.

Hence, the volume of vessel B = 3 / 2 × 20 = 30 litre

23. Ratio of milk and water is 80: 20 or 4: 1. In the resulting mixture, the ratio of milk and water is 1: 4.

Using the rule of alligation on milk, we have,



Therefore, the ratio of original and added part is 1: 3. Hence, the amount of water added = 30 × 3 = 90 litre.

24. Amount of milk left = 90 × 9/10 × 9/10 = 72.9 litre

Amount of water = 90 – 72.9 = 17.1 litre

Required ratio is 72.9: 17.1 or 81: 19

Alternatively: Ratio of milk left and total mixture = $(9/10)^2 = 81/100$

Therefore, the ratio of milk and water is 81: 19.

25. Let the number of students in X, Y and Z be x, y and z respectively.

Applying the rule of alligation, x/y = (81 – 76)/(76 - 70) = 5/6.

And y/z = (90 – 86)/(86 – 81) = 4/5

Therefore, x: y: z = 10: 12: 15.

Let x = 10a, y = 12a and z = 15a

Average marks of X, Y and Z = $\frac{70\times10a+81\times12a+90\times15a}{10a+12a+15a} = \frac{3020}{37} = 81\frac{25}{37} \approx 82$

26. As the quantity to be drawn from the Alloys A and B to form another alloy C must be a multiple of 13 and 11, let us choose the quantity drawn as LCM of 13 and 11; which is 143.

Amount of copper in Alloy C = 4 × 11 + 5 × 13 = 109 kg.

Amount of zinc in alloy C = 9 × 11 + 6 × 13 = 99 + 78 = 177 kg.

Hence the ratio of copper and zinc = 109: 177

27. A mixture of two varieties of wheat is sold at Rs 28 per kg, at a profit of 40%. Hence, the cost price of the mixture = (28)/(1.4) = Rs 20.

Let x and (x + 3) be the prices in rupees of the first and the second varieties respectively. The quantities are mixed in the ratio 3 : 4 (given)

By alligation equation, $\frac{q1}{q2} = \frac{p2-p}{p-p1} \Rightarrow \frac{3}{4} = \frac{(x+3)-20}{20-x} \Rightarrow$ x = 128/7.

28. Initial quantity of milk in the vessel = 0.2 (10) = 2 ml

Initial quantity of water in the vessel = 0.8 (10) = 8 ml

After x ml of milk was added, the ratio of milk and water would become 4: 1

x = 4 × 8 – 2 = 30. After y ml of water was added, ratio of milk and water would again become 1: 4.

y = 4 × (4 × 8) – 8 = 120 ⇒ x + y = 150.

29. Prices of 3 varieties of sugar are Rs 10, Rs 12 and Rs 18 per kg. Let them be mixed in the ratio x: y: z.

Hence, price of the mixture is (10x + 12y + 18z)/(x + y + z) = 14

⇒ 10x + 12y + 18z = 14x + 14y + 14z

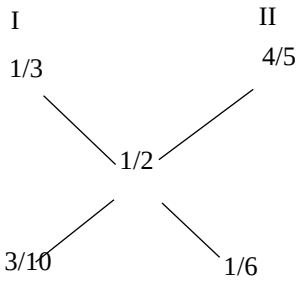
⇒ 4x + 2y – 4z = 0; ⇒ 2x + y – 2z = 0. This equation has infinite solution sets. Hence, from among the given options, the one which satisfies the equation will be the solution.

Option 4, i.e., 3: 4: 5 satisfies the equation.

30. Here, hydrochloric acid in the first container is 1/3 of total

And hydrochloric acid in the second container is 4/5 of total. The amount of hydrochloric acid required in the final solution is 1/2.

By rule of alligation, we have,



The ratio in which the amounts are taken from container 1 and container 2 is 3/10: 1/6 or 9: 5.

So, amount taken from the first vessel is 9/14 × 28 = 18 litre.

31. Let the amounts taken out from each vessel is 10 litre.

Amount of alcohol and water in the first vessel is 6 litre and 4 litre respectively.

Amount of alcohol and water in the second vessel is 8 litre and 2 litre respectively.

Amount of alcohol and water in the third vessel is 7 litre and 3 litre respectively.

Total alcohol = 6 + 8 + 7 = 21 litre

Required percentage = 21/30 × 100 = 70%.

32. We have $\frac{Alcohol\ left}{Total\ vol} = \frac{9}{25} = \left(\frac{3}{5}\right)^2$. So every time 2/5th of the alcohol was taken out.

So, 2/5 of total = 6 ⇒ Total = 15 litre.

33. The original mixture had 90 litres of milk and 10 litres of water. It had 9/10th of milk.

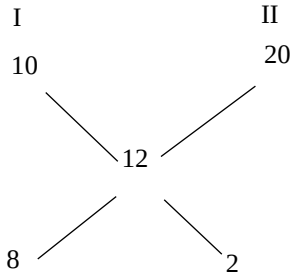
After the first withdrawal of 10 litres, 9 litres of milk and 1 litre of water is lost. So, the resulting mixture would have 81 litres of milk. On adding water, the vessel would have 81/100th of its contents as milk. We want the milk concentration to be at most 66 2/3%. (i.e., less than or equal to 66 2/3%), the milk quantity to be at most 66 2/3 litres. After the second replacement the vessel would have 729/1000th of its contents as milk. After the third replacement, the vessel would have 6561/10000th of its contents as milk i.e., 65.61 litres of milk (which is less than 66 2/3 litre). The least number of times the procedure has to be carried out is 3.

34. Let the initial volumes of milk in P and Q be p and q respectively. After the first transfer, P would have 0.2p and Q would have 0.8p + q. After the second transfer, P would have 0.2p

+ 0.6(0.8p + q) and Q would have 0.4(0.8p + q).

So, $\frac{0.2p + 0.6\ (0.8p + q)}{0.4\ (0.8p + q)} = \frac{23}{12}$ ⇒ p/q = 5/2.

35. The effective rate of interest = 960 × 100/8000 = 12%



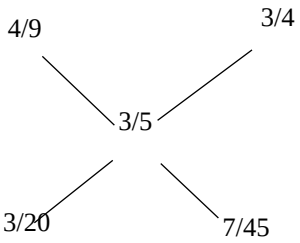
Hence, the amounts invested in two parts is in ratio 4: 1.

The amount invested in first part is Rs 4/5 × 8000 = Rs 6400

The amount invested in second part is Rs 1/5 × 8000 = Rs 1600

Required difference = 6400 – 1600 = Rs 4800.

36. By rule of alligation, we have,

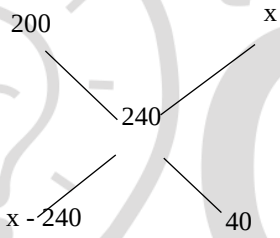


The ratio of the contents taken out is 3/20: 7/45 or 27: 28.

37. The cost price of R = 288/1.2 = Rs 240/kg

The cost price of P is Rs 200 and let the cost price of Q is Rs x.

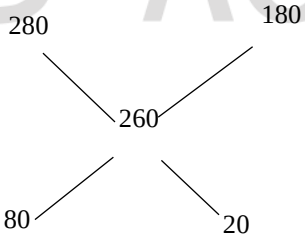
By Alligation, we have,



Given that, (x – 240)/40 = 1/2 ⇒ 2x – 480 = 40 ⇒ 2x = 520 ⇒ x = 260

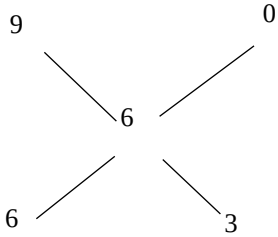
Now Q is obtained by mixing the varieties with cost price Rs 280/kg and Rs 180/kg.

Again, using the rule of alligation, we have,



Hence, the two varieties are mixed in the ratio 80: 20 or 4: 1.

38. By rule of alligation, we have,



The ratio of the replaced part and the non-replaced part is 6: 3 or 2: 1. Given that 9 litre are replaced, so the total volume = 9 × 3 = 27 litre.

39. Let the capacity of the vessel is x litre.

We have, $x - 17.1 = x(1 - 9/x)^2$

$\Rightarrow x^2 - 17.1x = x^2 - 18x + 81$

$\Rightarrow 0.9x = 81 \Rightarrow x = 90$ litre.

QUANTIFIERS CAT ACADEMY

Exercise – 6

1. $Day_1 + Day_2 + Day_3 + Day_4 = 4 \times 58 = 232$ .....(i)

$Day_2 + Day_3 + Day_4 + Day_5 = 4 \times 60 = 240$ .....(ii)

(ii) – (i), $Day_5 - Day_1 = 8$

We also know that Day_1 and Day_5 are in ratio 7: 8. Thus, Day_5 = 64 degrees

2. All of Sachin's innings could be viewed as having scored either 66 or a duck.

The total runs scored by Sachin i.e. 110 × 63 has been scored at 66 per innings. Thus, number of innings he has scored is $\frac{110\times63}{66} = 105$

Thus, number of innings he has not scored = 110 – 105 = 5

Alternative method:

Let n be the number of innings when Sachin got out for a duck. The rest of the (110 – n) innings contributed 3 runs each (avg. drops from 66 to 63) to these n innings such that they get 63 runs each (actually scored 0, but avg. is 63). So, (110 – n) × 3 = 63n i.e., n = 5

3. When the youngest is not considered, the average increases by 4 years. This means that the rest 7 had 4 years extra to give to the youngest. Hence youngest must have been 7 × 4 = 28 years less than the average of the group of 8.

When the eldest is not considered, the average decreases by 3 years. This means that each of the rest were 3 years less, which was contributed by the eldest. Hence eldest must have been 7 × 3 = 21 years more than the average of the group of 8.

Difference between the ages of oldest and the youngest = 28 + 21 = 49 years

Alternative method:

When the youngest or the eldest is not considered, the sum totals of the ages changes. Because of this, the average also changes. Since the change in the average is 4 – (–3) = 7, the sum total of the ages differs by 7 × 7 = 49, when the eldest and the youngest are not considered. Thus, the difference in the ages of eldest and youngest has to be 49.

4. Let Akram have taken x wickets before today's match.

Thus, runs conceded so far = 19.2 × x.

After today's match, average will be $\frac{19.2x+84}{x+7} = 19$

$\Rightarrow 19.2x + 84 = 19x + 133$ i.e., $0.2x = 49$ i.e., $x = 245$.

5. Sum of ages of 15 faculties in 2005 is 15 × 48 = 720.

In 2006, the sum of the ages of the same 15 faculties will be 720 + 15 = 735.

But three faculties attaining 60 in the year (i.e., considered 61 in 2006) left and three new of average age 36 years joined.

Hence, average age in 2006 is $\frac{735-61\times3+36\times3}{15} = 44$

6. Let A_H,,A_W and A_C denote the present ages of the husband, wife and the child respectively.

$(A_H - 3) + (A_W - 3) + (A_C - 3) = 3 \times 27$ i.e., $A_H + A_W + A_C = 90$.....(i)

$(A_W - 5) + (A_C - 5) = 2 \times 20$ i.e., $A_W + A_C = 50$.....(ii)

Substituting the value of $A_W + A_C$ from (ii) in (i) A_H = 40yrs

The above working of equations can also be thought of as follows, without using any pencil and jotting any equations. Since, three years ago the average age of husband,

wife and child was 27 years, the average of the present ages would be 30. Thus, total of their present ages = 90

Since, five years ago the average age of wife and child was 20 years, the average of the present ages would be 25. Thus, total of their present ages = 50. Thus, husband's age = 90 – 50 = 40.

7. 8 people spend Rs. 30 each so their average expenditure is Rs. 30. The ninth one spent Rs. 20 more than the average of all nine, so this 20 would have been distributed among the rest of the 8 persons' expenditure, $\frac{20}{8} = 2.5$. Hence the average expenditure by each = 30 + 2.5 = 32.5. The ninth one would have spent Rs. 32.5 + 20 = Rs. 52.5. Total money spent by all of them 32.5 × 9 = 292.5.

8. The inclusion of the teacher increases the average age of a class by 1.5. It means that whatever is the difference between the teacher's age and the class average without the teacher, it is divided among all the students and the teacher equally, thus increasing the average. This difference is given to be 30, then the total number of people $\frac{30}{1.5} = 20$. So, number of students is 19.

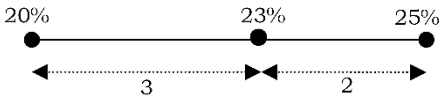
9. The fastest way to solve this is to consider the 10 consecutive odd numbers as ……, $(n-3)$, $(n-1)$, $(n+1)$, $(n+3)$, ……

This is so because in this case the average of the numbers will become n.

Thus, by the given condition, sum, $10n = 180 + n$ i.e. $n = 20$. This will also be the average of the 1st and the last number and hence the sum of the 1st and last number will be 2 × 20 = 40.

10. Monthly sales data is entered for a year and then the average monthly sales for the year is found. Thus, the total sales is divided by 12. When one three-digit figure is reversed, the maximum difference will be when the number is of the type 900, on reversing we get 009, hence the difference is 891. This 891 will be the total reduction in the sum, hence the average will come down by $\frac{891}{12} = 74.25$.

11. **Method 1:** Using Alligation



The ratio of the two cost prices is 2: 3. The cost price of the cow is $\frac{3}{5} \times 1300 =$Rs. 780

Method 2:

Let the cost of the cow be Rs. x, then the cost of the calf would be Rs. $(1300 - x)$. Thus,

25% of x + 20% of $(1300 - x)$ = 23% of 1300

5% of x = 3% of 1300 i.e., x = Rs. 780

12. **Method 1:**

By selling the mixture at Rs.10/litre, he earns a profit of 25%. It means that the cost price of the mixture must have been Rs.8/litre. But the cost of milk is Rs. 10/litre. Hence the mixture had 20% water and 80% milk.

The actual amount of water he mixed was 5 litres, which was 20% of the mixture. So, the amount of mixture is 25 litres.

Method 2:

If a milk-man is selling a mixture at the same rate at which he purchased milk, and the ratio of milk to water mixed is a: b, his profit is going to be $\frac{b}{a}$, in terms of fraction equivalence. Why? Profit is earned only on the part equivalent to water i.e., b and cost is incurred only on the part equivalent to milk i.e., a.

In this case rate of purchase of milk and rate of selling mixture is same i.e., Rs. 10/litres. This, since profit is 25%, i.e., $\frac{1}{4}$, ratio of milk and water in the mixture is 4: 1. Since water is equal to 5 litres, milk is 20 litres and mixture is 25 litres.

13. **Method 1:**

By selling the mixture at Rs.13.75/litre, a profit of 25% is earned. So, cost price of mixture must be Rs.11/litre. But the cost of soda in the mixture is Rs. 12/litre. This implies that every litre of mixture should have $\frac{11}{12}^{th}$ part of soda and $\frac{1}{12}^{th}$ part water. Ratio of water and soda is 1: 11.

Method 2:

The method 1 is preferable any day, because it does not require one to write anything. But in case the oral method is not clear or too demanding then use this method…

Let quantity of soda be x and that of water be y. Since SP = $\frac{5}{4}$ × CP,

$13.75 \times (x + y) = \frac{5}{4} \times 12 \times x$, i.e., $13.75 \times y = 1.25 \times x$ i.e., $\frac{y}{x} = \frac{1.25}{13.75} = \frac{5}{55} = \frac{1}{11}$

14. Let the concentration of milk in the second solution is x.

Method 1: Using weighted average

$\frac{5}{9} = \frac{0.4\times1+x\times1}{1+1}$ i.e. $x = \frac{32}{45}$

Ratio of milk and water in the second solution is 32: 13.

Method 2: Using equations…

Since equal quantities of the two solutions were taken and the ratio of milk and water in the first solution was 2: 3, let us assume 5 litres of both solutions were taken and also assume that quantity of milk and water in the second solution is x and $(5 - x)$ respectively. Thus, in the mixture, $\frac{\text{milk}}{\text{water}} = \frac{2+x}{3+(5-x)} = \frac{5}{4}$ i.e., $9x = 32$ i.e., $x = \frac{32}{9}$

Thus, ratio of milk and water in second solution is $\frac{32}{9} : \left(5 - \frac{32}{9}\right)$ i.e., 32: 13

15. **Method 1:**

5 litres of the solution will have 3 litres of milk and 2 litres of water. 10% of 5 litres i.e., 0.5 litres of milk is added to the original solution. Volume of water remains the same i.e., 2 litres and volume of milk becomes 3.5 litres. Now ratio of milk and water is 3.5: 2 i.e., 7: 4.

Method 2:

Since pure milk is added, proportion of water will vary inversely with the volume. As volumes are in the ratio 10: 11, i.e., volume is increased, proportion of water will decrease and will become $\frac{2}{5} \times \frac{10}{11} = \frac{4}{11}$. Thus, ratio of milk and water is 7: 4.

16. For removal of solution and replacement by water,

Final Concentration of milk = Initial Concentration × (1 – Fraction Removed)

$\frac{2}{5} = C_i \times \left(1 - \frac{20}{100}\right) \Rightarrow C_1 = \frac{1}{2}$ i.e. ratio of milk and water is 1: 1.

Alternatively:

Consider 5 litres of initial solution of milk and water. After removal and replacement, the volume will still be 5 litres of which 2 litres is milk and 3 litres is water. The last activity was 1 litre of water was added. So, the quantity of water just before adding, but after removal of solution, was 2 litres and ratio of milk and water was 1: 1. When solution is removed, ratio of milk and water does not change and thus initially also the required ratio was 1: 1.

17. Since water is being replaced, working on proportion of milk in the solution, $\frac{75}{100} = \frac{80}{100}\left(1 - \frac{x}{40}\right)$ i.e., x = 2.5 litre.

18. Let x be the initial proportion of milk in the solution.

$$\frac{1}{210} = x \times \frac{9}{10} \times \frac{10}{11} \times \frac{11}{12} \text{ i.e. } x = \frac{2}{3}$$

Hence ratio of milk and water in the original solution = 2: 1

19. **Method 1:**

When 2 litres of water is added, the total volume of the solution becomes 10 + 2 = 12 litres. Now when 2 litres of solution is removed, the ratio of milk and water does not get affected. Hence the ratio of milk and water before removal is the same as the ratio after removal i.e., 2: 3.

Milk in 12 litres of solution $\frac{2}{5} \times 12 = \frac{24}{5}$ litres. The milk quantity remained unchanged when water was added to the solution. Milk in 10 litres solution was also $\frac{24}{5}$ litres.

Hence water in 10 litres of the solution $= 10 - \frac{24}{5} = \frac{26}{5}$litres

Ratio of milk and water in the original solution $\frac{24}{5} : \frac{26}{5} = 12:13$

Method 2:

Working backwards, removing 2 litres of solution does not change the proportion of milk and water. Thus, ratio of milk and water before removal of solution, but after adding water, is also 2: 3.

When pure water is added to a solution of milk and water, proportion of milk is inversely proportional to the volumes. Since volume is increased from 10 litres to 12 litres, the proportion of milk will reduce and becomes $\frac{2}{5}$.

Thus, if initial proportion of milk is x, then $x \times \frac{10}{12} = \frac{2}{5}$ i.e., $x = \frac{12}{25}$. Thus, initial ratio of milk and water is 12: 13.

20. **Method 1:**

Let the amount of milk in the 8 litres of solution is x.

Expected concentration $= \frac{x}{8} \times \left(1 - \frac{3}{8}\right) = \frac{5x}{64}$

If water is added first, the volume of solution increases to 11 litres but the amount of milk in it remains the same. So, concentration of milk is $\frac{x}{11}$. After 3 litres is removed from the mixture, the concentration of milk remains the same because removing part

of the solution only changes the volume, not the concentration of any of the constituents.

Ratio of the expected concentration of the milk to the actual concentration of the milk

$= \frac{5x}{64} : \frac{x}{11} = 55:64$

Method 2:

The proportion of milk only changes when water is added. And not when part of the solution is removed. Further the proportion of milk varies inversely with the volume when water is added.

When water had to be added, it was expected that 3 litres of water be added to 5 litres of solution i.e., an increase in the volume from 5 litres to 8 litres but actually 3 litres of water was added to 8 litres of solution i.e., an increase in the volume from 8 litres to 11 litres. Thus, if x is the initial proportion of milk in the solution, $\frac{\text{expected proportion of milk}}{\text{actual proportion of milk}} = \frac{x \times \frac{5}{8}}{x \times \frac{8}{11}} = \frac{55}{64}$.

21. When milk is delivered to any household, the proportion of milk does not change. It changes only when water (or milk) is added.

After delivering to the first household, 1 litre of water is added to 3 litres of pure milk. Thus, percentage of milk will become 75%.

After delivering to the second household, 1 litre of water is added to 2 litres of 75% milk solution. Since pure water is added, percentage of milk will vary inversely with volume. Since volume increased from 2 litres to 3 litres, the percentage of milk will reduce to $75\% \times \frac{2}{3} = 50\%$.

After delivery to third household, 1 litre of *milk* is added to 1 litre of 50% milk solution. Thus, percentage of water will now vary inversely with the volume. Since volume increased from 1 litre to 2 litre, percentage of water will decrease to $50\% \times \frac{1}{2} = 25\%$. Thus, percentage of milk = 75%.

22. Let the volume of the original solution is x. Since solution is replaced with water, working on proportion of milk,

$$\frac{25}{231+25} = \frac{2}{5} \times \left(1 - \frac{15}{x}\right)^3 \text{ i.e. } \frac{25}{256} = \frac{2}{5}\left(1 - \frac{15}{x}\right)^3$$

$$\left(1 - \frac{15}{x}\right)^3 = \frac{125}{512} \text{ i.e. } \left(1 - \frac{15}{x}\right) = \frac{5}{8} \text{ i.e., } x = 40 \text{ litres}$$

23. Proportion of milk changes only when water is added and not when solution is removed. Further since pure water is added, proportion of milk is inversely proportional to the volume.

If the original volume of the solution is x litres, each time water is added, the volume is increased from x litres to $(x + 15)$ litres. Thus, each time water is added, proportion of milk will decrease and would become (proportion of milk before adding) $\times \frac{x}{(x+15)}$.

Proportion of milk when water is added for first time = $\frac{2}{5} \times \frac{x}{(x+15)}$

Proportion of milk when water is added for second time = $\frac{2}{5} \times \frac{x}{(x+115)} \times \frac{x}{(x+15)}$

$= \frac{2}{5} \times \left(\frac{x}{x+15}\right)^2$

Similarly, proportion of milk when water is added for third time = $\frac{2}{5} \times \left(\frac{x}{x+15}\right)^3$, which is given as $\frac{25}{25+231}$. Thus, $\frac{2}{5} \times \left(\frac{x}{x+15}\right)^3 = \frac{25}{256} \Rightarrow \left(\frac{x}{x+15}\right) = \frac{5}{8} \Rightarrow x = 25$.

24. 16.66% profit is $\frac{1^{th}}{6}$ of the cost price. Thus, ratio of Earnings to Cost should be 7: 6.

Let x litres of water be mixed with 1 litres of milk. Further let cost of milk be Rs. 5/ litre, thus cost of Bisleri water is Rs. 1/ litre.

$\frac{\text{Earnings}}{\text{Cost}} = \frac{5\times(1+x)}{5\times1+1\times x} = \frac{7}{6}$ i.e., $30 + 30x = 35 + 7x$ i.e., $x = \frac{5}{23}$

Thus, ratio of milk and water being mixed is 1: $\frac{5}{23}$ i.e., 23: 5.

25. When A and B are mixed: The concentration of milk in solution D = $\frac{0.4\times2+0.5\times1}{2+1} = \frac{13}{30}$

When B and C are mixed: The concentration of milk in solution E = $\frac{0.5\times1+0.6\times2}{2+1} = \frac{17}{30}$

Since D and E are mixed in the ratio 1: 1, the concentration of milk in the solution will be the simple arithmetic mean, $\frac{\frac{13}{30}+\frac{17}{30}}{2} = \frac{1}{2}$

So, ratio of milk and water is 1: 1.

Alternately, we could have reduced the scenario to just one mixing of A, B and C as follows…

Solution D has ratio of A and B as 1: 2 and solution E has ratio of B and C as 2: 1. Since equal quantities of D and E are mixed, let's say 3 litres of D and 3 litres of E are mixed (3 litres because each of 1: 2 and 2: 1 add up to 3). Thus, mixing of D and E implies that 1 litre of A, (2 + 2) = 4 litres of B and 1 litre of C are being mixed. Thus, proportion of milk in the final mixture = $\frac{0.4\times1+0.5\times4+0.6\times1}{1+4+1} = \frac{3}{6} = \frac{1}{2}$. Thus, ratio of milk and water in final mixture is 1: 1.

26. Average age of students: Average age of class including teacher = 8: 9

Age of teacher: Average age of students = 3: 1= 24: 8

If the average age of students is 8, then the age of teacher is 24 and this increases the average age of class by 1. The teacher has 16 extra which she distributes among the students and herself, giving 1 to each to bring up the average by 1. So, there must have been 15 students.

27. The average is 83% in six subjects. If the highest in any subject is 91%, the lowest could be obtained if 91% marks are scored in 5 subjects. So, there will be an extra 5

× (91 – 83) = 40 marks. Hence the sixth subject could be at 40 marks less than the average i.e., 83 – 40 = 43%.

28. Let's put some values. Of the 10 numbers, if 5 are negative, their average is –5, so sum = –25. The rest 5 will be positive. Their sum needs to be 25 to give an average of 0. Hence their average will be 5. Of the 10 numbers, if 8 are negative, their average is –5, so sum = –40. The rest 2 will be positive. Their sum needs to be 40 to give an average of 0. Hence their average will be 20. So, the average of the positive numbers cannot be found out uniquely until and unless the number of negative or positive numbers is specified.

29. If A, B and C represent the average marks scored by the three divisions,

A + B = 60 × 2 = 120.....(i)

B + C = 75 × 2 = 150.....(ii)

C + A = 62 × 2 = 124.....(iii)

Adding (i), (ii) and (iii) (A + B + C) = 197.....(iv)

Substituting (i) in (iv), C = 77%

Substituting (ii) in (iv), A = 47%

Substituting (iii) in (iv), B = 73%

Alternate Short-cut:

The two pairs which have the higher scores (75% and 62%) is B & C and A & C. The division common to the two pairs, C, would have the highest scores.

The two pairs which have the lower scores (60% and 62%) is A & B and A & C. The division common to the two pairs, A, would have the lowest scores.

This will work only when there are three quantities to compare and we have their pair-wise comparison.

30. In finding the three averages, the total runs scored is the same. It is just the denominator that changes. If I_p, I_o and I_d are the number of total innings played, number of innings batsman is out and number of innings batsman is out for a duck, respectively, then...

Thus, $\frac{\text{Total Runs}}{I_O} = 45$.....(i)

$\frac{\text{Total Runs}}{I_o - I_d} = 54$...(ii)

$\frac{\text{Total Runs}}{I_p} = 36$.....(iii)

Thus, I_p: I_o: $(I_o - I_d) = \frac{1}{36} : \frac{1}{45} : \frac{1}{54}$ i.e., $\frac{1}{4} : \frac{1}{5} : \frac{1}{6}$ i.e., 15: 12: 10.

Thus, required ratio 15: (15 – 12): (12 – 10) i.e., 15: 3: 2

31. Let's assume that the number of students who got more than 70% is 100. Since their average is 74%, and the average of students who scored more than 60% is 68%, so each one of these 100 students contributes 6% to the students who are between 60 and 70% i.e., a total of 600. If we want to minimize the number of students, we

should give maximum possible to each student scoring between 60% and 70%. So, if each one of these students secured 60%, to bring the average up to 68%, we need to give 8% to each. Number of students would then be $\frac{600}{8} = 75$. So at least 75 students out of a total of 100 + 75 = 175 students should have secured between 60% and 70% i.e., a percentage of $\frac{75}{175} = \frac{3}{7} = 42.84$

Alternate Solution:

Let number of students scoring between 60% and 70% be x and their average marks scored be M%. So, the sum of their scores is $x \times M$%

Let the number of students scoring more than 70% be y and their average marks is given as 74%. So, the sum of their scores is $y \times 74\%$

Thus, the total number of students scoring more than 60% is $x + y$ and the sum of their scores is $(x \times M\% + y \times 74\%)$. Also, it is given that the average of these students is 68%. From this we can say that the sum of their scores is $(x + y) \times 68\%$.

Equating the sum of scores found by the two approaches,

$x \times M\% + y \times 74\% = (x + y) \times 68\%$

$y \times 6\% = x \times (68\% - M\%)$ i.e., $\frac{y}{x} = \frac{68-m}{6}$

Since we want to find the minimum value of $\frac{x}{x+y}$, i.e., the maximum value of $\frac{x+y}{x}$ i.e., $1 + \frac{y}{x}$. Hence $\frac{y}{x}$ has to be maximum, so M has to be minimum i.e., 60%.

With M = 60%, $\frac{y}{x} = \frac{68-60}{6} = \frac{4}{3}$ i.e., $\frac{y+x}{x} = \frac{7}{3}$ i.e., $\frac{x}{x+y} = \frac{3}{7} = 42.84$.

32. The average of 40 consecutive natural numbers would be the average of the middle two numbers of the sequence. Since any two consecutive numbers will have one odd and one even, the average will be a fraction. Since the average after removing a number is 45, the previous average would have been either 44.5 or 45.5. This means that the two middle numbers would have been 44 and 45 or 45 and 46. Since 40 consecutive numbers are required so we get two sequences:

25, 26, ...44, 45, ...63, 64 and 26, 27, ... 45, 46,.....64, 65

In the first sequence, the average goes up by 0.5 when a number is deleted. This means that the rest of the 39 numbers had given an extra 0.5 to this number i.e. this number must have been 0.5 × 39 less than the average. Average was 44.5 so the number erased = 44.5 – 19.5 = 25

In the second sequence, the average goes down by 0.5 when a number is deleted. This means that the number had given 0.5 to each of the rest of the 39 numbers i.e. this number must have been 0.5 × 39 more than the average. Average was 45.5 so the number erased = 45.5 + 19.5 = 65.

33. When three consecutive numbers are erased from a list of consecutive numbers, the maximum change in the average will be 1.5. Since the new average is approximately 21.75, the average of the series before numbers were erased would have been 21.75 ± 1.5 i.e. between 20.25 and 23.25

Also, the average of a set of consecutive numbers has to be either an integral value or an (integer + 0.5). Hence average of the series before the numbers were erased could have been 20.5, 21, 21.5, 22, 22.5 or 23. Accordingly the series of numbers starting from 1, could have been till 40 or 41 or 42 or 43 or 44 or 45. But when 3 numbers are erased, the number of numbers remaining has to be a multiple of 19 (because the average of the rest of the numbers is of the form $\frac{***}{19}$). Thus, the original number of numbers has to be 41 from the above list (only then will the number of numbers remaining, after 3 are erased, be 38 which is a multiple of 19). Originally the series is of 41 numbers with an average of 21. Three numbers are erased and average becomes $21\frac{15}{19}$ i.e., average of the rest of the 38 numbers increases by $\frac{15}{19}$ i.e., the three numbers must have 'borrowed' $\frac{15}{19} \times 38 = 30$ from the 38 others so that their average would rise up to 21. Hence the average of the three erased numbers has to be 11.

34. Overall profit percentage will be the weighted average of the two profits where the cost incurred on the entire lot will be the weight. Since cost price of an orange and a mango is in the ratio 1 : 3, and they were bought in equal number, the total cost incurred on oranges is, say, x and total cost incurred on mangoes is say $3x$

Overall profit percentage $\frac{20\times x+30\times 3x}{4x} = 27.5\%$

Alternately, if you do not want to view the question as that of weighted average and just solve it using fundas of Profit and Loss…

Since equal numbers of oranges and mangoes are purchased, let's say the person just bought 1 orange and one mango at the rate of Rs. 1 and Rs. 3. Thus his total cost is Rs. 4 and total profit is 20% of Rs. 1 + 30% of Rs. 3 = 0.2 + 0.9 = 1.1. Thus, profit percentage = $\frac{1.1}{4} \times 100 = 27.5\%$

35. Since both covered the same distance and reach the finishing line at the same time, so both must have had the same average speed. The average speed of A is 25 kmph. So average speed of B should also be 25 kmph. B covers some distance at 20 kmph and some at 30 kmph. Since 25 is the arithmetic mean of 20 and 30, the time for which B has run at the two speeds is the same. Since time is same, distance run will be proportional to the speeds and the ratio of the distances run at the two speeds will be in the ratio 2: 3.

This question is just checking that you are aware of the fact that in using weighted average to find average speed, the weights are the 'time' run at various speeds and not the 'distance' covered at various speeds.

36. Let the percentage of iron in the two alloys be P_1 and P_2. Let the weight of the piece cut off from the alloys be x.

Then $\frac{(6-x)P_1+xP_2}{6} = \frac{(12-x)P_2+xP_1}{12}$

Cross-multiplying, $12P_1 - 2xP_1 + 2xP_2 = 12P_2 - xP_2 + xP_1$

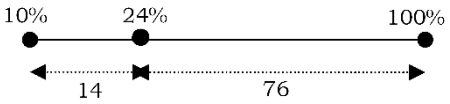
Re-arranging, $12(P_1 - P_2) = 3(P_1 - P_2)$.

Since P_1 is not equal to P_2, the term $(P_1 - P_2)$ can be cancelled and we get $x = 4$.

Short-cut:

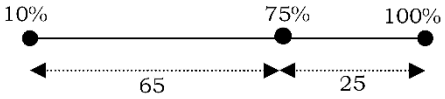
When would the weighted average of P_1 and P_2 in a mixture be equal to the weighted average of P_1 and P_2 in another mixture? Only when the ratio of the weights in the two cases are equal. Thus, $\frac{6-x}{x} = \frac{x}{12-x} \Rightarrow 72 - 18x + x^2 = x^2$ i.e., $18x = 72$ i.e., $x = 4$.

37. We can consider that the second-grade cotton is a mixture of first grade cotton and waste. Thus, working with percentage of waste in the alligation..



Thus, the ratio of mixing of first grade cotton and pure waste to result in second grade cotton is 76: 14 i.e., 38: 7. This can also be interpreted as: in (38 + 7) = 45 kgs of second grade cotton, there would be 38 kgs of first grade cotton. Thus in 27 kgs of second grade cotton, there would be $27 \times \frac{38}{45} = \frac{114}{5} = 22.8$ kgs of first grade cotton.

38. We can consider that fresh grapes is a mixture of dry grapes and water. Thus, working with percentage of water in the alligation.



Thus, the ratio of mixing of dry grapes and pure water to result in fresh grapes is 5: 13. This can also be interpreted as: in (5 + 13) = 18 kg of fresh grapes, there would be 5 kg of dry grapes. Thus, to obtain 15 kg of dry grapes, we would need 18 × 3 = 54 kg of dry grapes.

39. This is a logical question rather than mathematical.

Since the sum of the numbers in the two ratios is 5, we could assume the total volume in each to be $5k$. Thus, total amount of milk in the two vessels combined = $2k + 4k = 6k$

Since no content spills out, after the removal and transfer, the volumes in the two vessels is still going to be $5k$ (the fraction removed from each also necessarily has to be the same, but is not of importance here). In the first vessel, since ratio of milk and water is 1: 1, milk will be $2.5k$. Thus, rest of the milk, $6k - 2.5k = 3.5k$ will be in the second vessel. The rest of the solution in second vessel, $5k - 3.5k = 1.5k$ will be water. Thus, ratio of milk and water in second vessel is 3.5: 1.5 i.e., 7: 3.

40. Each time the operation is done, the total volume of the solution increases by 10 litres.

Working on proportion of milk:

When part of a solution is flushed away, proportion of milk in the solution does not change. Only when water is added, does the proportion of milk change. Further since pure water is added, proportion of milk varies inversely with the volume.

In the first operation, when water is added, volume is increased from 20 litres to 40 litres.

In the second operation, when water is added, volume is increased from 30 litres to 50 litres.

And so on…

Thus, proportion of milk after four operations = $1 \times \frac{20}{40} \times \frac{30}{50} \times \frac{40}{60} \times \frac{50}{70} = \frac{1}{7}$

Further after four operations, the volume of the final solution is 70 litres. Thus, volume of milk = $\frac{1}{7} \times 70 = 10$ litres.

41. Continuing after 4 operations, proportion of milk after five operations = $\frac{1}{7} \times \frac{60}{80} = \frac{3}{28}$

Thus, proportion of milk and water after five operations = 3: 25.

Alternate Solutions: Working on quantity of milk: When one is working on quantity of milk, the workings will be exactly similar to the above, but the way one has to think is very different. While working on quantity of milk, when water is added, quantity of milk does not change, but when part of the solution is removed, milk is also removed and quantity of milk decreases.

If $\frac{1}{x}^{th}$ part of a solution of milk and water is removed, $\frac{1}{x}^{th}$ part of the quantity of milk present is also removed. In the first operation, 10 litres are removed from 30 litres, i.e., $\frac{1}{3}^{rd}$ of the solution is removed. So $\frac{1}{3}^{rd}$ of the amount of milk present will also be removed and only $\frac{2}{3}^{rd}$ of the milk present will remain.

In the second operation, 10 litres are removed from 40 litres i.e., $\frac{1}{4}^{th}$ of the solution is removed. So $\frac{1}{4}^{th}$ of the amount of milk present will also be removed and only $\frac{3}{4}^{th}$ of the milk present will remain. And so on for successive removals.

Thus, volume of milk left after four operations = $30 \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} = 10$ litres.

Volume of milk present after 5 operations = $10 \times \frac{6}{7} = \frac{60}{7}$. Total Volume after 5 operations will be 30 + 10 × 5 = 80 litres. Thus, the rest of it will be water i.e., amount of water = $80 - \frac{60}{7} = \frac{500}{7}$. Thus, ratio of water to milk after 5 operations = 500: 60 i.e., 25: 3.

42. This question is unique one, in the sense that it is asking the ratio of *water* in *vessel 1* to the ratio of *milk* in *vessel 2*. Irrespective of the way transfers take place, if two vessels, each has initially pure milk and pure water respectively (they could also be of different volumes) and if after any number of transfers (in any manner), the final volumes of the two vessels are the same as the initial volumes as each had initially, the ratio of *water* in one vessel to that of *milk* in other has to be 1: 1.

43. In such questions where part of solution is transferred from one vessel to another, an amazingly smart way of tackling would be to use the following:

1. Work on any one part, either milk or water and not on both. After complete number of operations, the volume is same as original volume, 10 litres in this case. Thus, if you know the amount of either water or milk, you can also know the volume of the other.

2. Let's say we are working on amount of milk. We know that across the two vessels, at any point of time, amount of milk is 10 litres. Thus, if we know the amount of milk in one vessel, we know the amount of milk in the other vessel.

3. When $\frac{1}{x}^{th}$ part of a solution of milk and water is removed, $\frac{1}{x}^{th}$ part of the quantity of milk present is also removed.

From each of the vessel, $\frac{1}{10}^{th}$ of the solution is drawn out. Thus $\frac{1}{10}^{th}$ of the milk present in the vessel will also be drawn out and $\frac{9}{10}^{th}$ will remain behind. Thus…

Quantity of milk in first vessel after 1st operation = $\underbrace{\frac{9}{10}^{th} \times 10}_{\text{milk remaining}} + \underbrace{\frac{1}{10}^{th} \times 0}_{\text{added from other vessel}} = 9$

If vessel 1 contains 9 litres of milk, vessel 2 will have 1 litre of milk.

Quantity of milk in first vessel after 2nd operation = $\frac{9}{10}^{th} \times 9 + \frac{1}{10}^{th} \times 1 = 8.1 + 0.1 = 8.2$

If vessel 1 contains 8.2 litres of milk, vessel 2 will have 1.8 litre of milk.

Quantity of milk in first vessel after 3rd operation = $\frac{9}{10} \times 8.2 + \frac{1}{10} \times 1.8 = 7.38 + 0.18 = 7.56$

Amount of milk in vessel 1 after 3 operations = 7.56 and amount of water will be 10 – 7.56 = 2.44. Thus, ratio of milk and water in vessel 1 will be 756: 244 i.e., 189: 61.

By the logic stated in the answer to the earlier question, the ratio of milk and water in the other vessel has to be 61: 189.

44. When 20% of the solution is removed, it does not change the proportion of milk, it just reduces the volume to 80%. Thus, when the part of the solution removed is replaced with solution having proportion of milk as $\frac{1}{4}$, the ratio of the weights in all the three instances of replacing is 4: 1.

After first replacement, milk concentration = $\frac{\frac{3}{4}\times 4+\frac{1}{4}\times 1}{5} = \frac{13}{20}$

After second replacement, milk concentration = $\frac{\frac{13}{20}\times 4+\frac{1}{4}\times 1}{5} = \frac{57}{100}$

After third replacement, milk concentration = $\frac{\frac{57}{100}\times 4+\frac{1}{4}\times 1}{5} = \frac{253}{500}$

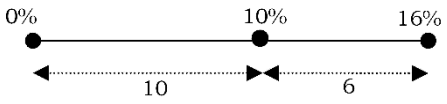
Ratio of milk and water in the solution now = 253: 247

Short-cut:

The initial solution had percentage of milk as 75% and $\frac{1}{4}^{th}$ of it is, successively, being replaced by 25% milk solution. The operation is done thrice. In such a situation the final percentage of milk will be $25\% + \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} \times (75\% - 25\%) = 25\% + \frac{64}{125} \times 50\%$, which in fraction terms is $\frac{1}{4} + \frac{64}{125} \times \frac{1}{2} = \frac{1}{4} + \frac{32}{125} = \frac{253}{500}$.

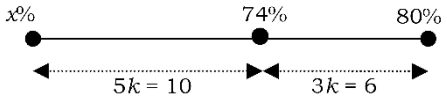
45. Method 1:

We should realize that Tin is not present in Brass. Thus, starting with Tin, percentage of Tin in Brass is 0%, in Bronze is 16% and that in a mixture of Brass and Bronze is 10%.



Thus, by using alligation/weighted average, we can find that Brass and Bronze is mixed in the ratio 3: 5.

Now working on either of Copper percentage or Zinc percentage, we will work on Copper, when Brass having x% Copper is mixed with Bronze having 80% Copper in the ratio 3: 5, we get an alloy having 74% Copper. Again, using alligation/weighted average, we can find the percentage of Copper in Brass.



Thus x% = 74 – 10% = 64%. Thus, Brass has 64% Copper and rest 36% Zinc. Thus, ratio of Copper to Zinc in Brass is 16: 9

Method 2:

Let the amount of Bronze in the fused mass be 100 gm. Then the amount of tin will be 16 gm in it. This 16 gm is 10% of the total fused mass hence weight of total fused mass is 160 gm. This implies that the weight of Brass in the fused mass is 60 gm. Final percentage of Zinc in the fused mass is 16%. Let x be Zinc percentage in Brass.

$\frac{\frac{4}{100} \times 100 + \frac{x}{100} \times 60}{160} = \frac{16}{100}$ $i.e.\ x = 36$. Hence in Brass, Zinc is 36% and Copper is 100 - 36 = 64%. Ratio of Copper to Zinc = 16: 9.

Exercise 7

1. Let initially the sides be x, y and z. Initial volume = xyz
After the change sides will be 1.2x, 1.2y and 1.2z.
New volume = 1.728(xyz). ∴ Increase in volume is 72.8%.

2. The percentage of charcoal = [100 – (75 + 10)] % = 15 %
Quantity of charcoal = 15 % of 24 kg = 0.15 × 24 = 3.6 kg

3. Let total number of votes be x. Then, 30% of x + 15000 = 70% of x
⇒ 15000 = 0.4x ⇒ x = 37500.
The required number = 70% of 37500 = 0.7 × 37500 = 26250

4. Net percentage change = a + b + ab/100 = 30 + 30 + (30 × 30)/100 = 69%.

5. Suppose, he has 100 fruits. He sells 30 fruits and throws away 40% of 70 i.e., 28 fruits. Now he was left with 42 fruits. Next day, he sells 50% of 42 = 21 fruits. So, he throws 21 + 28 = 49 fruits. Hence, the required percentage = 49%.

6. Let the original price of the bicycle be Rs. x. Then,
x × 12/11 × 13/12 × 14/13 = 1274 ⇒ = 91 × 11 = Rs. 1,001.

7. Let the time he takes to answer 1 question is 1 minute. So, he takes 12 minutes to read a passage. Total time spent by him = 12 × 4 + 5 + 8 + 8 + 6 = 75 minute.

If the total time is reduced by 10%, then total time = 75 × 0.9 = 67.5 minutes.

This reduction of 7.5 minutes should be in the reading time. So, the new reading time = 48 – 7.5 = 40.5 minute. The ratio of original time to new time is 48: 40.5.

Therefore, the ratio of original speed to new speed is 40.5: 48

Increase in speed = 7.5 and percentage increase = 7.5 × 100/40.5 = 18.5%.

8. Expenditure in this case will be 0.96 × 0.92 = 0.8832times the earlier expenditure.

Thus, the expenditure will decrease by (1 – 0.8832) × 100 = 11.68%.

9. Let the total students are 100.

Number of boys = 70

Number of boys playing cricket = 10% of 70 = 7.

Total number of students who play cricket = 14.

Percentage of boys playing cricket = 7 × 100/14 = 50%.

10. Let there be 100 products in the stockpile. Hence, products from M1 = 40, from M2 = 30 and from M3 = 30. Number of defective products from M1 = 0.03 × 40 = 1.2, from M2 = 0.01 × 30 = 0.3 and from M3 = 0.05 × 30 = 1.5. Therefore, total number of defective products = 3, and percentage of defective products = 3%.

Alternative method:

Total percentage defectives = 3% of 40% + 1% of 30% + 5% of 30% = 3% of total.

11. We have 7(x + y) = 4x + 8y where x and y are the production for first and second quarter. The equation gives x: y = 1: 3.

12. Let total score = x

Passing marks = 0.39x + 58 = 0.55x – 22 ⇒ 80 = 0.16x ⇒ x = 80/0.16 = 500.

13. Weight of first lump = x kg

Weight of second lump = (20 – x) kg

Now, 31.25x + 30(20 – x) = 617.5 ⇒ x = 14 kg

Total gold = 75% of 14 + 85% of 6 kg = 15.6 kg

Now percentage of gold = 15.6 × 100/20 = 78%.

14. We have (A – 20) = 0.4(B + 20), i.e. A – 0.4B = 28 and (B – 40) = 0.4(A + 40), i.e., B – 0.4A = 56. Solving, we get A = Rs. 60

15. Let 100 be the total number of people, college will ask for donations.

∴ People already solicited = 60.

Amount raised from the people solicited = 600 × 60 = 36000

Now 36000 constitutes 75% of the amount. Hence, remaining 25% constitutes 12000. ∴ Average donation from remaining people = 12000/40 = 300.

16. Assume x be population of the town.

Number of people who work = 0.37x × 0.76 + 0.63x × 0.54 = 0.6214x

⇒ 0.6214x = 124280 ⇒ x = 2,00,000.

17. Let weight of silver and copper be x and y respectively, in alloy A.

Silver in alloy B and C is x + 15 and x + 9 respectively and copper in alloy B and alloy C is y each.

From B: $\frac{x+15}{x+y+15} = \frac{90}{100}$ ⇒ x – 9y = - 15

From C: $\frac{x+9}{x+y+10} = \frac{84}{100}$ ⇒ 4x – 21y = –15

∴ x = 12, y = 3. ∴ Percentage of silver in A = $\frac{12}{15} \times 100 = 80\%$

Exercise 8

1. Let the number of votes polled be x.

Number of votes secured by Ashok = 70x/100 = 0.7x

He won by a majority of 0.4x votes. Therefore, 0.4x = 168 ⇒ x = 420.

2. Let Ravi's salary after the increment be 4x. Therefore, before the increment it was 3x and the total family income was 15x. After Ravi's increment, the family income is 16x. Required percentage = 4x × 100/16x = 25%.

3. Let the price of the TV set is Rs 100. The new price = Rs 80.

Now, if it has to be same as before, then the price should be increased by Rs 20.

Required %age = 20 × 100/80 = 25%.

4. Let the initial price of the article be Rs 100

Decrease in the price = Rs 20

Price after the decrease = Rs 80

Increase in the price = Rs 16

Price after the increase = Rs 96

Therefore, the price effectively decreases by 4%.

5. Let the third number is 100

Therefore, the first number is 60 and the second number is 50.

Required %age = $\frac{60}{50} \times 100 = 120\%$.

6. Let the total marks be x and the pass marks be p

0.38x = p + 18 ——— (1)

0.27x = p - 37 ——— (2)

Solving (1) and (2) gives 0.11x = 55 ⇒ x = 500 and p = 0.38x – 18 = 172

Therefore, the required percentage = 172 × 100/500 = 34.4%.

7. Let the salary of the family is Rs 100 and the expenditure on household items = Rs 20.

New expenditure of the family = 20 × 1.2 = Rs 24 and the new salary of the family = Rs 110.

Required %age = 24×100/110 = 21 9/11%.

8. Let the manufacturing cost in 2021 be Rs 100

Transportation cost in 2021 = Rs 30

Total cost in 2021 = Rs130

Manufacturing cost in 2022 = 100 × 1.2 = Rs 120

Transportation cost in 2022 = 33.33% of 120 = 1/3 × 120 = Rs 40

Total cost in 2022 = Rs 160

Required %age = $\frac{30}{160} \times 100 = 18.75\%$.

9. Let the male population and the female population in 2021 be x and y respectively.

Population of males in 2022 = 3x/4 and of females is 6y/5. As 44.44% or 4/9 of total population are females, so 5/9 of the total population will be males. Therefore, ratio of males and females in 2022 is 5: 4

So, $\left(\frac{3x}{4}\right) = \frac{5}{4} \times \frac{6y}{5} \Rightarrow$ x = 2y or x: y = 2: 1. Therefore, males were 2/3 of 66.67% of the total population.

10. Let the length and the breadth be ℓ and b respectively. ℓ ≥ b.

If ℓ = b, percentage increase in the perimeter is 15%. Also, if ℓ > b, percentage I increase is more than 15%. Therefore, the percentage increase can be 16%.

11. Price of 4 balls = Price of 1 pair of gloves

Price of 1 pair of pad = Price of 3 pairs of gloves = Price of 12 balls

Price of 1 bat = Price of 2 pairs of pads = Price of 24 balls

Price of a ball as a percentage of price of a bat $\frac{1}{24} \times 100 = 4.166\%$

12. At the end of the year, value of building = value of land

500000(1 – x%) = 300000(1 + x%) i.e., 5 – 5x % = 3 + 3x % i.e., 8x % = 2 i.e., x % = 1/4 i.e., 25%.

13. Let the value of the land be x, value of the building = 1000000 – x

$\Rightarrow \frac{6x}{5} \times x + \frac{9}{10} \times (1000000 - x) = 1020000$ i.e., x = 400,000

Alternately:

Say the land also depreciates by 10% along with the building. Then the combined valuation should have been 9,00,000. But the valuation is 10,20,000 i.e. 1,20,000 higher. This is because instead of increasing the value of land by 20%, we have decreased the value of land by 10%. Thus 30% of the land = 1,20,000 i.e. original value of land = 4,00,000.

14. A x% increase and a x% decrease always results in a overall percent decrease. Thus, a p% increase and a p% decrease will lower my salary. And a q% increase and a q% decrease will yet again decrease the salary. Thus, for whatever values of p and q, Y is lesser than X.

15. Surface Area α (Radius)^2

$\frac{Final\ surface\ area}{Initial\ surface\ area} = \frac{144}{100}\ i.e., \frac{(Final\ Radius)^2}{(Initial\ radius)^2} = \frac{144}{100}$

$\Rightarrow \frac{Final\ radius}{Initial\ radius} = \frac{12}{10} = \frac{6}{5}$. Hence percentage increase in the radius = 20%.

Again, Volume of a sphere α (Radius)^3.

$\Rightarrow \frac{Final\ volume}{Initial\ volume} = \frac{(Final\ Radius)^3}{(Initial\ radius)^3} = \frac{216}{125} = 1.728.$

16. Let the base and height of the triangle is 'b' and 'h' respectively.

The area of the triangle is 1/2 × b × h.

As the height increased by 14.28%, the new height is 8h/7. Let 'B' is the new base. Therefore, 1/2 × 8h/7 × B = 8/5 × (1/2 × b × h)

⇒ B = 7b/5 = b + 2b/5.

Therefore, the breadth should increase by 40%.

17. Let the production of coal in 2005 is 100.

Production of coal in 2006 = 110.

Production of Lignite in 2005 = 25% of 100 = 25

Production of Lignite in 2006 = 20% of 110 = 22

Change in production of lignite in 2006 as compare to 2005

$= \frac{(22-25)}{25} \times 100 = -12\%.$

18. Let total number of units of product A and B be *x*. Hence the number of units of A and B will be 2x/5 and 3x/5.

Amount contributed by A = 2x/5 × 5000 = 2000x0

$2000x$ is 25% of total value of inventory. So, value of rest of the inventory will be 75% i.e., $2000x \times 3 = 6000x$

Value of each unit of B = $\frac{6000x}{\frac{3x}{5}} = Rs\ 10000.$

19. If there were 100 voters, 60 supported A and 40 supported B. On polling day,

60 – *x*% of 60 + *x*% of 40 = 50 i.e., *x*% of 20 = 10 i.e., *x* = 50%

20. Comparing Maruti and Ikon: Please note that mileage is km per litre. And since this of Maruti is 20% more than that of Ikon, in 1 litre of petrol, the distances covered by Maruti and Ikon will be in the ratio 6: 5. To travel the same distance, the consumption of petrol of Maruti and Ikon will be in the ratio 5: 6.

Comparing Ikon and Santro: The consumption i.e., litres/km of Ikon is 20% more than that of Santro. Thus, to travel the same distance, the consumption of petrol of Ikon and Santro will be in the ratio 6: 5.

Thus, consumption to travel same distance by Maruti, Ikon and Santro will be 5: 6: 5 and since cost is directly related to consumption, the cost of driving a Maruti and Santro over same distance will be the same.

21. If the car gives 20% more mileage per litre of petrol, it covers 6/5 times the original distance for the same fuel. That means it uses $5/6^{th}$ of the original amount of fuel used for the same distance covered.

Expenditure = Price of fuel × Quantity of fuel

New Expenditure = (11/10 × Price of fuel) × (5/6 × Quantity of fuel) = 11/12 × Expenditure. The new expenditure is 8.33% less than the previous expenditure.

22. In this case the labour charges remain the same. So, we should make use of this fact.

Say, the total cost of the watch is Rs.4, then labour charge will be Re.1. After the increase, the labour cost i.e., Re.1 will be 20% of the increased cost i.e., the increased total cost is Rs.5 i.e., an increase of Re. 1 from the earlier cost of Rs. 4. The increase in the cost Re.1 is the increase in material cost. Thus, material cost increased from Rs. 3 to Rs. 4 i.e., an increase of 33.33%.

23. Let's say the cost of the car is 100, then cost of engine is 20 and the cost of the rest of the car is 80. After the increase, this 80 is 75% in the car cost. So increased car cost is 80 × 4/3 = 320/3.

25% of this is the engine cost, which will be 320/3 × 1/4 = 80/3. So, percentage increase is $\frac{\frac{80}{3}-20}{20} \times 100 = 33.33\%.$

24. Let the number of correct questions = *x*, number of incorrect questions = y

Total number of attempted questions = $x + y$

Hence his score = *x*. If he had attempted 60% more questions, he would have attempted $\frac{3(x+y)}{5}$ more questions. If he would have made 20% fewer errors, another y/5 would have got added to his score.

Hence, $x + \frac{3(x+y)}{5} + y/5 = 2x,\ i.e., x = 2y.$

Number of wrong questions is half of the number of correct questions, so 2/3rd of the attempted questions is correct, which is 66.66%.

25. 90% of the voters voted and 60 were illegal. Thus, there were ((90% of total) – 60) legal votes. Half way mark of these is ((45% of total) – 30). Of the legal votes, Obama secured 154 more than half and McCain secured 154 less than half. Thus, (45% of total) – 30 + 154 = (47% of total) i.e., 2% of total = 124 i.e., total votes = 6,200. And legal votes = 5,580 – 60 = 5,520.

QUANTIFIERS CAT ACADEMY

Exercise 9

1. Suppose Rehman buys (LCM of 15, 20 and 35) = 420 apples.

Total cost of apples bought at 15 for a rupee = 420/15 = Rs 28

Total cost of apples bought at 20 for a rupee = 420/20 = Rs 21

∴ Total C.P = Rs. (28 + 21) = Rs. 49

S.P for (420 + 420) 840 apples = 840 × 2/ 35 = Rs 48

So, %age loss = $\frac{49-48}{49} \times 100 = 2.04\%$.

2. Let the cost of each apple be Rs x

Cost of 3 apples = Rs 3x

Selling price of 3 apples = Cost price of 4 apples = Rs 4x

⇒ Profit on selling 3 apples = Rs x

⇒ %age profit = $\frac{x}{3x} \times 100 = 33.33\%$

3. SP (5m) = CP (5m) + Profit (5m)

As Profit (5m) = SP (2m),

SP (5m) = CP (5m) + SP (2m)

⇒ SP (3m) = CP (5m) ⇒ SP/CP = 5/3

⇒ %age profit = $\frac{5-3}{3} \times 100 = 66.67\%$

4. Let the cost of each gm to the trader be Rs 1.

Cost price of 1000 gm = Selling price of 900 gm.

Cost of 900 gm = Rs 900

Selling price of 900 gm = Rs 1000

⇒ %age profit = $\frac{1000-900}{900} \times 100 = 11.11\%$.

5. Let the cost price of A be Rs x

Selling price of A = Cost price of B = Rs 1.1x

Selling price of B = Cost price of C = Rs 1.32x

Given that 1.32x = 660 ⇒ x = 500. So, A bought it at Rs 500.

6. Let the cost price of Kiran be Rs x. Selling price of Kiran = Rs 0.8x

If he sold it at 10% profit, his selling price = Rs 1.1x

Given that 1.1x = 0.8x + 300 ⇒ x = 1000

7. CP = SP + Loss

⇒ CP (15kg) = SP (15kg) + SP (3kg)

⇒ CP (15kg) = SP (18 kg) = Rs 540 (given) ⇒ selling price per kg = 540/18 = Rs 30.

8. Suppose that Shyam purchased 3 kg for Rs 1800

Cost price of 1 kg = Rs 600

He sold one kg at a loss of 20%. Hence his selling price of this kg = Rs 480

His total selling price must be (1.2) × 1800 = Rs 2160

Hence selling price of the remaining 2 kg must be Rs 1680.

Cost price of 2kg = 2 × 600 = Rs 1200.

So, Profit percentage = $\frac{1680-1200}{1200} \times 100 = \frac{480}{1200} \times 100 = 40\%$.

9. When two items are sold at the same selling price with one being sold at x% profit and the other being sold at x% loss, on the overall a loss is always made and the overall loss percentage is given by $x^2/100\%$. As Balu sold both TV sets at the same price and x = 10, he must have made an overall loss of $10^2/100 = 1\%$.

10. Let the cost price of the watch be Rs 100. Let the marked price be Rs x.

Selling price = Rs 0.8x. Also given that the selling price = Rs 125

⇒ 0.8x = 125 ⇒ x = 125/0.8 = 156.25

So, Percentage of mark-up = 56.25%.

11. Suppose 2 articles have the same cost price and one is sold at x% profit and the other is sold at y% loss. The overall profit/loss percentage is given by 1/2 (difference of x and y)%. As x = 20 and y = 10, overall Profit/loss percentage = 5% profit.

12. Let the cost price of the item be Rs x. Profit = Rs 0.2x

Now, x + 0.2x = 60 ⇒ x = 50

To gain 30%, profit must be Rs 0.3x, so, the selling price must be Rs 1.3x = Rs 65.

13. Let the cost price of each in be Rs x

Profit made on selling 5 m = cost price of 2 m = Rs 2x

Cost price of 5 m = Rs 5x

⇒ %age profit = $\frac{2}{5} \times 100 = 40\%$.

14. Total cost price = 80 × 12·50 = Rs 1000

Total profit = 80 × 4.50 = Rs 360

⇒ Total selling price = Rs 1360

Total selling price of 20 articles = Rs 360

Selling price of each of the remaining articles = (1360 – 360)/60 = Rs 16.67.

15. Let the cost price = Rs 100

Marked price = Rs 125

As discount is 30%, so, the selling price = 125 × 0.7 = Rs 87.5

Therefore, the loss is 100 – 87.5 = Rs 12.5 or 12.5%.

16. Here, the cost price = Rs 360 and profit is Rs 40, therefore, selling price = Rs 400

Let the marked price is Rs x. As the discount percentage is 11 1/9% or 1/9 of marked price, so the selling price = 8/9 of marked price.

⇒ 8x/9 = 400 ⇒ x = 450.

17. 0.82CP + 990 = 1.15CP ⇒ 990 = 0.33CP ⇒ CP = Rs 3000

To make a profit of 10% he should sell at 3000 × 1.1 = Rs 3300.

18. Let the initial cost be Rs 100.

It went up by 40% i.e., new cost price = Rs140

Initial Selling Price = Rs 120. (as the profit is 20%)

Present Selling Price = 140 × 1.2 = Rs168

Percentage increase in selling price = (168 – 120) × 100/120 = 48 × 100/120 = 40%.

19. Let the cost price of 1 mango = Rs 1

Cost Price of 100 mangoes = Rs 100 …. (1)

Profit percentage expected = 25%

Therefore, proposed sale price per mango = Rs 1.25 …… (2)

20 mangoes are rotten; 80 mangoes are available for sale.

If these mangoes are sold at the original sale price of Rs1.25 per mango, the money received from the sale of 80 mangoes = 80 × 1.25 = Rs 100 …… (3)

As (1) and (3) are equal, there is neither profit nor loss.

20. Let the correct weight be 100 kg

Let its cost be Rs 100. So, S.P = Rs 120

Weight usually weighed = 80 kg

Present weight is 20% more = 96 kg

Selling Price = Rs 120

Cost of 96 kg = Rs 96

⇒ %age profit = $\frac{120-96}{96} \times 100 = \frac{24}{96} \times 100 = 25\%$.

21. Let the selling price of one television be x.

Total money made = 50x × 1/2 = 25x

(Since one TV is given free with each one bought by customer.)

Given total cost = 10,000 × 50 = 500000

Loss = 15x

Selling price + loss = cost price ⇒ 25x + 15x = 500000 ⇒ 40x = 500000

⇒ x = 12500. Therefore, selling price of each television = Rs 12500.

22. Let the total cost be Rs 100. Costs of P, Q and R are Rs 10, Rs 20 and Rs 20 respectively.

Let costs of others is 100 - (10 + 20 + 20) = Rs 50

Increase in each of the costs of P, Q and R is 2

Increase in the total cost is Rs 15

Increase in the cost of the others = 15 – 3 × 2 = Rs 9

Percentage increase in the cost of the others = 9 × 100/50 = 18%.

23. Suppose Rakesh purchased 2x chocolates.

He bought x chocolates at 15 paise each and x chocolates at 20 paise each.

Total cost price = 15x + 20x = 35x paise

Selling price of each chocolate = 20 paise

Total selling price = 20 × 2x = 40x paise

Profit = 5x paise. So, 5x = 1000 ⇒ x = 200 ⇒ 2x = 400.

24. Number of articles sold = 20% of 50 = 10

Cost of 5 articles = Rs 1200

Cost of 1 article = Rs 240

Cost of 10 articles = Rs 2400

Selling price of 10 articles = 2400 + 1200 = Rs 3600

Selling price of 1 article = Rs 360

Number of articles remaining = 40.

Total value of the remaining articles at selling price = 40 × 360 = 14400.

25. Let the cost prices for Ajay and Dinesh be Rs a and Rs d respectively.

Cost price of Balu = Rs 1.560.

So, Ajay's article's value increases by Rs 0.56a.

Cost price of David = Rs 0.8d.

Selling price of David = Rs 0.56d

⇒ 0.56a + 0.56d = 28000 ⇒ a + d = 50000

26. Let the marked price be Rs x.

Let the cost price be Rs 100. Profit = Rs 20

Selling Price = Rs 120

Discount = Rs 0.2x

Given, x - 0.2x = 120 ⇒ x = 150

Hence, the marked price is 50% above the cost price.

27. Let the cost of a car be 100x.

After a discount of 100000, the profit = 15%

⇒ Selling price of the car = 115x, and Marked price of the car = 115x + 100000

Given that the M.P of the car is 25% more than the C.P of the car

⇒125x = 115x + 100000 ⇒ 10x = 100000 ⇒ x = 10000

Marked price of the car = 115x + 100000 = Rs 1250000.

28. Let the cost price to Feroze be Rs 100

Feroze marks up the price to Rs 130

He sells it at 20% discount; at Rs 104

Sohail marks up the price to 20% more than Rs 100, equal to Rs 120

The least sale price = 104. To sell it at Rs 104, percentage discount to be offered

$= \frac{16}{120} \times 100 = \frac{40}{3} = 13\frac{1}{3}\%$.

29. Let the cost price of Girish be Rs 100.

Marked price of Girish = Rs 160

Let the maximum number of discounts be n.

$\Rightarrow 160 \times (0.9)^n \geq 100$ .....(1)

Maximum value of n satisfying (1) is 4.

30. If CP and SP are the original cost and selling price, then CP and 1.2 × SP would be the cost and selling price after the changes.

As profit increases by 50%, 1.2 × SP – CP = 1.5 × (SP – CP) i.e., 0.5 × CP = 0.3 × SP

⇒ SP = 1.66 × CP i.e., a profit of 66.66%.

31. Since profit % is $1/3^{rd}$, assuming CP as $3x$, the SP would be $4x$.

Initial SP = $4x$ – 160, Initial CP = $3x$ – 120.

$\frac{Initial\ SP}{Initial\ CP} = \frac{4x - 160}{3x - 120} = \frac{4 \times (x - 40)}{3 \times (x - 40)} = \frac{4}{3}$.

Initial SP and CP are also in the ratio 4:3, so initial profit percentage = 33.33%.

Alternately,

You should have immediately noticed that the profit would increase by Rs. 40, which is exactly 33.33% of the increase in the cost i.e., of Rs. 120. Since the final profit percentage is 33.33% and the profit percentage on the increase in cost is also 33.33%, the original profit percentage also has to be 33.33%.

32. Since CP increases by Rs. 50 but SP increases only by Rs. 40, the profit decreases by Rs. 10. This is equal to 12.5% of Initial profit i.e., $1/8^{th}$ of Initial profit i.e., initial profit = Rs. 80. Final profit = 80 – 10 = Rs. 70.

33. If cost price increases by 20%, selling price also has to be increased by 20% to maintain the profit percentage.

20% of the initial selling price = Rs.32, so initial selling price = Rs.160

Increased selling price = 160 + 32 = Rs.192.

34. Consider a trader is making p% profit on a cost of Rs. x. Say, the cost increases by Rs. y. If the trader increases the SP by just Rs. y, he would have just managed to recover the increase in cost. Thus, his profit amount would have been the same, as before the increase. But the profit percentage would decrease as he is making the same amount of profit on an increased cost price. To continue earning a profit percentage of p%, he would not only have to recover this additional cost, Rs y, but also earn a profit of p% on this addition cost.

Since the cost increased by Rs. 50, an increase of Rs. 50 in the SP just recovers the cost. The additional increase of Rs. 30 in the SP is to maintain the earlier profit percentage, say p%, and as stated above this is p% of the increase i.e., p% of 50

Thus 30 = p% of 50 i.e., p% = 60%. The margin percentage corresponding to a profit percentage of 60% is $\frac{60}{160} \times 100 = 37.5\%$.

35. Since we have to find the original price of one dozen of oranges, let it be p.

Then, as per the data given, $\frac{240}{p-1} - \frac{240}{p} = 1$ i.e., $\frac{1}{p-1} - \frac{1}{p} = \frac{1}{240}$.

Solve this equation to get p = 16.

36. Let us say the cost of the car = 100, then cost of the engine = 20

The cost of the engine increases by 10%, so final cost of engine = 22

Final cost of the car = 102 i.e., an increase of 2%. So, selling price should also increase by 2% to maintain profit percentage.

37. If the margin percentage is 10%, then profit percentage $= \frac{10}{100-10} \times 100 = 11.11\%$

If the cost increases by Rs. 45000, to maintain the profit percentage, the selling price should increase by Rs. 45000 + 11.11% of 45000 = Rs. 50000.

38. I. Two successive discounts of 10% and 20% is equivalent to a single discount of $-10\% - 20\% + \frac{10 \times 20}{100} = -28\%$

III. Three successive discounts of 10% each will result in a multiplying factor of 0.9^3 i.e., 0.729 i.e., an effective discount of 27.1%

IV. Two successive discounts of 15% each is equivalent to a single discount of –15% – 15% + $\frac{15 \times 15}{100}$= –27.75%

For a customer, getting a single discount of 30% is better than any of the above.

39. We know, discount, $22.22\% = \frac{2}{9}^{th}$. This means that if the total quantity of the goods being bought is 9, 2 out of the 9 are discounted i.e., are free. Since 9 is the total, x = 9 – 2 = 7 and y = 2.

40. Here, $\frac{MP_A \times \frac{7}{8}}{MP_B \times \frac{11}{12}} = \frac{SP_A}{SP_B}$ i.e., $\frac{MP_A}{MP_B} = \frac{11}{12} \times \frac{8}{7} \times \frac{7}{11} = \frac{2}{3}$

41. Since profits reduced by 37.5% because of the discount, 37.5% of profit = 30.

$\frac{3}{8} \times$Profit = 30 i.e., Profit = 80.

If cost price = Rs. 450 and profit was Rs. 80, then selling price = 450 + 80 = Rs. 530 which is also the list price.

42. If the MRP is assumed as Rs. 100, the cost to the trader would be Rs. 75. He has to make a profit of 10% on this Rs. 75 i.e., he has to sell it at Rs. 82.5. Thus, he can offer a discount of Rs. 17.5 on the MRP of Rs. 100 i.e., a discount of 17.5%.

43. Cost price to the retailer $= \frac{88}{100} \times 2400 = 24 \times 88$

If profit is 9.09%, selling price $= \frac{12}{11} \times 24 \times 88 = 96 \times 24$

Since discount is 4% of list price, $\frac{24}{25} \times$list price = 96 × 24 i.e., list price = Rs 2400

44. Since the profit % is less than the mark-up, obviously the reading of the balance is less than the actual weight. Say the balance reads x gm for 500 gm of weight kept.

If the rate of purchase of goods is Rs. 1 per gram, cost to the shop-keeper is Rs. 500 and he earns Rs. 1.2 × x in return. Thus, $\frac{1.2\times x}{500} = \frac{9}{8}$ i.e., x = 468.75 gm.

45. Assuming that a customer buys 100 gm of spices, each gram costing Re.1.

Food Shop: If spices have 10% impurities, in every 100 gm, 90 gm would be spices and the rest will be impurities. This is available at cost price, so it would cost Rs. 100. So Loss to the customer = $\frac{10}{100} \times 100 = 10\%$

Mega shop: For every 100 gm, the customer gets only 90 gm. Since there is a markup of 1%, so he pays Rs. 101 for quantity worth Rs. 90. Here since he is paying Re. 1 extra for the same quantity, his loss would definitely be more than the 1st case. So, we don't need to calculate it.

Family shop: If he adds 5% impurities, in every 100 gm, 95 gm would be spices and the rest will be impurities. This is available at 5% mark up, so it would cost Rs. 105.

Loss to the customer = $\frac{10}{105} \times 100 = 9.5\%$

Hence, the customer should go to Family shop.

Exercise 10

1. If CP = Rs. 100, MP = Rs. 120.

Average SP = Rs $\left(\frac{1}{4} \times 108 + \frac{1}{2} \times 120 + \frac{1}{4} \times 90\right)$

= Rs. (27 + 60 + 22.5) = 109.5

Hence, profit percentage = $\frac{109.5 - 100}{100} \times 100 = 9.5\%$.

2. Let the cost of 1 apple be 'a', cost of 1 mango be 'm' and the cost of 1 orange be 'o'

Therefore, (2a + 3m + 4o) – (a + 2m + 2o) = 6 ...(i)

And (3a + 3m + 5o) – (a + 2m + o) = 8 ...(ii)

2 × (ii) – (i): 3a + m + 6o = 10.

To make a profit of 5% one should sell 3 apples,

1 mango and 6 oranges at Rs. 1.05 × 10 = Rs. 10.5.

Percent discount = $\frac{12 - 10.5}{12} \times 100 = 12.5\%$.

3. Suppose he invests Rs. 150 in each of the lots.

So total investment = Rs. 300.

He will get 5 dozen of first lot and 3 dozen of second lot. In all he will have 8 dozen, selling price of which will be 40 × 8 = Rs. 320

∴ Profit percentage = $\frac{320 - 300}{300} \times 100 = \frac{20}{3} = 6.67\%$

4. Let the list price of the book be Rs. 100. The booksellers cost price (CP) is Rs. 58. He earns 20% profit on his selling price (SP)

⇒ 0.2 SP = (SP – CP) ⇒ SP = CP / 0.8 = 58/0.8 = 72.5

Hence, maximum discount that can be offered = (100 – 72.5) = 27.5%.

5. Let the cost price of the washing machine be Rs. 100 and list price be Rs. x.

Then, x – x × 20/100 = 112

⇒ 0.8x = 112 ⇒ x = Rs. 140

When the shopkeeper gives a discount of 25%, then selling price = 140 – 0.25 × 140

= 140 – 35 = Rs. 105. Hence, there will be a profit of 5%.

6. Loss = CP – SP.

1 dozen SP = 6 dozen CP – 6 dozen SP

7 dozen SP = 6 dozen CP. Therefore, %age loss = 1/7 × 100 = 14.28%.

7. SP of television = 3200 × 0.75 × 0.85 = Rs 2040.

8. Total selling price = Rs. 24,000 and in this type of transaction there is always loss.

Therefore, %age loss = $20^2/100$ = 4%.

Now, total cost price = 24000/0.96 = Rs 25000

Therefore, Loss = 25000 – 24000 = Rs. 1,000.

9. Let the CP be Rs. x

∴ SP = Rs. 1.2 x

New CP = Rs. 0.8x

New SP = Rs. (1.2x – 5)

So, 1.25 (0.8x) = 1.2x – 5 ⇒ x = 1.2x – 5. ∴ x = 25.

10. Let CP of one article = Rs. 1

CP of 5 articles = Rs. 5.

SP of 5 articles = CP of 4 articles = Rs. 4.

Hence SP < CP ⇒ loss = 5 – 4 = 1 ⇒ %age loss = 1/5 × 100 = 20%.

11. Suppose the initial cost price be Rs. x and the selling price be Rs. y. Initial profit = Rs. y – x. After the increase in manufacturing, cost price = Rs. 1.2x

New profit = Rs. (y – 1.2x)

∴ (y – x) – (y – 1.2x) = 20

⇒ y – x – y + 1.2x = 20 ⇒ 0.2x = 20 ⇒ x = 100.

12. The selling price of the item = 850 × 0.8 = Rs. 680.00

∴ Profit percentage = $\frac{(680 - 500)}{500} \times 100 = \frac{180}{500} \times 100 = 36\%.$

13. Let the CP of each T-shirt be Rs. x

CP of the (3 + 1) T-shirts to the shop = 4x

SP of the 3 T-shirts = 1.6 × 3x = 4.8x

Profit percentage = $\frac{0.8x}{4x} \times 100 = 20\%.$

14. Let the weight of 'p' apples be equal to 1 kg and the weight of 'q' oranges be equal to 1 kg.

Let the cost price and selling price of 1 apple be 'C_1' and 'S_1' respectively.

Let the cost price and selling price of 1 orange be 'C_2' and 'S_2' respectively.

Therefore, $10 \times C_2 = p \times C_1$ ....(i)

$q \times C_2 = 12 \times C_1$ ....(ii)

$15 \times S_2 = p \times S_1$ ....(iii)

Let the selling price of 1 kg of oranges be equal to that of 'x' apples.

Therefore, $q \times S_2 = x \times S_1$ ....(iv)

Dividing the equation (i) by (ii), we get that 10/q = p/12 ⇒ p × q = 120.

Dividing the equation (iii) by (iv), we get that 15/q = p/x ⇒ x = 8.

15. Let 'x' and 'y' be the cost price of 1 pen and 1 pencil respectively.

Let the selling price of 1 pen be 'z'.

Cost price of 3 pencils and 5 pens = 5x + 3y

Therefore, 5z = 1.2(5x + 3y) or z = 0.24(5x + 3y).

Also, 2z = 0.75(2x + 6y) or, 2(0.24(5x + 3y)) = 0.75(2x + 6y) or, 5x = 17y.

Selling price of 6 pens = 6z = 1.44 (5x + 3y) = 1.44 × (20y) = 28.8y

Cost price of 6 pens and 4 pencils = 6x + 4y = 24.4y

Profit percent = $\frac{(28.8y - 24.4y)}{24.4y} \times 100 = 18.03\%.$

16. Let x kg be the quantity sold at 17% profit, and the price of sugar be Rs. 1/kg

∴ 1.17x + (100 – x)1.07 = 110 ⇒ 0.10x = 3 ⇒ x = 30.

17. Piano is sold for Rs 325 × 0.8 = Rs 260 after giving 20% discount. After additional discount, it was sold for Rs. 234.

So, second discount percentage = $\frac{260-234}{260} \times 100 = \frac{26}{260} \times 100 = 10\%.$

18. Let the CP of 1000 g be Rs 100. ⇒ CP of 850 g = Rs.85

SP of 850 gm when he sells at 5% loss = 100 – 5 = Rs. 95

Percentage profit (m) = (95 – 85) × 100/85 = 200/17

SP of 850 g when he sells at 5% gain =100 + 5 = Rs. 105.

Percentage profit, n = (105 – 85) × 100/85 = 400/17.

Therefore, m + n = 600/17.

19. Let us look at the two equations. Let (5 pens + 7 pencils + 4 erasers) cost Rs. x, and (6 pens + 14 pencils + 8 erasers) will cost Rs. 1.5x.

In the second case, had Rajan decided to buy 10 pens instead of 6, it would have cost him Rs. 2x.

∴ (10 pens + 14 pencils + 8 erasers) = Rs. 2x.

Now, subtracting the second equation from the third, we get 4 pens cost Rs. 0.5x.

So, 5 pens will cost Rs. 0.625 x. This is the amount that I have spent on pens.

Hence, fraction of the total amount paid = 0.625 = 62.5%.

20. Revenue in 1 hr = $100\,(50 + x)\,\frac{(100-20-1.5x)}{100}$

Cost = $40\,(50 + x)\,\frac{(100+2x)}{100}$

Profit = $2000 - 2.3x^2 - 75x$. If profit > 0, x < 17.38 ⇒ x = 17.

21. He must have suffered only 10% loss in one item and earned 20% profit on the other item. Otherwise, he cannot make an overall profit.

Let the S.P. of both the items be Rs. 100x each.

Item	SP	% Profit	CP
1	100x	- 10	111.11x
2	100x	20	83.33x
Total	200x	-	194.44x

∴ Overall percentage profit = $\frac{(200-194.44)x}{194.44x} \times 100 = 2.86\%.$

22. Let C.P. of the book be Rs. x

Since loss = 20%, so, S.P. = Rs. 0.8x

If S.P. = Rs. (0.8x + 3), profit = 30%.

Hence, 1.3x = (0.8x + 3) ⇒ x = 6

If it is sold for Rs 6.60, then the profit = Rs 6.6 and %age profit = 0.6 × 100/6 = 10%.

23. Total cost = 9x + 5x + 11x = 25x

20% of total cost = 20% of 25x = 5x

Manpower cost: Profit = 5x: 5x = 1: 1

24. Trader's net profit = 200 × 100/800 = 25%.

25. Let's say the shopkeeper buys at Re. 1 per gram.

He buys 1000 gm, but pays for 1,100 g. He now mark-up his CP by 10%, hence he charges at the rate of Rs. 1.1 per gram. Hence, he makes a revenue of Rs. 1,100. So, he neither makes a profit nor a loss.

26. Let the actual value be 100x.

After he used the counterfeit weight, the value becomes 80% of the actual value or 80x. Also, after adding 20% impurities to the value now, it becomes 120% of 80x = 96x. And because of faulty balance it becomes 1.25 × 96x = 120x

So, %age profit = $\frac{120x - 80x}{80x} \times 100 = 50\%$.

Exercise 11

1. Let the rate of interest be R% p.a.
Interest on Rs 4000 = Rs 500
So, 500 = 4000 × R × 2/100 ⇒ R = 6.25%
Interest on Rs 5000 = Rs 625
Let the required time be T years.
Therefore, 625 = 5000 × T × 6.25/100 ⇒ T = 2.

2. We have, Amount = $P(1+ R/100)^t$

⇒ Amount = $1000 \times (1 + 20/100)^3$ = Rs 1728.

3. Let the sum be Rs P.

Given that P (1 + 4 × 8/100) = 6600 ⇒ P = Rs 5000

4. If the sum triples, the interest obtained will be twice the sum. This takes 4 years. If the sum becomes 5 times, the interest must be four times the sum. This takes a total of 8 years.

5. Let the sum of Rs P, triple in 4 years at R% p.a.

⇒ $P(1 + R/100)^4 = 3P$

⇒ $(1 + R/100)^4 = 3$ ..... (i)

Let it takes 'n' years to become 9 times.

Therefore, $P(1 + R/100)^n = 9P$

⇒ $(1 + R/100)^n = 9 \Rightarrow [(1 + R/100)^4]^{n/4} = 3^2$

⇒ $3^{n/4} = 3^2 \Rightarrow n/4 = 2 \Rightarrow n = 8$ years.

6. Interest for the fourth year = Amount at the end of the first 4 years - Amount at the end of the first 3 years

$= 4000(1 + 10/100)^4 - 4000(1 + 10/100)^3 = 4000(1.4641 - 1.3310)$

$= 4000(0.1331)$ = Rs 532.40

7. Let the sum is Rs 100. As the rate percent is 40% p.a., so, it is 10% for 3 months.

Amount after one year = $100(1 + 10/100)^4 = 100 \times (1.1)^4 = 146.41$.

Therefore, effective rate of interest = 46.41%

8. Let the sum is Rs P and the rate of interest is R%.

We have $P(1 + R/100)^2 = 28800$ ..... (i)

And $P(1 + R/100)^3 = 34560$ ..... (ii)

Dividing (ii) by (i), we get, 1 + R/100 = 1.2 ⇒ R = 20%. Put it in (i), we get

$P \times 1.2 \times 1.2 = 28800 \Rightarrow P$ = Rs 20000.

9. Let the sum be Rs P and the rate percent is R%.

Interest for the 2nd year = $P(1 + R/100)^2 - P(1 + R/100) = 3300$

⇒ $P(R/100)(1 + R/100) = 3300$ ..... (i)

Interest for the 3rd year = $P(1 + R/100)^3 - P(1 + R/100)^2 = 3630$

⇒ $P(R/100)(1 + R/100)^2 = 3300$ ..... (ii)

Dividing (ii) by (i), 1 + R/100 = 1.1 ⇒ R = 10%. Substituting this value in (i), we get

P = Rs 30000.

10. Value of Rs 15000 at the end of the first year = Rs 18000

As Sanjay repaid Rs 12000, he has to repay the value of the balance of Rs 6000 at the end of the second year. He has to repay 6000 × 1.2 i.e., Rs 7200.

11. Let each instalment be Rs x

Value of Rs 26400 at the end of the first year = 26400 × 1.2 i.e., Rs 31680

As Praveen repaid Rs x at the end of the first year, he has to repay the value of the balance of Rs (31680 - x) at the end of the second year.

⇒ 1.2 (31680 - x) = x ⇒ x = Rs 17280.

12. The interest on a sum remains the same each year under simple interest.

Therefore, the total interest = 3 × 1000 = Rs 3000

13. Let the principal amount is Rs x.

Given that simple interest = Rs 25410.

Now, 25410 = x × 11/2 × 6/100 ⇒ x = Rs 77000.

14. Let the sum is Rs x.

We have, 37 × x × 6/100 – 33 × x × 6/100 = 54

⇒ 4 × x × 6/100 = 54 ⇒ x = Rs 225.

15. If the sum becomes sixteen times, then it means it has increased by 1500%.

Let the time taken is 't' years.

We have, t × 30 = 1500 ⇒ t = 50 years.

16. Let the rate percent is R%. Time period is 7 years.

We have, R × 7 = 84 ⇒ R = 12%.

17. Let the sum be Rs x.

Then Amount = Rs 10x ⇒ Interest = Rs 9x.

Let the time period be r years and rate of interest be r%.

⇒ 9x = x × r × r/100 ⇒ r = 30%.

18. Let the total amount invested be Rs 2x

Amount invested in each scheme = Rs x

Total S.I = x × 1 × 15/100 + x × 1 × 20/100 = 1400

⇒ 0.35x = 1400 ⇒ x = Rs 4000

Therefore, the total amount invested = 2x = Rs 8000.

19. Let the sum is Rs x and the rate percent is R%.

We have, $x (1 + R/100)^2 = 6000$ ..... (i)

And $x (1 + R/100)^3 = 7200$ ..... (ii)

Dividing (ii) by (i), we get 1 + R/100 = 1.2 ⇒ R = 20%.

20. Let the sum is Rs x and the rate of interest is R% p.a.

We have $x [(1 + R/100)^3 – (1 + R/100)^2] = 2420$

$\Rightarrow x (1 + R/100)^2 \times R/100 = 2420$ ..... (i)

And $x [(1 + R/100)^4 – (1 + R/100)^3] = 2662$

$\Rightarrow x (1 + R/100)^3 \times R/100 = 2662$ ..... (ii)

Dividing (ii) by (i), we get, $1 + R/100 = 1.1 \Rightarrow R = 10\%$.

21. Let the sum be Rs 100.

Amount after eight years = Rs 400

Amount after sixteen years = Rs 1,600 i.e., it takes 8 + 8 = 16 years for the sum to become sixteen times of itself.

22. Let the sum be Rs x.

Then, the amount after three years = 2.197 × x

$\Rightarrow 2.197 \times x = x (1 + R/100)^3 \Rightarrow (1.3)^3 \times x = x (1 + R/100)^3$

$\Rightarrow 1 + R/100 = 1.3 \Rightarrow R/100 = 0.3 \Rightarrow R = 30\%$.

23. If a sum takes T years to become N times under compound interest, interest being compounded annually at R% p.a.

$\Rightarrow (1 + R/100)^T = N \Rightarrow R = (N^{1/T} – 1) \times 100$

$\Rightarrow R = [(1 + 33.1/100)^{1/3} – 1] \times 100 = 10\%$.

24. In all the cases, the rate of interest per year is the same. If this is the case, then the interest will increase with the number of times compounding is done per year. Therefore, option (1) will result in the maximum interest.

25. Let the principal amount is Rs x. As the rate of interest is 10% p.a., so it will be 20% for 2 years. The amount given is Rs 10080.

Now $1.2x = 10080 \Rightarrow x = 10080/1.2 =$ Rs 8400.

If the rate is 10% p.a. compounded annually, then $A = P (1 + R/100)^2$

$\Rightarrow A = 8400 \times (1.1)^2 =$ Rs 10164.

Hence, compound interest = 10164 – 8400 = Rs 1764.

26. Let P is the principal and r% is the rate percent.

We have, $P (1 + r/100)^3 = 138240$ .....(i)

And $P (1 + r/100)^5 = 199065.6$ .....(ii)

Divide (ii) by (i), we get, $(1 + r/100)^2 = 1.44 \Rightarrow r = 20\%$

Put in (i), we get $P = 138240/(1.2)^3 =$ Rs 80000.

27. If principal is Rs 10000, the amount under compound interest after 3 years will be

= 10000 × 1.1 × 1.2 × 1.3 = Rs 17160

If principal is Rs 10000, the amount under 20% simple interest after 3 years will be

= 10000 × 1.6 = Rs 16000.

Therefore, the interest in the second case will be 17160 – 16000 = Rs 1160 less.

28. The compound interest for the $(n + 1)^{th}$ year is the same as the amount for one year on a principal equal to the n^{th} year interest.

So, $1996.5 = 1885[(1 + (r/100)] \Rightarrow r = 10\%$ p.a.

29. The compound interest for the $(n + 1)^{th}$ year is the same as the amount for one year on a principal equal to the n^{th} year interest.

$\Rightarrow 1464.1 = (1 + R/100) \times 1331 \Rightarrow R = 10\%$.

Interest accrued for the first two years when Rs 12000 is invested in the same scheme $= 12000 \times [(1.1)^2 - 1] = 12000 \times 0.21 =$ Rs 2520.

30. In 3 years, Rs 10000 at 20% simple interest will become $10000 \times 1.6 =$ Rs 16000.

In the next 2 years, Rs 16000 will become under compound interest $= 16000 \times (1.2)^2$

= Rs 23040.

31. Let the amount that Rs 8000 becomes after 4 years be x.

Then $(x + 14440)(1.1)^2 = 30{,}250$ Or $x + 14440 = 30250/1.21 \Rightarrow x + 14440 = 25000$ $\Rightarrow x = 10560$. Now, let the rate of simple interest for the first 4 years be r%,

Then $8000(1 + 4r/100) = 10{,}560 \Rightarrow 320r = 2560 \Rightarrow r = 8\%$

32. Let P_1 is the sum invested in scheme A and P_2 is the sum invested in scheme B.

So, $P_1 + P_2 = 34000$ ——— (1)

Now, $P_1 \times (1 + 20/100)^2 = 9 \times P_2(1 + 60/100)^2$

$\Rightarrow P_1 \times (1.2)^2 = 9 \times P_2 \times (1.6)^2$

$\Rightarrow P_1 = P_2 \times 16 \Rightarrow P_1 : P_2 = 16 : 1$

Therefore, the amount invested in scheme B is $1/17 \times 34000 =$ Rs 2000.

33. Let the first, second and third sums be a, b and c respectively.

Now, $a(1 + 5/100) = b(1 + 5/100)^2 = c(1 + 5/100)^3 = k$ (say)

$\Rightarrow a : b : c = k/(21/20) : k/(441/400) : k/(9261/8000) = 20/21 : 400/441 : 8000/9261$

$\Rightarrow a : b : c = 8820 : 8400 : 8000 = 441 : 420 : 400$.

34. Let Suhaas borrowed Rs 'a'. As he has paid back after one year, so $a \times 1.28 = 38400$

$\Rightarrow a =$ Rs 30000.

Let the amount borrowed by Bhanu is Rs 'b'.

Given that $a + b = 54000 \Rightarrow 30000 + b = 54000 \Rightarrow b =$ Rs 24000

Now, $24000 \times (1 + 4r/100) = 38400 \Rightarrow 1 + 4r/100 = 1.6 \Rightarrow 4r/100 = 0.6 \Rightarrow r = 15\%$.

35. If compounding is done annually r = 20%, n = 2,

Let P = 100, $A = 100 \times (1.2)^2 =$ Rs 144

If compounding is done half-yearly, r = 10%, n = 4, $A = 100 \times (1.1)^4 = 146.41$

So, the difference = 146.41 – 144 = Rs 2.41

Now, If P = 100 then, difference = 2.41

If difference is 1084.5, $P = 1084.5 \times 100/2.41 =$ Rs 45000.

Exercise 12

1. Let the rate percent is R%.

We have $2.4c (1 + R/100)^{18} = 19.2c$

$\Rightarrow (1 + R/100)^{18} = 8 \Rightarrow (1 + R/100) = 2^{1/6}$ ..... (i)

The value that 4 crores would amount to in 24 years at the same rate

$= 4c (1 + R/100)^{24} = 4c (2^{1/6})^{24}$ = Rs 64 crore.

2. Here the difference in compound interest and simple interest is Rs 30.

$\Rightarrow P (r/100)^2 = 30$. Also, $Pr/100 = 600$

$\Rightarrow 600 \times r/100 = 30 \Rightarrow r = 5\%$

$\Rightarrow P \times 5/100 = 600 \Rightarrow P$ = Rs 12000.

3. The difference between the compound and simple interest for 3 years is given by

$P [(R/100)^3 + 3 (R/100)^2]$

$\Rightarrow P [(20/100)^3 + 3 (20/100)^2] = 3200$

$\Rightarrow P \times 16/125 = 3200 \Rightarrow P$ = Rs 25000.

Alternate method:

Under CI, amount for 3 years $= (1.2)^3 \times P = 1.728P$

Under SI, amount for 3 years $= (1 + 3 \times 0.2) \times P = 1.6P$

Difference in amounts = Difference in interests.

Hence, $1.728P – 1.6P = 3200 \Rightarrow 0.128P = 3200 \Rightarrow P = (3200/0.128)$ = Rs 25000.

4. He borrows Rs 2500000. At the end of the first year, it becomes

$2500000 \times (1 + 12/100)$ = Rs 2800000

He repays Rs 500000 ——— (1)

Rs 2300000 is the principal for the 2nd year.

This becomes $230000 \times (1 + 12/100)$ = Rs 2576000 which is repaid by the person, at the end of 2nd year.

In the first year, (on his earnings side), he earns 20% on Rs 2500000 = Rs 500000.

Out of this, (1) is repaid. Investment remains as Rs 250000.

This earns 20% during the second year. So, it becomes Rs 3000000. Out of this Rs 2576000 is repaid. So, he makes, 3000000 – 2576000 = Rs 424000

5. Loan amount = Rs 24000, Simple Interest = 12% per annum

After 1 year, interest $= 0.12 \times 24000$ = Rs 2880

Interest for the second year $= 0.12 \times 19200$ = Rs 2304 (Since 20% of the principal has been repaid)

Interest for the third year $= 14400 \times 0.12 = 1728$

Interest for the fourth year $= 9600 \times 0.12 = 1152$.

Total interest = 2880 + 2304 + 1728 + 1152 = 8064.

6. Let the value of each instalment is Rs 100. The first instalment will be paid 4 years before the due date, the second will be paid three years before the due date, the third will be paid two years before the due date, the fourth will be paid one year before the due date and the last instalment will be paid on the due date itself.

The future value of the first instalment after 5 years = 100 + 100 × 8 × 4/100

= Rs 132

The future value of the second instalment after 5 years = 100 + 100 × 8 × 3/100

= Rs 124

The future value of the third instalment after 5 years = 100 + 100 × 8 × 2/100

= Rs 116

The future value of the fourth instalment after 5 years = 100 + 100 × 8 × 1/100

= Rs 108

The future value of the fifth instalment after 5 years = Rs 100

Total loan paid off = 132 + 124 + 116 + 108 + 100 = Rs 580

So, if the due amount is Rs 580, the instalment = Rs 100

If the due amount is Rs 1450, the instalment = 100 × 1450/580 = Rs 250.

Alternate Method:

In case of simple interest, we have the direct formula to calculate the instalment.

Instalment = $\frac{100A}{100T + \frac{RT(T-1)}{2}}$, where A is the amount due, R is the rate percent and T is the time period.

Therefore, instalment = $\frac{100 \times 1450}{100 \times 5 + \frac{8 \times 5 \times 4}{2}} = \frac{145000}{580} = 250.$

7. Here Principal = Rs 100000

At the end of first year, it amounts to 100000(1 + 8/100) = Rs 108000

Out of this he repays Rs 10000. So, amount due at the end of 1st year = 108000 – 10000 = Rs 98000.

At the end of the second year this amounts to 98000 [1 + 8/100] = Rs 105840.

Out of this he repays Rs 10000. Therefore, at the beginning of the third year amount due from him = 105840 – 10000 = Rs 95840.

8. Let the principal amount is Rs P.

CI for 2nd year = P $(1 + 5/100)^2$ – P (1 + 5/100)

CI for 3rd year = P $(1 + 5/100)^3$ – P $(1 + 5/100)^2$

Difference = P $(1.05)^3$ – $2(1.05)^2$ – P × 1.05 = 42

⇒ P = Rs 16000.

9. The value of the first instalment (at the time the sum was borrowed) = 1200/1.2 = Rs 1000

The value of the second instalment (at the time the sum was borrowed) = 1152/$(1.2)^2$ = Rs 800

The value of the third instalment (at the time the sum was borrowed)

= 2592/$(1.2)^3$ = 1500

Sum borrowed = Total value (at the time the sum was borrowed) = Rs 3300

10. Let the principal be P.

Let 'I' be the interest for one year

P + 3I = 2832 ——— (1)

P + 5I = 3120 ——— (2)

Subtracting (1) from (2), I = 144 and P = Rs 2400

11. Let the rate of interest be R% p.a.

Extra interest = 10000 × 2 × R/100 = 4000 ⇒ R = 20%.

12. Let the rate of interest is R%.

If the sum is doubled, it means it increased by 100%.

So, R × 10 = 100 ⇒ R = 10%.

Let it takes 't' years to become five times i.e., increase of 400%.

So, 10 × t = 400 ⇒ t = 40 years

13. Let the sum is Rs P and the rate of interest is R%.

We have P × R × 4/100 = P ⇒ R = 25%

Again, P × 25 × 5/100 = 12500 ⇒ P = Rs 10000

Required interest = 10000 × 25 × 3/100 = Rs 7500.

14. Let the sum that Rohan lent to P be Rs x and sum he lent to Q = Rs (100000 – x).

We have 2 × [10x/100 + 20 × (100000 – x)/100] = 32000 ⇒ x = Rs 40000.

15. Let us say he lends Rs P. Let us say the least number of years it takes the sum to fetch an interest at least equal to itself be N.

We have, P × [(2/100) + (4/100) + (6/100) + ……(2N/100)] ≥ P

⇒ N(N + 1) ≥ 100

If N = 9, N (N + 1) < 100

If N = 10, N (N + 1) > 100 ⇒ N = 10.

16. Amount borrowed from Balu = Rs 20000

Amount borrowed from Chetan = Rs 40000

Let the amount that he added is Rs x.

Now, (60000 + x) × 0.18 – 20000 × 0.1 – 40000 × 0.2 = 4400

⇒ 10800 + 0.18x – 2000 – 8000 = 4400

⇒ 0.18x = 3600 ⇒ x = Rs 20000.

17. Let the sum is Rs P and the rate of interest is R% p.a.

We have, P × [(1 + R/100)^2 – (1 + R/100)] = 14400 …… (i)

P × [(1 + R/100)^3 – (1 + R/100)^2] = 17280 …… (ii)

Dividing (ii) by (i), we get, 1 + R/100 = 1.2 ⇒ R = 20%.

18. Let the rate of interest be r% then, r% of interest of nth year

= Difference of interest for n^{th} and $(n + 1)^{th}$ year.

⇒ r% of 1256 = 1413 – 1256 = 157 ⇒ r = 157/12.56 = 12.5%.

19. Let the sum be Rs P and the total interest for the second and third years be Rs X.

Interest for the first year = 0.2P

Interest for the second year = (Amount at the end of two years) - (Amount at the end of one year)

= P [(1.2)^2 – 1.2] = 1.2 × P × 0.2

Interest for the third year = P $[(1.2)^3 - (1.2)^2]$ = 1.2 × P × 0.24

⇒ 0.2 P = 4000 ⇒ P = 20000. Now X = 0.528 P ⇒ X = Rs 10560

20. Let the sum be P and the rate be r%

⇒ P $(1 + r/100)^2$ = 1200 ..... (i)

And P $(1 + 2r/100)^2$ = 1323 ..... (ii)

Dividing (ii) by (i), we get P $(1 + 2r/100)^2$/ P $(1 + r/100)^2$ = 1323/1200

⇒ $[(100 + 2r)/(100 + r)]^2$ = 441/100 ⇒ r = 5 5/19%.

21. Rs 20000 deposited at the beginning of the first year becomes, in 2 years, an amount equal to 20000 × $(1.05)^2$ = Rs 22050 (i.e., at the beginning of the third year) Rs 20000 deposited at the beginning of the second year becomes 20000 × 1.05 = Rs 21000 in 1 year.

So, the amount at the beginning of the third year = 22050 + 21000 = Rs 43050

Now Rs 10000 is withdrawn, so, there is Rs 33050 in the account on which 5% interest for one more year is earned. Final amount = 33050 x 1.05 = Rs 34702.5

22. Value of Rs 72000 at the end of the first year = Rs 86400. Since Rs x was repaid at the end of the first year, in order to clear the loan at the end of the second year, the amount to be repaid must be

(86400 – x) + 0.2 (86400 – x) = 57600

⇒ 1.2 × (86400 – x) = 57600

⇒ x = Rs 38400.

23. Let the sum be Rs 2P. Rs P lent at simple interest fetches Rs 0.2 P as interest each year. Rs P lent at compound interest fetches Rs 0.24 P as interest in the second year and Rs 0.3456 P as interest in the fourth year.

Given that 0.24 P - 0.2P = 400 ⇒ P = 10000

Required difference = 0.3456P – 0.2P = Rs 1456

24. Let the value of the machine two years ago Rs x then,

x × 0.9 × 0.9 = 24300 ⇒ x = Rs 30000.

25. We have, 150% = 3/2 times and 240% = 12/5 times. Thus, assuming amount invested in Platinum scheme as $12x$, amount invested in Gold scheme can be found as $8x$ and that invested in Silver scheme can be found as $5x$. Hence 10% × $8x$ + 12% × $5x$ + 15% × $12x$ = 3200 i.e., $3.2x$ = 3200 i.e., x = 1000. Hence amount invested in Silver scheme is 5 × 1000 = 5000.

26. If total SI earned in first 2 years is $50k$, then the interest in first year, even in case of CI, is $25k$. The interest earned in 2nd year in case of CI is $26k$. The extra k is the interest on first year's interest ($25k$). Thus, the rate of interest is $k/25k$ i.e., 4%

Difference between SI and CI earned in first 2 years = $(r/100)^2$ × P.

Thus, 16 = $(4/100)^2$ × P ⇒ P =10,000

Thus, SI in first year = 4% × 10,000 = 400. Thus, SI and CI in second year will be 400 and 416 i.e., a sum of 816.

27. This case is like the case where the time period is 2, the rate of interest being half of the annual rate i.e., 4%. The difference between compound and simple interest in that case is $Pr^2/100^2$ = 5000 × $4^2/100^2$ = 8.

28. Let the compound interest earned in 17th year be Rs. x.

Then the CI earned in 18th year = x + 20% of x = $1.2x$ (using the concept of interest on interest)

Similarly, CI earned in 19th year = $1.2x$ + 20% of $1.2x$ = $1.2^2 \times x$

And finally, CI earned in 20th year = $1.2^3 \times x$, which is given to be Rs. 2160.

$\Rightarrow x = 2160 \times 5/6 \times 5/6 \times 5/6$ = Rs 1250.

29. For a difference of 1 year, CI can be computed as SI.

Hence, from the 2nd year to the 3rd year, interest earned = (675 – 625) = Rs. 50 on Rs. 625. Hence, the rate of interest = 50 × 100/625 = 8% p.a.

30. Let p be the principal value, r% be the rate of interest

We have $p \times r^2/100^2 = 160$ ..... (i)

Also given that $2880 = p \times r \times 2/100 \Rightarrow pr = 2880 \times 100/2 \Rightarrow pr = 144000$ ..... (ii)

Using (ii) in (i), we get, $r \times 144000 = 160 \times 100 \times 100 \Rightarrow r = 11\ 1/9\%$.

31. Here $CI = P[(1 + R/100)^2 - 1]$

$\Rightarrow P[(1 + 10/100)^2 - 1] = 420$

$\Rightarrow P(1.21 - 1) = 420 \Rightarrow P \times 0.21 = 420 \Rightarrow P$ = Rs 2000.

Now, SI = 2000 × 10 × 2/100 = Rs 400.

32. Here SI for 2 years = Rs 354 ⇒ SI for one year will be Rs 177.

Let's assume the sums of Narayan and Murthy are x and y respectively.

We have, $6x/100 + 7y/100 = 177 \Rightarrow 6x + 7y = 17700$ ..... (i)

Also given that $x/4 = y/5 \Rightarrow x = 4y/5$, put it in (i), we get,

$6 \times 4y/5 + 7y = 17700 \Rightarrow 24y/5 + 7y = 17700 \Rightarrow y = 1500$ and $x = 1200$

Hence, total sum (x + y) = 1500 + 1200 = Rs. 2,700.

33. Difference of sum after 2 and 3 years = 1,728 – 1,440 = Rs. 288

Rs. 288 is the simple interest on Rs. 1,440 for one year.

$288 = 1440 \times r \times 1/100 \Rightarrow r = 20\%$.

34. Let three parts be a, b and c where a > b > c. Therefore, by the simple interest formula $1.04a = 1.08b = 1.12c$ and $a + b + c = 2189$. Solving above equations, we get c = 703.

35. Let, the amount that was invested by Rohit in bank be 'C'.

$1.2(1.2C - 200) = 400 \Rightarrow 1.44C = 240 + 400 \Rightarrow C = 640/1.44$ = Rs 4000/9.

36. Interest earned on first part = 3500 × 3 × 8/100 = Rs 840.

Interest earned on second part = $3500 \times (1.1)^2 - 3500$ = Rs 735

Therefore, the required difference = 840 – 735 = Rs 105.

37. The required difference = $5000 \times (1 + 5/100)^4 - 5000 \times (1 + 10/100)^2$

= $5000 \times (1.2155 - 1.21)$ = Rs 27.50

38. Amount at the end of the 1st year after repayment = 6000 × 1.05 – 1200

= Rs. 5,100.

Therefore, amount at the beginning of the 2nd year will be Rs. 5,100.

Amount at the end of the 2nd year after repayment = 5,100 × 1.05 – 1200

= Rs. 4,155. Therefore, amount at the beginning of the 3rd year will be Rs. 4,155.

39. Let the value of each instalment be x.

$\Rightarrow x/1.02 + x/(1.02)^2 + x/(1.02)^3 + x/(1.02)^4 = 33000 \Rightarrow x$ = Rs 8666.6

Exercise 13

1. Number of Man days required to complete the job = 900. Time taken by 45 men to complete it = 900/45 = 20.

2. Total time for which 18 men work = 240 hours.
Number of man hours required to complete the job = 18 × 240-man hours.
Number of days taken by 15 men working 9 hours a day to complete it
= (18 × 240)/(15 × 9) = 32.

3. Number of man hours required to complete the job = 20 ×10 ×12 = 2400
Number of men required to complete a job twice as large = (2 × 2400)/(30 × 8) = 20.

4. Given that 10B = 20G ⇒ 1B = 2G
Now, 10B + 20G = 20G + 20G = 40G
So, if 20 girls can do the work in 10 days, 40 girls will do the same work in
(20 × 10)/ 40 = 5 days.

5. Here (3M + 4W) × 10 = (24M + 2W) × 2
⇒ 15M + 20W = 24M + 2W ⇒ 9M = 18W ⇒ 1M = 2W
Let us consider 3 men and 4 women. They are equivalent to 3 × 2W + 4W = 10W and they can do the work in 10 days.
Also, 11M + 3W = 22W + 3W = 25W.
So, if 10W can do the work in 10 days, 25W will do the work in (10 × 10)/25 = 4 days.

6. Here we have

Men	Days	Work
20	10	1/5
x	20	4/5

Now, using $M_1 \times D_1 \times W_2 = M_2 \times D_2 \times W_1$, we get,
20 × 10 × 4/5 = x × 20 × 1/5
⇒ x = 40. Therefore, 40 – 20 = 20 additional men are required.

7. We have (X + 10) × 100 = X × 120 ⇒ 100X + 1000 = 120X ⇒ 20X = 1000 ⇒ X = 50.

8. Here we have

Men	Days	Work
10	15	1
x	10	5

Now, using $M_1 \times D_1 \times W_2 = M_2 \times D_2 \times W_1$, we get,
10 × 15 × 5 = x × 10 ⇒ x = 75.

9. Here 1M = 3W ⇒ 15W = 5M
Now 9 men can do the work in 15 days. So, 5 men will do the same work in 15 × 9/5 = 27 days.

10. We have, $M_1 \times D_1 \times H_1 \times W_2 = M_2 \times D_2 \times H_2 \times W_1$
⇒ 1 × 4 × 10 × 1 = 1 × x × 8 × 1
⇒ x = 5 days.

11. Here we have

Men	Days	Hours	Work
3	3	3	1
9	x	1	1

Now, using $M_1 \times D_1 \times H_1 \times W_2 = M_2 \times D_2 \times H_2 \times W_1$, we get
$3 \times 3 \times 3 \times 1 = 9 \times x \times 1 \times 1 \Rightarrow x = 3$

12. Total work = 30 × 12 × 50 = 18000-man hours.
Amount of work done in 15 days putting 10 hours a day = 15 (10) (50) = 7500-man hours.
Work remaining = 18000 - 7500 = 10,500-man hours.
Number of men remaining = 40
Number of hours = 10 per day.
Number of days needed = 10500/(40 × 10) = 26 1/4 days.
Therefore, they require 11 1/4 more days to complete the work.

13. Here we have

Men	Days	Work
48	60	2/5
x	60	3/5

Now, using $M_1 \times D_1 \times W_2 = M_2 \times D_2 \times W_1$, we get,
$48 \times 60 \times 3/5 = x \times 60 \times 2/5 \Rightarrow x = 72$.
Therefore, 72 – 48 = 24 additional men are required.

14. We have, $(6m + 8w) \times 10 = (8m + 22w) \times 5$
$\Rightarrow (6m + 8w) \times 2 = 8m + 22w$
$\Rightarrow 12m + 16w = 8m + 22w \Rightarrow 4m = 6w \Rightarrow 2m = 3w$
Therefore, work = $(6m + 8w) \times 10 = 170$ w days
New work = 170 × 3 days
34 women will do it in 170 × 3/34 = 15 days.

15. Here we have

Men	Days	Work
80	84	20000
x	63	30000

Now, using $M_1 \times D_1 \times W_2 = M_2 \times D_2 \times W_1$, we get,
$80 \times 84 \times 30000 = x \times 63 \times 20000$
$\Rightarrow x = 160$.

16. Here, 2m = 5w and work = 5w × 15d = 75 women days
$\Rightarrow 4m + 5w = 2 \times (2m) + 5w = 2 \times (5w) + 5w = 15w$
So, an equivalent of 15 women completes a work of 75 women days in 5 days.

17. Work 1 man can do in 3 days = Work 4 boys can do in 2 days.
3 man days = 8 boy days ..... (1)
Work to be done = 72 × 16 boy days
= 72 × 16 × 3/8 = 72 × 6 mandays.

As work is to be completed in 48 days, x = (72 × 6)/48 = 9.

18. Let the rate of work of each man be x units / day.
The number of men who left after 56 days (i.e., 24 + 32) is N.
The total work is x × 40 ×96 = x × 40 × 24 + x × (40 +20) × 32 + x × (60 – N) × 20
⇒ N = 12

19. Let the required cost be Rs x
Now, 10 cows = 20 sheep ⇒ 10 cows + 40 sheep = 60 sheep
20 cows + 10 sheep = 50 sheep
Let the cost of grass for each sheep be Rs y/day.
So, 60y × 20 = 900 ⇒ y = 3/4.
Therefore, x = 50y × 18 = 50 × 3/4 × 18 = 675.

20. The cost of grass for 20 cows and 30 sheep for 30 days = Rs 720.
Given 30 sheep eat double the grass eaten by 20 cows.
⇒ Cost of grass eaten by 30 sheep in 30 days = 2/3 × 720 = Rs 480.
The cost of grass eaten by 1 sheep in one day = 480/(30 × 30) = 8/15
The cost of grass eaten by 20 sheep in 15 days = 8 × 20 × 15/15 = Rs 160.

21. 30 men can make 12 tables in 18 × 7 hours.
i.e., 4 men can make 3 tables in $18 \times 7 \times \frac{30}{4} \times \frac{3}{12} = \frac{3\times7\times15\times3}{4}$ hours.
And it is given that in the same time, 3 women can make 4 chairs.
Thus, 3 women can make 4 chairs in $\frac{3\times7\times15\times3}{4}$ hours.
i.e., 45 women can make 32 chairs in $\frac{3\times7\times15\times3}{4} \times \frac{3}{45} \times \frac{32}{4}$ = 126 hours.
Working at rate of 9 hours per day, this means $\frac{126}{9}$=14 days.

22. 30 men complete 1/7th of the work in 2 days.
Thus, 30 men will complete the remaining 6/7th of the work in 6 × 2 = 12 more days.
The work has to be completed in 10 more days and hence men needed will be $30 \times \frac{12}{10}$= 36. Hence 6 more men are needed.

23. The work can be thought of as 10 × 8 = 80 student–days.
First two days, all 10 were present and hence 20 student–day equivalent work is done.
On 3rd and 4th day, 2 people were absent, so on these two days, 2 × 8 = 16 student–day equivalent of work is done.
On the last two days, 3 people were absent. So, on these two days, 2 × 7 = 14 student–day equivalent of work is done.
A total of 20 + 16 + 14 = 50 student–days equivalent work is done and the balance 30 student–day of work was done after the 4th day till the second last day. With 10 students working, this period is equal to 3 days. Thus, total time taken = 2 + 2 + 3 + 2 = 9 days. Hence instead of planned 8 days, work took a total of 9 days i.e. an increase of 1/8th.
The above in single equation is (in terms of student–days)
10 × 8 = 10 × 2 + 8 × 2 + 10 × x + 7 × 2 i.e., 30 = 10 × x i.e., x = 3 days

Short–cut:
One just needs to check the work that was not done because of absenteeism. This is the work that will be left after 8 days. And due to absenteeism, 2 × 2 + 2 × 3 = 10 student–days work was not done. This 'pending' work can be done by 10 students in 1 extra day.

24. Number of pumps needed

$$= 18 \times \underbrace{\frac{25}{40}}_{\substack{\text{high watt}\Rightarrow\\ \text{less pumps}}} \times \underbrace{\frac{1736}{2170}}_{\substack{\text{less water}\Rightarrow\\ \text{less pumps}}} \times \underbrace{\frac{12}{15}}_{\substack{\text{less height}\Rightarrow\\ \text{less pumps}}} \times \underbrace{\frac{10}{14}}_{\substack{\text{more days}\Rightarrow\\ \text{less pumps}}} \times \underbrace{\frac{7}{9}}_{\substack{\text{more hours}\Rightarrow\\ \text{less pumps}}} = 4$$

25. Since number of lamps is to be found, $98 \times \frac{5}{4} \times \frac{10}{30} \times \frac{76.50}{12.25} = 255$

26. Rather than work in time taken (which cannot be added when the three pumps are working simultaneously), let us work along the lines of the rate of pumping water by the three pumps (the rates can be added when pumps are working simultaneously). Also, since the comparison is with the medium pump, let rate of pumping water by the medium pump be x units. Thus, rate of pumping water by the large pump is $\frac{3}{2}x$ and that of small pump is $\frac{2}{3}x$. To arrive at these fractions, don't use a lot of time or thoughts – the numbers used in the ratio are 2 and 3 and the larger pump will have a higher rate and the smaller pump will have a lower rate.
Thus, when all the three pumps are working simultaneously, the effective rate is $\frac{3}{2}x + x + \frac{2}{3}x = \frac{19}{6}x$. Since this rate is $\frac{19}{6}$ times the rate of the medium pump, the fraction of time that is needed will be $\frac{6}{19}$.

27. Let x men leave on the 31th day and $2x$ men join back on the d^{th} day. Thus,
On days 1 to 30 i.e., 30 days there were 95 men in camp
On days 31 to $(d - 1)$ i.e. $(d - 31)$ days, there were $(95 - x)$ men in camp
On days d to 200 i.e. $(200 - d + 1)$ i.e. $(201 - d)$ days, there were $(95 + x)$ men in camp.
Thus, $95 \times 30 + (95 - x) \times (d - 31) + (95 + x) \times (201 - d) = 95 \times 200$
i.e., $95d - 95 \times 31 - xd + 31x + 95 \times 201 - 95d + 201x - xd = 95 \times 170$
i.e., $95 \times 170 + 232x - 2xd = 95 \times 171$ i.e., $232x = 2xd$ i.e., $d = 116$.

Short–cut:
The food saved by the 'few' men not being in the camp, was equal to the extra food consumed by 'extra' men present in the camp from the day 'twice' as many men joined back to the last day of the camp. And since 'twice' as many men came back, the 'extra' men = 'few' men. Hence the days when there were fewer men = days when there were extra men.
Let the men join back on day d^{th} day. Thus, the extra men are present on days d to 200 i.e. $(200 - d + 1)$ days i.e. $(201 - d)$ days. And the camp has fewer men for days 31 to $(d - 1)$ i.e. $(d - 31)$ days.
By logic explained in earlier paragraph, $201 - d = d - 31$ i.e., $2d = 232$ i.e., $d = 116$.

28. Here, 1 man = 2 children
So, 8C + 12M = 4M + 12M = 16M
So, 16 men can do the work in 9 days, therefore, 12 men will finish it in 16 × 9/12 = 12 days.

Exercise 14

1. Let A can do 1 unit/day. As he completes the work in 24 days, so, the total work is 24 units. When A and B together do this work, they take 12 days. In 12 days, A will do 12 units of the work. The remaining work of 12 units will be done by B. So, B can also do 1 unit/day. Therefore, B will take total 24 days.

2. Part of the job that can be done by both in a day = 1/16
Part of the job that can be done by them in 4 days = 4 × 1/16 = 1/4
Remaining part of the job = 3/4
Time taken by Ajay to complete it = 18 × 3/4 = 13.5 days.

3. Let the total work is 180 units.
P and Q will do 15 units/day
Q and R will do 12 units/day
P and R will do 10 units/day
Adding these equations, we get, 2 (P + Q + R) = 37
⇒ P + Q + R = 37/2 units/day.
Work done by P, Q and R in 3 days = 3 × 37/2 = 55.5 units
Remaining work = 180 – 55.5 = 124.5 units.
P and Q will take 124.5/15 = 8.3 days.

4. Part of the job done by A in a day = 1/16
A worked for a total of 8 days. Therefore, A completed 8 × 1/16 = 1/2 of the job
Hence, B can complete the remaining 1/2 of the job in 4 days.
Therefore, B alone can complete the entire job in 8 days.

5. Part of the tank filled per minute when all the three pipes are opened
= 1/10 + 1/20 – 1/40 = 1/8.
Therefore, they would take 8 minutes to fill the tank.

6. Let the time taken by R to empty the tank be r minutes.
We have, 1/24 + 1/36 – 1/r = 1/18 ⇒ r = 72 minutes.

7. Work done by Anand, Bhanu and Chandra in 1 day = 1/4
Work done by Anand in 1 day = 1/8
Work done by Chandra in 1 day = 1/16
So, the work done by Bhanu in 1 day = 1/4 – (1/8 + 1/16) = 1/16.
⇒ Bhanu takes 16 days.

8. Let the total work is 60 units
P + Q = 5 units/day ..... (i)
Q + R = 3 units/day ..... (ii)
R + P = 4 units/day ..... (iii)
Adding the above three equations, we get, 2(P + Q + R) = 12 units/day
⇒ P + Q + R = 6 units/day
So, P, Q and R will take 60/6 = 10 days to complete the work.

9. Let the total work is 120 units.
Somu can do 2 units/day and Ramu can do 3 units/day.
So, the amount will be divided between Somu and Ramu in 2: 3.
Amount received by Somu = 2/5 × 450 = Rs 180.

10. Work done by Gautham in 1 hour = 1/4
Work done by Karan in 1 hour = 1/12
Work done by both (in 2 hours) = 1/4 + 1/12 = 1/3
⇒ total work is done in 3 × 2 = 6 hours.

11. Let the total work is 36 units.
X can do 3 units/day and Y can do 2 units/day

Work done by X and Y in two days = 5 units.
So, work done by X and Y in 14 days = 7 × 5 = 35 units.
Remaining one unit work will be done by X in 1/3 days. Therefore, total time taken is 14 1/3 days.

12. Part of the tank filled in an hour by both taps = 1/10 + 1/15 = 1/6. So, Part of the tank filled in an hour by both taps = 6/10 = 3/5th of the tank.

13. Let the total capacity of the tank is 12 litre.
Pipe can fill 4 litre/hour
Actual rate of filling = 3 litre/ hour. So, the leak is draining 1 litre/hour.
Total time taken by leak is 12 hours.

14. Let the total capacity of the tank is 60 litre.
The two pipes can fill it at 2 litre/min and 3 litre/min.
The emptying pipe can drain at 1 litre/min
Total water filled in one minute = 2 + 3 – 1 = 4 litre/min
Time taken to fill the tank = 60/4 = 15 minutes.

15. As A and B worked for 32 days on the whole, work done by them = 32/40 = 4/5.
Therefore, Remaining work i.e., 1/5th of the work is completed by C in 24 days.
So, C takes 24 × 5 = 120 days to complete the work.

16. Let the total work = 120 units.
Akbar can do 3 units/day and Ajay can do 2 units/day.
As the work is completed in 30 days, so, Akbar di 30 × 3 = 90 units.
The remaining work of 30 units is done by Ajay in 15 days.
So, Ajay joined 30 – 15 = 15 days after Akbar.

17. P works for a total of 9 days. Q works for a total of 18 days.
R works for a total of 12 days. Let the times taken by Q and R to complete the job be q days and r days respectively.
Therefore, 9/27 + 18/q + 12/r = 1
As q ≤ 54, therefore, r ≥ 36. Only option (2) is not possible.

18. Let the efficiencies of P, Q and R be 3x gadgets/day, 4x gadgets/day, 5x gadgets/day.
3x × 6 + 4x × 8 + 5x ×10 = 400 ⇒ x = 4.
Required number of gadgets = 6x × 6 + 12x × 8 + 5x × 10 = 728

19. Portion of wall built per day = 1/8th = 12.5%
20% of this (i.e., 2.5% of the wall) falls off.
Therefore 10% of the wall is completed every day.
To determine whether the whole of the 10th day is needed for work, consider work done in 9 days.
In 9 days 90% of the wall is built. On the 10th day, one-tenth has to be built. So, he takes = (1/10)/(1/8) = 4/5 days. Therefore, he takes 9 4/5 days to construct the wall.

20. Let Sunil can do 6 units/day, Raman can do 2 units/day and Kapil can do 1 unit/day.
Work done by all three in 30 days = 30 × (6 + 2 + 1) = 270 units.
Work done by Sunil and Raman in next 18 days = 18 × 8 = 144 units.
Therefore, the total work = 270 + 144 = 414 units
Raman alone will take 414/2 = 207 days to complete it.

21. Let the maximum efficiency of the man is 16 units/hour
As with maximum efficiency, he can complete the work in 150 hours, so total work = 150 × 16 = 2400 units
Work done in 8 hours = 2 × (16 + 8 + 4 + 2) = 2 × 30 = 60 units

Therefore, to complete 2400 units, the time taken = 8 × 2400/60 = 320 hours.

22. Let the efficiency of each man is 1 unit/day.
Total work done in 5 days = 1 + 4 + 12 + 32 + 80 = 129 units.
Now, if the work done by each of them is constant, then let the work is completed in 'n' days.
So, 1 + 2 + 3 + ……. + n = 129
⇒ n (n + 1)/2 = 129 ⇒ n (n + 1) = 258
⇒ $n^2 \approx 256$ or n ≈ 16.
We find that 15(15 + 1) < 258 < 16(16 + 1)
Work is completed on the 16^{th} day.

23. Let the work done by Ram on first day be x, so, second day he does 2x work.
Also, as A takes 40% less time than Ram, the ratio of time taken by Saleem and Ram is 0.6: 1 ⇒ 3: 5.
Therefore, the ratio of efficiencies of Saleem and Reema = 5: 3.
On first day Saleem did 5x/3 and on second day 10x/3.
Total work = x + 2x + 5x/3 + 10x/3 = 8x
Combined efficiency of Saleem and Ram, when there is no change = x + (5x/3) = (8x/3)
Number of days required = Work/efficiency = (8x)/(8x/3) = 3
Therefore, Saleem and Ram can together do it in 3 days.

24. As the ratio of work done is 2: 3: 5, so Vivek can do 2/10 or 1/5 of the work in 12 days. So, he will do the complete work in 12 × 5 = 60 days.
From the ratio 1: 2: 3, it can be decided that Rameshwar does in 30 days and Bhuvan does in 20 days.
The work done by all three in 8 days = 8/60 + 8/30 + 8/20 = 48/60 = 4/5.

25. Let the capacity of the tank is 72 litre.
Pipe A can fill 4 litre/min, Pipe B can fill 3 litre/min and pipe C can empty 2 litre/min
Water filled till 10: 15 a.m. is 15 × 4 – 5 × 2 = 50 litre.
Remaining tank = 72 – 50 = 22 litre.
Now, the water filled by all the pipes together in one minute = 4 + 3 – 2 = 5 litre
So, time taken to fill the remaining tank = 22/5 = 4.4 minutes.
Therefore, the tank will be full at 10: 19: 24 a.m.

26. Let the work totally take x days from the start. Niranjan stopped working after y days and Rajesh stopped working y days before the completion of the work. Vinayak worked throughout.
Therefore, y/10 + (x – y)/10 + x/12 = 1 ⇒ [6y + 4(x – y) + 5x]/60 = 1
⇒ 9x + 2y = 60
So, the only possible solution such that x > y and x and y are integers, is x = 6, y = 3.

27. Let the total work is 60 units.
Krishna can do 3 units/day and Rama can do 2 units/day.
Together, they can do 5 units/day. But as the work is completed in 6 days, so the work done per day is 10 units. So, Gopi can do 5 units/day.
Therefore, ratio of efficiency of Krishna, Rama and Gopi is 3: 2: 5.
Amount received by Gopi = 5/10 × 720 = Rs 360.
The amount received by Gopi daily = 360/6 = Rs 60.

28. Ratio of the wages of P, Q, R and total = 3: 4: 5: 12. This is also the ratio of the work completed by P, Q and R. So, P completed $1/4^{th}$ of the job. Also, the job was completed in 5 days. Therefore P, working, alone, can complete the job in 20 days.

29. Alok and Sachin were to complete a piece of work in 20 days. They lost 1/3rd of the pay. It implies that they could not complete one-sixth of the work after working for 20 days.
Alok alone can do it in 40 days. Let Sachin alone can do it in x days.
Therefore, 20/40 + 20/x = 5/6 ⇒ 20/x = 5/6 – 1/2 ⇒ 20/x = 1/3 ⇒ x = 60 days.

30. Initially tank is full. Total time to empty it is 60 minutes.
In 18 minutes, the part emptied = 18/60 = 3/10th of the tank.
At that point, the pipe, that can fill the tank in 30 minutes, is opened.
When both are working the tank is filled in 1/30 – 1/60 = 1/60 i.e., 60 minutes.
Therefore, the tank is filled in 3/10 × 60 + 18 = 36 minutes.

31. Let the capacity of the tank is 60 minutes.
Pipe P can fill 4 litre/min, pipe Q can fill 3 litre/min and pipe R can empty 2 litre/min.
In 6 minutes, the water filled is 8 + 6 – 4 = 10 litre.
Therefore, in 30 minutes, the water filled is 10 × 5 = 50 litre.
In the next 2 minutes, pipe P will fill 8 litres. Now, only 2 litres are left to fill. It will be filled by pipe Q in 2/3 minutes. Therefore, the total time taken is 32 2/3 minutes.

32. Let total capacity of the tank is 48 litres. The two taps can fill the tank with efficiency of 3 litre/min and 1 litre/min whereas the leak can empty 2 litre/min.
The capacity of the tank below the leak is 3/4 × 48 = 36 litre.
Time taken to fill the 36 litre is 36/4 = 9 min.
Time taken to fill the remaining 12 litre = 12/2 = 6 min. Therefore, the total time taken is 9 + 6 = 15 minutes.

33. Let the time taken by X and Y to independently fill the tank be x minutes and y minutes respectively.
2x/3 + y/3 = 14 and 2y/3 + x/3 = 16
Solving these, x = 12 and y = 18
Part of the tank filled when X and Y work together in a minute = 1/12 + 1/18 = 5/36
Therefore, the time taken to fill the tank = 36/5 = 7.2 min.

34. Raman and Rajan can do the half work in 18 days, so, they can do the complete work in 36 days. Similarly, Rajan and Rajiv can do the total work in 24 days. Let the total work is 72 units.
Therefore, Raman and Rajan can do 2 units/day and Rajan and Rajiv can do 3 units/day. As, Rajiv alone can do the work in 36 days, so he can do 2 units/day.
Therefore, Rajan can do 1 unit/day and Raman can also do 1 unit/day. So, the time taken by Raman to complete the whole work alone is 72 days.

35. Rate of work and time taken to do a work are in inverse proportion. Hence, Kaushik is one and a half times more efficient than Ravi implies, Ravi takes one and a half time more time than Kaushik, to do the same work. Kaushik takes 20 days. Hence Ravi takes 20(1 + 1.5) = 50 days to do the same work. In 10 days, amount of work completed by both together = 10/20 + 10/50 = 7/10th of the work.

Exercise 15

1. The ratio of efficiency of Peter and Pan is 3: 2. Let Peter can do 3 units/day and Pan can do 2 units/day. Together they can do 5 units/day.
The total work = 36 × 5 = 180 units.
So, Pan alone will take 180/2 = 90 days to complete the work.

2. Let the work done by Pradeep, Abishek and Antony in one day be a, b and c respectively.
Then, a = 2/3 (b + c) ..... (i)
a + c = 2b ..... (ii)
Also, c = 1/45 ..... (iii)
From (i), 3a – 2b = 2c or 3a – 2b = 2/45
And from (ii), 2b – a = 1/45
Adding, we get 2a = 3/45 ⇒ a = 1/30 and b = 1/36.
So, Pradeep takes 30 days and Abishek takes 36 days.

3. Let the total work is 64 units.
Jadeja can do 8 units/day, Bhangar can do 4 units/day, Balaji can do 2 units/day and Dravid can do 1 unit/day.
Now Dravid will take 16 days to complete 1/4th i.e., 16 units of work.
Dravid and Balaji will complete next 16 units in 16/3 days.
Dravid, Balaji and Bhangar will do the next 16 units in 16/7 days and all four will complete the remaining 16 units in 16/15 days.
Therefore, the total time taken = 16 + 16/3 + 16/7 + 16/15 = 24 24/35 days.

4. Let the total work is 60 units. Machines P, Q and R can do 3 units/day, 2 units/day and 1 unit/day respectively.
Work done by all three machines together in 5 days = 5 × 6 = 30 units.
Work done by Q and R in next three days = 3 × 3 = 9 units.
Remaining work = 60 – 39 = 21 units and machine P will complete it in 7 days.
Total work done by machine P = 3 × 5 + 21 = 36 units.
Therefore, work done by P = 36/60 or 3/5th of the total work.

5. Let the works done by each worker in group A and each worker in a group B in a day be a units and b units respectively.
Given 3a + 6b = 1/20 ..... (i)
And 8a + 4b = 1/10 ..... (ii)
Solving (i) and (ii), we get, a = 1/90 and b = 1/300
So, a + b = 1/72.
Therefore, the work will be done in 72 days.

6. Let the total work is 72 units and Chakri and Bharat can do 3 units/day and 2 units/day respectively. The work done in first 5 days (including the holiday) is 3 + 2 + 3 + 2 = 10 units.
Therefore, work done in 35 days is 10 × 7 = 70 units.
Now, it is the 8th cycle and Bharat starts the work in every even numbered cycle.
The remaining work is 2 units and Bharat will take one day to complete it.
Therefore, total days = 36.

7. Let the total work is 60 units. Sonia can do 3 units per day and Priyanka and Anjali can do 2 units/day.
Work done in first 3 days = 3 + (3 + 2) + (3 + 2 + 2) = 15 units
Therefore, to complete 60 units of work, time required = 3 × 4 = 12 days.

8. The man would work at 1 unit/day. Men joining on the 2nd day, 3rd day and 4th day would work at 2 units /day, 3 units/day and 4 units/day respectively.
The total work = (1) + (1 + 2) + (1 + 2 + 3) + (1 + 2 + 3 + 4) = 20 units.
Required time = 20/1 = 20 min.

9. Prakash, Pranay and Pramod together complete $(1/20 + 1/30 + 1/60)^{th}$ or $1/10^{th}$ of the work per day. Together, the three take (x + 6) days to complete. So, x + 6 = 10,
⇒ x = 4
The work remaining after the departure of Prakash can be completed by Pranay and Pramod in (y + 6) days.
Therefore, (y + 6) (1/30 + 1/60) = 1 – x/10
⇒ (y + 6) × 3/60 = 1 – 4/10 ⇒ y = 6
Now, all three worked for x days, two people worked for y days and one person worked for z days (say).
⇒ x/10 + 3y/60 + z/60 = 1
⇒ 4/10 + 6/20 + z/60 = 1 ⇒ z = 18
Therefore, total days required = x + y + z = 4 + 6 + 18 = 28.

10. Let x be the number of men initially engaged.
Let d be the number of days for which they worked. Hence, x men completed half of the work in d days …… (1)
Because 2x men joined, total number of working men = x + 2x = 3x.
This new group completes the remaining work 6 days earlier; i.e., 3x men complete half of the work in (d – 6) days --------- (2)
As the product of men and number of days is the same in both cases, being equal to work done, so, x × d = 3x (d – 6);
⇒ d = 3 (d – 6) ⇒ 2d = 18 -------- (3)
Now, 2d is the duration required by x men to complete the work. Hence 18 days will be taken to complete the work.

11. The time from 10:00 a.m. to 6:00 p.m. = 8 hours.
So, one of the pipes worked for 8 hours and the other pipe worked for x hours.
We have, 8/12 + x/18 = 1 ⇒ x = 6. So, pipe Q was closed for 2 hours.
Or, x/12 + 8/18 = 1 ⇒ x = 20/3. So, pipe P was closed for 8 – 20/3 = 4/3 hours.
So, the minimum possible time is 4/3 hours.

12. Let the rate of filling of the first tap be x litres/hr
Let the rate of filling of the nth tap be denoted by t_n
So, $t_2 = x$, $t_3 = 2x$, $t_4 = 4x = 2^2x$, $t_5 = 8x = 2^3x$. Therefore, $t_n = 2^{n-2}x$.
Now, $t_6/t_9 = 2^4x/2^7x = 1/8$.
Therefore, time taken by the ninth tap = 1/8 × 80 = 10 minutes.

13. To fill $3/4^{th}$ part, the filling pipe takes 3 hours.
To empty $3/4^{th}$ part, the emptying pipe takes 2 hours.
When both are operated simultaneously, they will empty $3/4^{th}$ part of the full tank in 1/2 – 1/3 = 1/6 i.e., 6 hours.

14. Rate of filling of the pipe is 5 litre/ hr, so, it will take 40 hours to fill the tank. But due to the leak, the time taken is 120 hours.
So, actual rate of filling was 200/120 = 5/3 litre/hour
The rate of drain of leakage = 5 – 5/3 = 10/3 or 3.33 litre/hour.

15. Let the total work is 120 units.
A + B = 10 units/day ……. (i)
A + C = 6 units/day ……. (ii)
Also A + B + C = 15 units/day …… (iii)
Adding (i) and (ii), we get. 2A + B + C = 16 units/day …… (iv)
Subtracting (iii) from (iv), we get A = 1 unit/day
Therefore, A can complete the whole work in 120 days.

16. Let the total work is 108 units.
Atul can do 9 units/day, Yogesh can do 3 units/day and Riyaz can do 2 units/day.
Work done in 6 days = 6 × (9 + 3 + 2) = 84 units.

Riyaz worked alone for the last 8 days and he did 8 × 2 = 16 units of work.
The remaining 8 units work was done by Yogesh and Riyaz together and they took 8/5 days.
Therefore, the total time taken is 6 + 8/5 + 8 = 78/5 = 15 3/5 days.

17. Let the total work is 24 units. A can do 3 units/day, B can do 2 units/day and C can do 1 unit/day.
Work done of first three days = 5 + 3 + 4 = 12 units.
Therefore, to do 24 units, time required is 3 × 2 = 6 days.

18. Let the total work is 60 units and A can do 2 units/day and B can do 3 units/day.
In the last 5 days, only B worked and he did 15 units.
The remaining 45 units work is completed by A and B together and they will take 45/5 = 9 days. Hence, A and B worked together for 9 days.

19. Let the slower pipe takes x hours to fill the reservoir.
We have, $1/x + 1/(x - 5) = 1/6$
$\Rightarrow x^2 - 5x = 12x - 30$
$\Rightarrow x^2 - 17x + 30 = 0$
$\Rightarrow (x - 15)(x - 2) = 0 \Rightarrow x = 15$ hours.

20. As A starts puffing, the length of the cigarette that gets burnt in one cycle
= (2 × 3) + 3 + (3 × 3) + 3 = 6 + 3 + 9 + 3 = 21 mm.
84 mm will get burnt in four such cycles.
Time taken in one cycle = 2 + 3 + 3 + 3 = 11s.
So, total time = 4 × 11 = 44s.

21. Let total work be 60 units. 1st tap fills 3units per minute, 2nd tap fills 4 units per minute and waste pipe empties 5 units per minute. So, when opened together all three
will fill (3 + 4 – 5) = 2 units per min. Hence in first 9 min, 18 units of tank is filled.
Since the waste pipe is closed
∴The remaining 42 units will require 42/(3 + 4) = 6 minutes to fill.

22. The water supply does not start until the water level reaches the 5,000-gallon mark, which will take 10 hours. Once the level reaches that mark, there is an inflow at the rate of 300 gallons per hour and an outflow at the rate of 500 gallons per hour. So effective emptying rate will be 200 gallons per hour. At this rate, the tank will be emptied in another 25 hr. So the total time taken to empty the tank is 35 hour.

23. Total amount of work = 60 man-hours
From 11 am to 5 pm, 6 technicians = 36 man-hours
From 5 pm to 6 pm, 7 technicians = 7 man-hours
From 6 pm to 7 pm, 8 technicians = 8 man-hours
From 7 am to 8 pm, 9 technicians = 9 man-hours
Total = 60 man-hours

24. Chandra and Mayank complete 63% work in 7 days. So, they complete 9% work in a day. Now, since the ratio of their efficiency Chandra and Mayank is 5: 4, so one of them completes 5% work a day, and other one 4% a day. Obviously, the person doing 5% work a day is the fastest worker. Time taken by the fastest worker = 20 days.

25. Let the capacity of the tank be 180 units. (LCM of 10, 12, 15 and 18)
Amount of water drawn in one minute by Outlet Pipe A = 18 Units
Outlet Pipe B = 15 Units
Outlet Pipe C = 12 Units
Outlet Pipe D = 10 Units
Let the total time taken to empty the tank with the given conditions be 't' minutes.
Amount drawn by A = 1 × 18 = 18 units

Amount drawn by B = 2 × 15 = 30 units
Amount drawn by C = (t–5) × 12 = (12t – 60) units
Amount drawn by D = t × 10 = 10t units ⇒ 18 + 30 + 12t – 60 + 10t = 180
⇒ 22t = 192 ⇒ t = 96/11 = 8 8/11minutes.

26. Let the work done by a man, a woman and a child in a day be m, w and c respectively.
Then, m = 3c and w = 2c
∴ 20m + 30w + 36c = 60c + 60c + 36c = 156c
Total units of work done in second case = (15m + 21w + 30c) × 7 × 18
= (45c + 42c + 30c) × 7 × 18 = 117c × 7 × 18
Hence, the required total wages = $\frac{78}{156c} \times 117c \times 7 \times 18 = Rs\ 7371$.

27. If the ratio of diameters of 3 inlet pipes of second tank is 1: 2: 3, the ratio of their cross-sectional areas will be 1: 4: 9. Suppose cross-sectional area of inlet pipe of first tank be k. Then the cross-sectional areas of three pipes of second tank will be 4k, 16k and 36k respectively. Total cross-sectional area of pipes of second tank is 56k.
Time taken in filling the second tank = 10 × k/56k × 2 × 2 = 5/7 min.

QUANTIFIERS CAT ACADEMY

Exercise 16

1. Let speed (in m/s) of a scooter, a car and a train be y, 4y and 16y respectively. Let they all cover equal distance of 'd' meters. Time taken by each of the three is d/y, d/4y and d/16y respectively.
∴ Required ratio of time taken is given as: Scooter: Car: Train = d/y: d/4y: d/16y
1: 1/4: 1/16 or 16: 4: 1.

Alternatively: If the distance travelled by two or more objects is same, then the ratio of their speeds and times are in inverse proportion.

2. Let the usual speed of the man be 's' m/min and usual time be 't' minutes.
When speed decreases to 4s/5, time taken will be 5t/4. (Since speed is inversely proportional to time taken).
We have, 5t/4 – t = 10 ⇒ t = 40. Therefore, the usual time taken = 40 minutes.

3. Let the normal speed be x km/hr and normal time taken be 't' hours.
Total distance = xt km
$\Rightarrow \frac{xt}{\frac{3x}{4}} = t + \frac{45}{60} \Rightarrow$ t = 2.25 hour.

4. Let x be the distance between house and office and 't' be the normal time taken.
x/5 = t + 1/12 ..... (i)
And x/6 = t – 1/12 ..... (ii)
Subtracting (ii) from (i), we get, x (1/5 – 1/6) = 1/6
⇒ x/30 = 1/6 ⇒ x = 5 km.

5. Let the distance be x km.
So, x/15 – x/25 = 8 ⇒ x = 300 km.

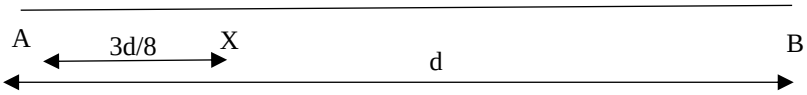
6. Let the distance covered by the man each time be 'd' km and time taken by him to cover 'd' km at a speed of 10 km/hr be 't' hours.
Then, d/t = 10 and d/(t – 2) = 15
Equating the value of d, we get, 10 × t = 15 × (t – 2) ⇒ t = 6 hours
Therefore, the required speed = d/(t – 1) = 10t/(t – 1) = 10 × 6/5 = 12 km/hr

7. Half of the journey = 464 km and 3/5th of the time = 6 hours.
Remaining distance = 464 km and Remaining time = 10 – 6 = 4 hours
Speed for the remaining journey = 464/4 = 116 km/hr

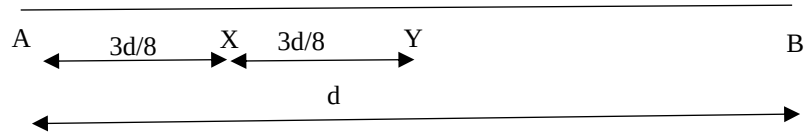
8. Since distance and time taken by the two men are same, their speed will be same. At 8 a.m., when the second man starts, the first man covers 1/2 of the distance in 2 hr, i.e., he can cover 1/4 distance in 1 hr. In the next hour, both cover 1/4 distance and hence meet at 9 a.m.

9. Let the total distance be x. So, the man travels a distance 3x/5 at a speed 3a. Therefore, total time taken to travel this distance = 3x/15a = x/5a.
He then travels a distance 2x/5 at a peed 2b. Hence, time taken to travel this distance = 2x/10b = x/5b.
Therefore, the total time taken in going from A to B = x/5a + x/ 5b
Now he travels from B to A and comes back. So total distance travelled = 2x at a constant speed 5c.
Hence, time taken to return = 2x/5c
Since the time taken in both the cases remains the same, we can write,
x/5a + x/ 5b = 2x/5c ⇒ 1/a + 1/b = 2/c

10. AB is the tunnel and 'd' km be its length.



Let the current position of the cat be X. If it runs towards A, it would reach A at the same time as the train reaches A. However, if it runs towards the other end B, it would
reach point Y at the same time when the train reaches A. Hence, point Y would be at a distance of 3d/8 km from X.



As the cat and the train would reach B simultaneously, the cat would cover the rest 2d/8 = d/4 km distance in the same time that the train takes to cover the whole tunnel i.e., d km. Therefore, the speed of the train = 4 × the speed of the cat.
Hence, ratio of the speeds of the train and cat is 4: 1.

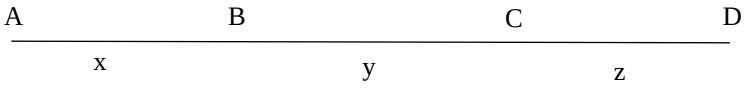
11. Total time taken by B to cover 60 km = 60/50 = 6/5 hr
It stops at station C for 1/4 hr. Now in 6/5 + 1/4 = 29/20 hours, train X travels 70 × 29/20 = 101.5 km.
This means they do not cross each other by the time train Y finishes its stop at station C. Let they meet after t h r.
Then, 70t + 50(t – 1/4) = 180 ⇒ t = 192.5/120 hour
Distance from A will be 70 × 192.5/120 = 112 km (approximately).

12. Overall, the train was late by 1 hr 30 min. Out of this the train was standing for 45 min. So due to the reduction of the speed to 2/3rd of the original speed, the train delayed by 45 min only.
⇒ 3t/2 – t = 45 ⇒ t/2 = 45 ⇒ t = 90 minutes, where 't' is the normal time taken by the
train to cover the stretch from the point of accident to the destination. Had the accident occurred 60 km down the line, the train would have been delayed by 1 hr only. Due to the reduction in speed, the delay would have been 15 min only (1 hr – 45 min stoppage)
∴ Normal time to cover the remaining distance ⇒ t = 2 × 15 = 30 min.
It means the train usually covers 60 km in 60 min (90 – 30 min).
Speed of the train = 60 km/hr.
Total length of the stretch of the journey = 30 km + 90 km = 120 km.

13. Let the usual speed of Ankit be 'x' km/hr.
We have, $\frac{30}{2x} + \frac{30}{\frac{x}{2}} = \frac{60}{x} + \frac{75}{60} \Rightarrow \frac{15}{x} + \frac{60}{x} = \frac{60}{x} + \frac{75}{60} \Rightarrow$ x = 12 km/hr

14. Let the speed of the cyclist be x km/min, and that of the car y km/min. The car had travelled for 10 min and the cyclist for 10 + 15 = 25 min when he was caught up by the car. By this moment, they had covered the same distance.
Consequently, 25x = 10y. By the time the car on its return trip encountered the cyclist. The car had covered 50y km and the cyclist 65x km. The sum of these distances is equal to twice the distance between A and B.
Therefore, 65x + 50y = 38. Solving the system of equations, we find x = 0.2 km/min and y = 0.5 km/min. Speed of the car = 0.5 × 60 km/hr = 30 km/hr.

15. Let the starting point for all the three, the point at Hari gets off from the car, the point at which Ram picks up Shyam and the final point of the journey be A, C, B and D respectively.



Let AB = x, BC = y and CD = z.
So, x/5 = (x + 2y)/25 ⇒ y = 2x ..... (i)
And z/5 = (z + 2y)/25 ⇒ y = 2z ..... (ii)
From (i) and (ii), we get, x: y: z = 1: 2: 1
Therefore, x = z = 25 km and y = 50 km.
Hence, the required time = (x + y + 2y + z)/25 = (25 + 50 + 100 + 25)/25 = 8 hours.

16. Distance covered = 52.8 km and time taken = 46 min = 46/60 hr
Average speed = 52.8 × 60/46 = 68.86 km/hr

17. Time taken to cover 180 km by car = 180/60 = 3 hour
Time taken to cover 240 km by train = 240/80 = 3 hours
Time taken to cover 300 km by bus = 300/50 = 6 hours
Total time taken to complete the whole journey = 3 + 3 + 6 = 12 hours
Average speed = (180 + 240 + 300)/12 = 720/12 = 60 km/hr

18. Let distance (in km) covered at a speed 4 km/hr be x.

We have, x/4 + (50 – x)/6 = 50/5
⇒ (3x + 100 – 2x)/12 = 10 ⇒ x + 100 = 120 ⇒ x = 20.
Hence, the distance covered at a speed of 4 km/hr is 20 km.

19. Due to stoppages, the bus covers 9 km less in 1 hr.
Time taken to cover 9 km = 9/54 × 60 = 10 min.
Hence, on average the bus stops for 10 min every hour.

20. Length of train = 900 m and its speed = 54 km/hr = 54 × 5/18 = 15 m/sec
Hence, the time it takes to cross the pole = 900/15 = 60 seconds.

21. Speed of the train = 500/25 = 20 m/sec = 20 × 18/5 = 72 km/hr

22. The distance to be travelled by the train to cross a platform of length 750 m is 750 + 650 i.e., 1400 m. At 72 km/hr. or 20 m/s. the train takes 1400/20 = 70 seconds to cross the platform.

23. Here the lengths of the trains = 250 m and 200 m. So, the total distance = 250 + 200 = 450 m.
The relative speed of faster train = 90 – 36 = 54 km/hr = 54 × 5/18 = 15 m/sec
Therefore, the required time = 450/15 = 30 sec.

24. Relative to the boy the speed of the train is 99 + 9 i.e., 108 km/hr
= 108 × 5/18 = 30 m/sec
Hence, the time it takes to cross him = 750/30 = 25 sec.

25. Let the usual time taken by the man to reach his office be 't'. The speed is 3/4th the normal speed. Hence, time is 4/3rd of the normal time.
We have, (4/3t) – t = 20 ⇒ t/3 = 20 ⇒ t = 60 minutes. Had the person walked at 4/3 of his usual speed, time taken by him = 3t/4 = 3/4 (60) = 45 minutes.

26. Let the usual time taken by the boy be 't'.
Distance between his home and school = 5(t – 4/60) = 4(t + 2/60) ..... (1)
Solving (1) we have t = 28/60 hr.

Time taken by the boy to go to school if he walks at 6 km/hr = 5(28/60 – 4/60)/6 = 1/3 hr = 20 min.

27. When M meets 'N' for the 2nd time, sum of the total distances travelled by M and N = 7 + (7 + 7) = 21 m. (M covered from S to T, 7 m; N covered from T to S, 7 m; and they together covered ST 7 m.)
The time of travel from the start to the time of second meeting is the same for both.
Hence, distance = time × (sum of the speeds).
Time taken = 21/(3 + 4) = 3 seconds
Total distance travelled by 'M' alone = 3 × 3 = 9 m.

28. Difference of the lengths of bridges = 550 – 430 = 120 m
Difference of the time taken to cross the bridge = 36 - 30 = 6 seconds
Speed of the train = 120/6 = 20 m/sec = (20 x 18)/5 = 72 kmph

29. Let the length of the train travelling at 60 kmph be L.
$\Rightarrow \frac{L}{(60+12)\times\left(\frac{5}{18}\right)} = 20 \Rightarrow$ L = 400 m
Time taken by the train to cross another cyclist travelling in the same direction as the train, at 20 kmph = $\frac{400}{(60-20)\times\left(\frac{5}{18}\right)} = 36\,sec$

30. If 't' is the time taken to meet, and t_1, t_2 are the durations taken after the meeting to reach the respective destinations then, $t = \sqrt{t_1 \times t_2}$. But it is given $t_1 = t_2$.
Hence, $t = \sqrt{t_1^2}$ or $t = t_1 = t_2$ -------- (1)
But it is given t = 5, so $t = t_1 = t_2 = 5$ hours. M travelled for 5 hours at x kmph and for another 5 hours at (x - 1) kmph. Therefore, 5x + 5(x - 1) = 50 ⇒ x = 5.5 kmph

31. Distance ran by the thief in 10 minutes = 1km
Distance moved by the jeep in 10 minutes = 1 × 5 km
Distance between the jeep and the thief when the jeep found a gap in the median = 2.5 km
Additional time in which the jeep would overtake the thief = 2.5/(9 – 6)
= 5/6 hours = 50 minutes. Therefore, the total time is 60 minutes.

32. Let the speeds of Anand and Bala be x kmph and y kmph respectively. Distance traveled by them when they met are 6x km and 6y km respectively.
6y/x – 6x/y = 5 ⇒ y/x – x/y = 5/6 = 3/2 – 2/3.
Comparing both sides, y/x = 3/2.
Total time = 6y/x + 6x/y = 6 × (y/x + x/y) = 6 × (3/2 + 2/3) = 13 hours.

33. Let Anwar met his wife at some point A. As he has saved 15 minutes, so he must have saved 7.5 minutes each in travelling from A to office and office to A. It means he reached A at 4: 52: 30 pm. It means his wife walked for 52.5 minutes from office to A. So, the distance which Anwar's wife has travelled in 52.5 minutes can be travelled by Anwar in 7.5 minutes.
Therefore, the ratio of times taken by Anwar and his wife is 52.5: 7.5 or 7: 1.
Ratio of speed of Anwar and his wife is 1: 7.
Hence, the walking speed of Anwar's wife is 30/7 km/hr.

34. If the car was stationary, the gunfire would have been heard at an interval of 24 seconds. But because it is moving towards the source of the sound, sound of the second gunfire was heard after an interval of 22 seconds only. The distance sound would have travelled in 2 seconds is the distance travelled by the car in 22 seconds.
So, 330 × 2 = V × 22 where V is the speed of the car. Therefore, V = 30 m/sec

35.



Let the car develop an engine problem at point C. So, it covers CB at 3/4th of its original speed. New time taken = 4/3rd of the original time taken to cover CB.
Given that (4/3 – 1) × t = 80 ⇒ t/3 = 80 ⇒ t = 240 minutes, where, t is the original time taken to cover CB.
When the problem develops 50 km further on, (i.e., at point D).



Let t_1 be the time taken to travel the distance DB, if the car had moved with normal speed.
So, $t_1/3 = 80 – 20 = 60 \Rightarrow t_1 = 180$ minutes..... (2) (As the car reached 20 minutes sooner).
From (1) and (2), Time taken to cover 50 km = (240 -180) = 60 min = 1 hr
Speed of the train = 50 kmph.
AC = 300 – (240/60) × 50 = 300 – 200 = 100 km.

36. Let the times taken for traveling to the town by train and by car be x hours and y hours respectively.
Given, x + y = 16 and 2y = 16 - 4 = 12 ⇒ y = 6 and x = 10. Therefore, 2x = 20
Hence, he would lose 4 hours.

37. Time taken by person = 16/8 = 2 hours = 120 min. As he is 9 minutes late, so actual time should be 111 minutes. Now, if he wants to reach 15 minutes early, he should take 96 minutes or 96/60 = 8/5 hours.
Therefore, the speed = 16/(8/5) = 10 kmph.

38. Let the total distance covered be 100d km.
We have, 50d/15 + 30d/18 + 20d/45 = 9.8 ⇒ d = 1.8.
Therefore, 100d = 180 km.

39. Time after which they meet = 84/(36 + 27) = 4/3 hours
The first person will cover 36 x 4/3 = 48 km in this time. So, they will meet 48 km from P.

40. Let the speeds of the faster car and the slower car be x kmph and y kmph respectively.
Given that, 80/(x + y) = 1 ⇒ x + y = 80 ..... (i)
And 80/(x – y) = 4 ⇒ x – y = 20 .....(ii)
Solving (1) and (2), x = 50. Therefore, the speed of the faster car is 50 kmph.

41. The cat runs away from its initial position for 1 minute at a speed of 24 kmph.
Distance covered by cat in one minute = 24 × 5/18 × 60 = 400 m
Now, the dog is separated from the cat by a distance of 100 + 400 = 500 m and this lead has to be covered at a speed of 33 – 24 = 9 kmph = 9 × 5/18 = 5/2 m/sec.
Therefore, the time taken = 500/(5/2) = 200 seconds.
But the time is required from the start of the cat's run, 1 minute has to be added.
i.e., 200 + 60 = 260 seconds.

42. When 'B' starts from city P towards city Q, the distance which 'A' would already have covered = 54 × 3 = 162 km.
At 9 a.m., train B is separated from train 'A' by a distance of 162 km.
Train B overtakes train A after a time of 162/(72 – 54) = 162/18 = 9 hours.
In 9 hours, the distance travelled by B = 72 × 9 = 648 km.
Distance from city Q, when they meet = (1440 - 648) = 792 km

43. By 10:00 a.m., A would have travelled 120 km. At 10:00 a.m. distance between A and B = 145 km. In another hour A and B would have travelled 60 km and 40 km repeating both cars would have together travelled 100 km. So, at 11:00 a.m.,

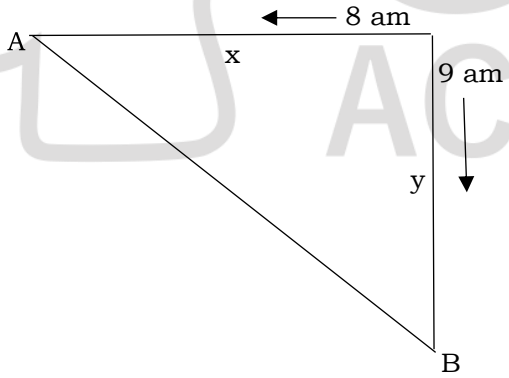
distance between A and B = 45 km. In the next half an hour, A would travel 30 km while B would be
stationary. By 11:30a.m, A would have travelled 210 km. Now it would stop for 20 minutes. At 11:30a.m, B would resume its journey and would be 15 km from S. It would reach S in another 22×5 = 110 minutes.
Therefore, the time of meeting = 11: 52: 30 a.m.

44. Anand and Ajay would meet for the first time during their onward journeys. They would meet for the second time during their return journeys. Time taken by Anand to travel from M to N = 8/3 seconds
Distance travelled by Ajay in this time = 8/3 × 5 = 40/3 m.
So, he would be 40/3 – 8 = 16/3 m from N when Anand reached N.
So, the distance between them would then be 8 – 16/3 = 8/3 m
Anand would cross Ajay in another (8/3)/8 = 1/3 seconds
Therefore, Ajay would have travelled a distance of 40/3 + 1/3 × 5 = 45/3 = 15 m when he crossed Anand for the second time.

45. Let the speeds of Anand and Ashok before the meeting be x kmph and y kmph respectively.
We have, 10/(x + y) = 1 ⇒ x + y = 10 ..... (i)
Distances travelled by Anand and Ashok before meeting are x km and y km respectively. As they reached their destinations simultaneously,
$y/(x-2) = x/(y+2) \Rightarrow y^2 + 2y = x^2 - 2x$
⇒ x – y = 2 ..... (ii)
Solving (1) and (2), x = 6.

46. Let his forward and return speeds be x kmph and y kmph respectively.
Average speed = 2xy/(x + y)
Given that, 2xy/(x + y) = (x + y)/2
$\Rightarrow (x+y)^2 - 4xy = 0 \Rightarrow (x-y)^2 = 0 \Rightarrow x = y$
So, 2xy/(x + y) = x
Hence, forward speed = Average speed = 1200/12 = 100 kmph.

47.



The distances covered by C and D are x and y respectively. Since C starts early compared to D, he covers more distance by noon compared to D. Hence x > y. If speed of each car is s.
x = 4s and y = 3s and $x^2 + y^2 = (150)^2$ (by Pythagoras theorem)
Taking x = 4s, y = 3s and solving for s we have, s = 30 kmph.

48. Let the distance it travelled without any problem be 'd' km.
Distance it travelled at the reduced speed = (600 - d) km.
Let its usual speed by 's' kmph.
Given, $\frac{d}{s} + \frac{600-d}{\frac{4s}{5}} = \frac{600}{s} + 1$ ..... (i)

And $\frac{d+150}{s} + \frac{450-d}{\frac{4s}{5}} = \frac{600}{s} + \frac{1}{2}$ ..... (ii)

Subtracting (i) from (ii), we get, $\frac{150}{s} - \frac{150 \times 5}{4s} = -\frac{1}{2}$

$\Rightarrow$ s = 75. Substituting it in (i), we get, d = 300.

49. Let the speed of his son be x kmph. As Ashwin returned home 20 minutes early, he saved a travel of 10 minutes to school from the meeting point and the return journey of 10 minutes. Distance he saved travelling each way = Distance travelled by his son by walk. His son would have walked for 50 minutes.
Therefore, x × 50/60 = 55 × 10/60 $\Rightarrow$ x = 11 kmph.

QUANTIFIERS CAT ACADEMY

Exercise 17

1. Here length of bridge = 10 km and the length of the train is 10/4 = 2.5 km.
Total distance travelled by train to clear bridge = 10 + 2.5 = 12.5 km
Time taken by train = 2.5 min
Speed = Distance/Time = 12.5/2.5 = 5 km/min = 5 × 60 km/hr = 300 km/hr

2. Relative speed of two boats = 5 + 10 = 15 km/hr
i.e., in 60 min they cover (together) = 15 km
∴ in 1 min they will cover together = 15/60 = 1/4 km.

3. Distance travelled by 1st train in 30 minutes = 80 × 1/2 = 40 km
So, they will meet after = (220 – 40)/(100 + 80) = 180/180 = 1 hour
Hence, the distance = 40 + 80 × 1 = 120 km.

4. We see that the trains take 18 hr and 8 hr respectively to complete their journeys.
Let the distance between the stations be LCM (18, 8), i.e., 72 km.
∴ Speed of the first train = 4 km/hr.
Speed of the second train = 9 km/hr.
By the time the train from A starts, train from B already would have travelled 9 km in 1 hr. Now the distance between the trains = 72 – 9 = 63 km. Relative speed of the trains = 9 + 4 = 13 km/hr.
∴ They will take 63/11 hours after 11 p.m. i.e., approximately at 3.51 a.m.

5. At 4.30 p.m. August Kranti Rajdhani Express crosses Baroda. On the particular day, Rajdhani Express, which is late by 10 min, would be 20 km away from August Kranti Rajdhani Express at 4.30 pm. Hence, they cross when they together travel 20 km.
Relative speed = 120 + 80 = 200 km/hr.
They cover 20 km in 6 min. Hence, they meet at 4.36 p.m.

6. We have,



Here, NS = 7000 m and SK = 10,000 m
Speed of the car of Mamta = 36 kmph = 10 m/s.
Case I: Buddha moves towards Nandigram and Biman moves towards Kolkata.
Case II: Biman moves towards Nandigram and Buddha moves towards Kolkata.
Time taken for Mamta and Buddha to meet from the instant Mamta starts moving = (7000 – 600 × 4)/14 = 2300/7 sec. (when the condition is as stated in Case I).
Time taken for Mamta and Buddha to meet from the instant Mamta starts moving = (7000 – 600 × 4)/6 = 4700/3 sec. (when the condition is as stated in Case II).
Time taken for Mamta and Biman to meet from the instant Mamta start moving = (7000 – 600 × 3)/7 = 8800/7 sec. (when the condition is as stated in Case I).
Time taken for Mamta and Biman to meet from the instant Mamta start moving = (7000 – 600 × 3)/13 = 400 sec. (when the condition is as stated in Case II).
The required time interval between the time when Mamta met Buddha and Mamta met Biman is 13 2/21 sec, therefore the condition is as stated in Case I.
Total time elapsed from the instant Biman started moving till the instant Mamta meets Biman = 600 + 8800/7 = 13000/7 sec.
Distance between Buddha and Biman when Mamta meets Biman = 13000× (4 + 3)/7 = 13000 m = 13 km.

7. We see that the car reduces its travel time by 30 min. So, it reduces its one-way travelling time by 30/2 = 15 minutes. Thus, it would have met the children 15 min earlier, i.e., 4.45 p.m. Now this means that the children walked from 4 p.m. to 4.45 p.m., i.e., 45 min.

8. Let the distance of Chennai from the point the car had over taken the bus, be 3x km. So, the car travels 3x km in 90 minutes. The car starts from Chennai at 9:00 a.m. and meets the bus once again in another 30 minutes. The car travelled only x km in 30 minutes. Obviously, the bus travelled only 2x km in 3 hours while the bus met the car again. So, the bus will travel 3x km in 4.5 hours. It will reach Chennai at 11:00 a.m.

9. The two cars meet 100/(60 – 40) = 5 hours later. During these 5 hours, the bird travels at 80 km/hr. So, the total distance travelled by the bird = 80 × 5 = 400 km.

10. For B, let the time taken to cover PQ, QT and TR be t_1, t_2 and t_3 respectively.
$t_1 : (t_2 + t_3) = 1 : 2$.
Also, since PQ = TR, therefore, $20 \times t_1 = 15 \times t_3 \Rightarrow t_3 = 4t_1/3$.
We know that QT = y = 10 km $\Rightarrow t_2 = 10/15 = 2/3$ hours.
Now, $\frac{t_1}{(t_2 + t_3)} = \frac{t_1}{\frac{4t_1}{3} + \frac{2}{3}} = \frac{1}{2}$
Hence, $2t_1 = 4t_1/3 + 2/3 \Rightarrow 2t_1/3 = 2/3 \Rightarrow t_1 = 1$ hour.
So, $t_1 = 1$ hour, $t_2 = 2/3$ hour, $t_3 = 4/3$ hour
$\Rightarrow t_2 + t_3 = 2$ hours.
Total distance PR = $t_1 \times 20 + (t_2 + t_3) \times 15 = 1 \times 20 + 2 \times 15 = 50$ km

11. At 7.30 a.m., Navjivan Express is at 50 km from A and at the same time, Howrah-Ahmedabad Express is at 20 km from B. Hence, distance between the trains at 7.30 a.m. is 30 km.
Relative speed = 50 + 40 = 90 kmph
Hence, time left = 30/90 = 1/3 hours = 20 minutes.

12. Let the speed of boat x km/hr. Then r = x – s $\Rightarrow$ x = r + s.
Time taken for return trip = $\frac{m}{s + x} = \frac{m}{r + 2s}$ *hour.*

13. Let 'b' and 'c' be the speed of boat in still water and current respectively.
Time taken in upstream = 2 × Time taken in downstream
$\Rightarrow \frac{AB}{b - c} = 2 \times \frac{AB}{b + c} \Rightarrow 2b - 2c = b + c \Rightarrow b = 3c$. The required ratio is 3: 1.

14. Let the distance between point Q and point P be 'x' m. Let the speed of A, B and the river be a, b and r respectively.
Therefore, x/(a + r) = 20/(b – r) …… (i)
Also, (x + 60)/(a + r) = 40/(b + r) …… (ii)
Dividing equation (i) by (ii) we get:
2x/(x + 60) = (b + r)/(b - r) …… (iii)
Given that b = 5r
Therefore, 2x/(x + 60) = 1.5 $\Rightarrow$ x = 180 m
Distance between points Q and R = 180 + 20 = 200 m.
Putting value of x = 180 in (i) we get, (a + r)/(b – r) = 9.
$\Rightarrow$ (a + r)/4r = 9 $\Rightarrow$ a = 35r = 7b. Therefore, Ratio of speed of A and B = 7: 1.

15. Let W be the width of the lake in meter and S_1, S_2 are speeds of the boats.
When, they meet for the first time, we get, $300/S_2 = (W - 300)/S_1$ …… (i)
Also, $(W - 300 + 200)/S_2 = (W - 200 + 300)/S_1$
$\Rightarrow (W - 100)S_1 = (W + 100)S_2$
$\Rightarrow 100 (S_1 + S_2) = W(S_1 - S_2)$ ……(ii)
From (i), $300 (S_1 + S_2) = W S_2$
Or, $S_1 + S_2 = WS_2/300$
In (ii), $W(S_1 - S_2) = 100S_2 W/300$
$\Rightarrow 300 S_1 - 300 S_2 = 100 S_2$
$\Rightarrow 300 S_1 = 400 S_2 \Rightarrow S_1/S_2 = 400/300 = 4/3$.

16. It is obvious that the cork is carried away by the flow of the water (i.e. it has the same speed as the water). So, if the swimmer is swimming away from the cork for half an hour (upstream), it will take him another half hour to swim back to the cork again (downstream, where he reaches the cork at the moment it is passing below the bridge). So, in that 1 hr time, the cork has moved 1 km towards to the bridge. The water in the canal flows at a speed of 1 km/hr.

17. Let x be rate of rowing by Rahul, and y be the rate of current in mph.
We have, $\frac{12}{x-y} - \frac{12}{x+y} = 6 \Rightarrow \frac{y}{x^2-y^2} = \frac{1}{4}$
$\Rightarrow y = \frac{x^2-y^2}{4}$ ..... (i)
When Rahul doubles his rowing rate, then we have,
$\frac{12}{2x-y} - \frac{12}{2x+y} = 1 \Rightarrow \frac{2y}{4x^2-y^2} = \frac{1}{12}$
$\Rightarrow y = \frac{4x^2-y^2}{24}$ ..... (ii)
Hence, from (i) and (ii), we have $2x^2 = 5y^2$
Putting $x^2 = 5y^2/2$ in (i), we get, $y = \frac{\frac{3y^2}{2}}{4} \Rightarrow y = 8/3$.

18. Let the speeds of Aku and Chika be y m/s and 3y m/s respectively and the speed at which the river flows be 'x' m/s. As per the question, the possible set of equations are:
(PQ – 30)/(y + x) = 30/(3y – x) ..... (i)
PQ/(y + x) = PQ/(3y – x) ..... (ii)
[If we assume flow of the river is from Q to P, then 3y + x = y – x, which is impossible.]
From (ii) 3y – x = y + x ⇒ y = x
Substituting in equation (i), we get, (PQ – 30)/2x = 30/2x ⇒ PQ = 60 meters.

19. Let AB = d km.
Let the speed of his boat in still water be x kmph. Let the speed of the river be y kmph.
x + y = d/6 ..... (i)
x – y = d/8 ..... (ii)
Solving (1) and (2), x = 7d/24 and y = d/24.
⇒ x: y = 7: 1.

20. First condition gives 4(b – s) = b + s ..... (I)
where, b = speed of the boat in still water (kmph) and s = speed of stream (in kmph)
Second condition gives 10/b = 2 ⇒ b = 5 ..... (II)
From (I), (II), we have 4(5 – s) = 5 + s ⇒ s = 3
⇒ Speed of the stream = 3 kmph

21. Let the speed of the stream be y, then the speed of the boat in still water will be y + 8
Upstream speed = y + 8 - y = 8 kmph. Hence, upstream time = (96/2)/8 = 6 hours
Time for downstream = 9 - 6 = 3 hours.

22. Let the seed of the boat in still water and the speed of the river be x kmph and y kmph respectively. Let the time taken for the downstream journey be 't' hours. Time taken for the upstream journey = (9 - t) hours.
We have, (x + y) t = 24
(x - y) (9 - t) = 24
In still water, y = 0 ⇒ x = 24/(9 – 5) = 24/4 = 6
(6 + y) t = 24 ..... (1)
(6 - y) (9 - t) = 24 ..... (2)
Solving (1) and (2), we get, y = 2.

23. Let the number of steps that the escalator moves for ten steps of Anil or Bala be 'e'.
Let the number of steps visible when the escalator is stationary be N.
$N = 60 + 6e = 140 - 14e \Rightarrow 20e = 80 \Rightarrow e = 4$
$N = 60 + 6e = 84$.

24. Let us say the number of steps that the escalator takes when A takes 120 steps is e.
In the time that A takes 120 steps, B takes 60 steps.
Let the number of steps on the escalator when it is stationary be N.
$N = 120 - e = 60 + e \Rightarrow e = 30 \Rightarrow N = 90$

25. Let the number of steps on either escalator be S
Let the speed of the person be x steps/sec and that of either escalator be y steps/sec.
$S = 90(x - y) = 18(x + y) \Rightarrow x = 3y/2$
So, $S = 45y$
Time taken to go up / down (in seconds) using either escalator, if it is switched off $= S/x = 45y \times 2/3y = 30$ sec.

26. Due to movement of the escalator, the speed has become 5/2 of original speed. Hence, time should become 2/5 of the original time, thereby saving 3/5 of original time.
Therefore, 3/5 of original time = 3 sec
$\Rightarrow$ Original time = 5 sec.
So, in 5 sec, Harish can cover $5 \times 2 = 10$ steps.
$\therefore$ There are 10 steps in the escalator.

27. Let the speed of the escalator be x steps/sec.
It is given that Rajat covers 11 steps in 35sec. So, in these 35 sec escalator will cover 35x steps.
Therefore, the total number of steps on the escalator must be $11 + 35x$. ...(i)
Similarly, Sharad takes 21 sec and covers 39 steps. So, in these 21sec escalator will cover 21x steps.
Therefore, total steps on the escalator must be $39 + 21x$. ...(ii)
Solving (i) and (ii) we get; $11 + 35x = 39 + 21x \Rightarrow x = 2$
Hence, total number of steps = 81.

28. In the time Samar takes 45 steps, Vivek takes 15 steps.
Therefore, for every 3 steps of Samar, Vivek takes 1 step.
Let the speed of Samar = 3 steps/sec and speed of Vivek is 1 step/sec
Let the speed of the escalator is x steps/ sec
Samar will take 15 seconds to reach the bottom and Vivek will take 40 seconds to reach the bottom.
We have, $40 + 40x = 45 + 15x \Rightarrow 25x = 5 \Rightarrow x = 1/5$
Total number of steps $= 40 + 40 \times 1/5 = 48$.

29. Shyama and Vyom are moving in the same direction. The speed of Shyama is more than Vyom and he is also taking more steps than Vyom to reach the top. It means that they are moving in the direction of the escalator i.e., the escalator is also moving upwards.
Let the speed of Shyama = 3 steps/sec and speed of Vyom is 2 steps/sec
Let the speed of the escalator is x steps/ sec
Shyama will take 25/3 seconds to reach the top and Vivek will take 10 seconds to reach the top.
We have, $25 + 25x/3 = 20 + 10x \Rightarrow 5x/3 = 5 \Rightarrow x = 3$
Total number of steps $= 25 + 25 \times 3/3 = 50$.

Exercise 18

1. Distance to be covered by A = (500 – 170) = 330 m.
In the time A covers 2 m, B covers 3 m. So, in the time A covers 330 m,
B covers 3/2 × 330 = 495 m
∴ When A reaches the winning post, B covers 495 m and therefore remains 5 m behind. Hence, A wins by 5 m.

2. Let the speeds of A, B and C be S_A, S_B and S_C.
S_A: S_B = 1000/960 = 25/24
S_B: S_C = 1000/975 = 40/39
S_A: S_c = 1000/936
Hence, A beats C by 64 m.

3. Let the speeds of P, Q and R be S_P, S_Q and S_R respectively.
∴ S_P: S_Q = 120: 100 = 6: 5 and S_Q: S_R = 120: 96 = 5: 4
⇒ S_P: S_Q: S_R = 6: 5: 4
Hence, the distance = 150 – 150 × 4/6 = 50 m.

4. Let the distance where they met be 'x' metres from the school.
So, the distance travelled by A = 200 m and that travelled by B = (2x + 200) m
Since the ratio of their speeds is 2: 3, the ratio of the distances travelled should also be 2: 3.
Hence, we have, 200/(200 + 2x) = 2/3
On solving, we get x = 50 m
The distance between their apartment and the school is given by (200 + x) = 200 + 50 = 250 m.

5. Suppose they normally meet at 9 a.m. at X. XY = 5 km. So, A takes 15 min to cover it and B takes 20 min to cover it. So, when A, B are at Y the time is 9.15 a.m. when B reaches X it would be 9.35 a.m. Hence, B is 35 min late on that day.

6. Situation (I):
In whatever time Karan covers a distance of 100 m, Arjun covers 90 m in the same time.
Situation (II):
Now Karan is 10 m behind the starting point. Once again to cover 100 m from this new point Karan will be taking the same time as before. In this time Arjun will be covering 90 meters only. This means that now both of them will be at the same point, which will be 10 meters away from the finish point. Since both of them are required to cover the same distance of 10 m now and Karan has a higher speed, he will beat Arjun by 1 metre.

7. Let L be length in metres of the race, which A finishes in t seconds.
Speed of A = L/t m/s
Speed of B = (L – 12)/t m/s
Speed of C = (L – 18)/t m/s
Time taken by B to finish the race = L/(L – 12)/t = Lt/(L – 12) sec
In this time, C covers (L – 8) m
$\frac{L-18}{t} \times \frac{Lt}{L-12} = L - 8 \Rightarrow$ L = 48 m

8. Let the speed of A and B be 50 m/sec and 40 m/sec, respectively.
Time taken by A to complete the race = 5000/50 = 100 sec
Time interval after which A crosses B every time = 400/(50 – 40) = 40 sec
Hence, in 100 seconds A will cross B two times.

9. Let length of the track = L

Therefore, required time = L/Relative speed = $\frac{L}{\frac{L}{16}-\frac{L}{28}} = \frac{28\times16}{28-16} = \frac{28\times16}{12} = \frac{112}{3}\ sec.$

10. Let x be the distance covered by A.
Time = Distance/Speed
⇒ x/15 = (14000 – x)/20
⇒ 35x = 14000 ×15 ⇒ x = 6000 m = 6 km.

11. A and B will meet at 5 – 3 = 2 points, irrespective of the radius of the circular track.

12. Required ratio = 5: 15 = 1: 3
(Since the time taken by both of them is same, distance covered will be directly proportional to their respective speeds.)

13. If they travel in same direction, they will meet at 10 – 3 = 7 distinct points.
If they travel in opposite direction, they will meet at 10 + 3 = 13 distinct points.
Hence, the ratio = 7: 13.

14. Charu and Chiku will meet for the first time after 3300/(25 + 15) = 82.5 seconds
So, they will meet for the third time after 82.5 × 3 sec = 247.5 seconds
By this time total distance covered by Charu is 25 × 247.5 m = 6187.5 m
Hence, they are a distance of 2887.5 m i.e., (6187.5 – 3300) m from point A.
But this is not the shortest distance. The shortest distance between the point A and the point at which they meet for the third time is (3300 – 2887.5) i.e., 412.5 m.

15. Chaman will be at point B for the fifth time after (30 + 4 × 60) = 270 seconds.
At this point Charu has covered 270 × 25 m = 6750 m i.e., he is at a distance of (6750 – 6600) m = 150 m from point A.
At this point Chiku has covered 270 × 15 m = 4050 m i.e., he is at a distance of (4050 – 3300) m = 750 m from point A.
Hence, the shortest distance between Charu and Chiku is 750 + 150 = 900 m.

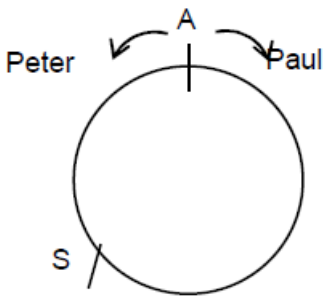
16. Ratio of speeds of the fastest and slowest runners is 2: 1. Hence they should meet at only one point on circumference i.e., the starting point (As difference in ratio in reduced form is 1). For the two of them to meet for first time, faster should have completed one complete round over slower one. Since two of them meet for the first time after 5 min, faster one should have completed 2 rounds (i.e., 2000 m) and slower one should have completed 1 round (i.e. 1000 m) in this time. Thus, faster one would complete the race (i.e., 4000 m) in 10 min.

17. Required time = 0.736 × 60/(2.25 + 3.5) = 0.736 × 60/5.75 = 7.68 minutes.

18. As options are independent of n
Let n = 2
Time taken for first round = 1/2 + 1 + 2 + 4 = 7.5 min
Time taken for second round = 8 + 16 + 32 + 64 = 120 min
Ratio = 120/7.5 = 16.

19. Akbar gives Birbal a start of 2 s and Birbal covers 10 m in these 2 s.
So, Birbal's speed is5 m/sec. He takes 18 seconds, more to cover the 90 m.
Akbar covered 100 m in 18 seconds. His speed = 100/18 m/sec = 5 5/9 m/sec.

20. Since Peter and Paul start at A and have the ratio of their speeds as 1: 2, first time when they meet say at S, the distances covered by them respectively were in the ratio 1: 2 i.e., Peter would have covered 200 m and Paul would have covered 400 m.



At the point S, the ratio of speeds will now become 2: 1. So, Peter will cover 400 m and reach A and Paul will cover 200 m and reach A. So, the 2nd meeting point will be at A, the 3rd meeting point will be at S and the 4th meeting point will be at A and so on.
So, by the time they meet for the 8th time i.e., at A, Peter would have covered 4 rounds.
Now, both are at A.
When Peter covers 1/4th round, (i.e., 150 m), Paul will cover 1/2 round, (i.e., 300 m)
Distance between them is either or 150 + 300 = 450 m or 600 – (150 + 300) = 150 m.

21. Let the speeds of the faster and the slower runners be x kmph and y kmph respectively. Time to meet for the first time is $3/(x - y)$ hours and this is given as 1 hour.
Hence $[3/(x - y)] = 1$ ..... (1)
Time taken, by the faster and the slower runners respectively, for one lap are $3/x$ and $3/y$ hours.
As the difference of the above times is given as 2 minutes, So, we have
$3/x = 3/y - 1/30$ ..... (2)
From (1), we get $y = x - 3$ which, when substituted in (2)
and solved for y, we get y = 15 kmph.

22. Since Pramod runs at half the speed of Prakash, they meet once for every round Pramod makes or for every 2 round Prakash makes.
Therefore, if they meet 11 times, Pramod makes 11 rounds and Prakash makes 22 rounds i.e., Prakash covers 11 times the circumference more than Pramod,
i.e., $11 \times 22/7 \times 7 \times 2$ m = 484 m

23. Let the speeds of P and Q be p km / hr and q km/hr respectively.
Same direction: Initial position of P was behind that of Q.
When P catches up with Q for the second time, the difference of the distance covered by them would be (XY + track length) i.e., 8 km.
So, $8/(p - q) = 40/60$ ..... (1) (when the two meet, p would have covered a greater distance, so, $p > q$)
Opposite direction: $(6 - 2)/(p + q) = 12/60$ ..... (2)
Solving (1) and (2) p = 16 and q = 4.

24. Here, L beats M by 40 seconds and M beats N by 40 seconds in the same race. Therefore, L beats N by 80 seconds and also by 125m. So, N can run 125 m in 80 sec.
To run 500m, N will take $80 \times 500/125 = 320$ seconds.
As M beats N by 40 seconds, so, M will take 320 – 40 = 280 seconds to run the race.

25. During the time Rahim covers 600 metres, Saleem covers 300 m, as Rahim's speed is double that of Saleem. During the time Rahim covers the balance 400 metres, Saleem covers 400 m, as their speeds are equal. Saleem had a head start of 200 m and he covers (300 + 400) = 700 m.

Hence, when Rahim is at finishing point, Saleem is at 200 + 700 = 900 m metres. Saleem takes 20 seconds to cover the balance 100 m. Saleem's speed = 5 m/s.

26. As Ajay gives Bala a start of atleast 20m, Bala has to run a maximum distance of 80m. As he is beaten by Bala by at most 20m, Ajay would have run a minimum of 80m when Bala finishes the race. As the speeds of both are distinct, Bala must have run less than Ajay when Ajay finished the race. Therefore, Ajay is faster than Bala. Hence, 4th option is not possible.

27. When Raja finishes the race, Rakesh would have run 200 - (20 + 20) = 160m
Therefore, ratio of the speeds of Raja and Rakesh = 200/160 = 5/4.
As Raja beats Rakesh by 20m and also beats him by 4 seconds, Rakesh must have taken 4 seconds to run the last 20 m.
So, speed of Rakesh = 20/4 = 5 m/sec
And speed of Raja = 5/4 × 5 = 6.25 m/sec.

28. Ram beats Shyam by one minute and Shyam beats Tarun by 30 seconds, so Ram beats Tarun by 90 seconds. Also, he beats Tarun by 250 meters.
So, Tarun runs 250 meter in 90 seconds.
He will run 1000 m in 360 seconds.
Hence, Ram will complete the race in 360 – 90 = 270 seconds.

29. If two or more runners have their speeds as a multiple of the speed of a runner N the time taken by these runners to meet for the first time at the starting point for the first time is the time taken by the N to complete one round. As the speeds of Harish and Suresh and Multiples of the speed of Girish, required time = Time taken by Girish to
complete 1 round = 1 minute.

30. As radius = 35 m, so, Circumference = 2 x 22/7 x 35 = 220 m
Time taken for them to meet for the first time at the starting point is LCM [220/20, 220/11] = LCM [11, 20] = 220 sec
To meet for the third time at the starting point from the start, they need 220 x 3 = 660 sec = 11 minutes.

31. As P's speed increases by the same amount as that by which Q's speed decreases, their relative speed remains unchanged.
Time taken by them to meet for the third time = 3 (Time taken by them to meet for the first time) = 3 × (48000/(10 + 30)) = 3600 sec.

32. Time taken by them to meet for the first time = 1800/(6 + 12) = 100 seconds
Distances covered by Ram and Shyam in this time are 600 m and 1200 m respectively. After the first meeting, their speeds would be exchanged. This would not affect time interval between consecutive meetings.
Therefore, in the next 100 seconds, Ram would have travelled 1200 m and would have reached his starting point and would meet Shyam there. In this manner Ram completed a round. In this manner he would have completed 3 rounds. When he would have completed another 1/4 rounds, Shyam would have completed 12/6 × 1/4 = 1/2 a round
Hence, the distance between them = 1800 × (1 – 1/2 – 1/4) = 450 m

33. Ratio of the speeds of Rohan and Sohan = 1000: 1000 - 200 = 5: 4
Let the speeds of Rohan and Sohan be 5x m/ minute and 4x m/ minute respectively.
1000/(5x + 4x) = 111 1/9 ⇒ x = 1
Required time = 1000/(5x – 4x) = 1000 seconds.